MICROSCOPIC DESCRIPTION OF PYGMY DIPOLE RESONANCE IN NEUTRON-RICH NUCLEI

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  − Realization of QRPA
  − Phonon-phonon coupling

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  − Giant and Pygmy dipole resonances for the Ca isotopes

Conclusions
Introduction

$E1$ strength in (spherical) atomic nuclei

![Diagram of nuclear energy levels with Two-phonon state, Pygmy Dipole Resonance, and Giant Dipole Resonance.](image)

Courtesy: N. Pietralla
Relevance of the PDR

1. The PDR might play an important role in nuclear astrophysics. For example, the occurrence of the PDR could have a pronounced effect on neutron-capture rates in the r-process nucleosynthesis, and consequently on the calculated elemental abundance distribution.


2. The study of the pygmy $E1$ strength is expected to provide information on the symmetry energy term of the nuclear equation of state. This information is very relevant for the modeling of neutron stars.


3. New type of nuclear excitation: these resonances are the low-energy tail of the GDR, or if they represent a different type of excitation, or if they are generated by single-particle excitations related to the specific shell structure of nuclei with neutron excess.

MAIN INGREDIENTS OF THE MODEL
Realization of QRPA

We employ the effective Skyrme interaction with the tensor terms in the particle-hole channel

\[
V(\vec{r}_1, \vec{r}_2)^C = t_0 \left(1 + x_0 \hat{P}_\sigma\right) \delta(\vec{r}_1 - \vec{r}_2) + \frac{t_1}{2} \left(1 + x_1 \hat{P}_\sigma\right) \left[\delta(\vec{r}_1 - \vec{r}_2)\vec{k}^2 + \vec{k}'^2 \delta(\vec{r}_1 - \vec{r}_2)\right] \\
+ t_2 \left(1 + x_2 \hat{P}_\sigma\right) \vec{k}' \cdot \delta(\vec{r}_1 - \vec{r}_2)\vec{k} + \frac{t_3}{6} \left(1 + x_3 \hat{P}_\sigma\right) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha \left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right) \\
+ i W_0 (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[\vec{k}' \times \delta(\vec{r}_1 - \vec{r}_2)\right]
\]

and

\[
V(\vec{r}_1, \vec{r}_2)^T = \frac{T}{2} \left\{[(\sigma_1 \cdot \vec{k}') (\sigma_2 \cdot \vec{k}') - \frac{1}{3} (\sigma_1 \cdot \sigma_2) \vec{k}^{'2}] \delta(\vec{r}_1 - \vec{r}_2) \\
+ \delta(\vec{r}_1 - \vec{r}_2) [(\sigma_1 \cdot \vec{k}) (\sigma_2 \cdot \vec{k}) - \frac{1}{3} (\sigma_1 \cdot \sigma_2) \vec{k}^2]\right\} \\
+ U \left\{(\sigma_1 \cdot \vec{k}') \delta(\vec{r}_1 - \vec{r}_2) (\sigma_1 \cdot \vec{k}) - \frac{1}{3} (\sigma_1 \cdot \sigma_2) [\vec{k}' \delta(\vec{r}_1 - \vec{r}_2) \vec{k}]\right\}
\]


Realization of QRPA

The Hamiltonian includes the surface peaked density-dependent zero-range force in the particle-particle channel.

\[ V_{\text{pair}}(\vec{r}_1, \vec{r}_2) = V_0 \left( 1 - \frac{\rho(r_1)}{\rho_c} \right) \delta(\vec{r}_1 - \vec{r}_2), \]

where \( \rho(r_1) \) is the particle density in coordinate space, \( \rho_c \) is equal to the nuclear saturation density. The strength \( V_0 \) is a parameter fixed to reproduce the odd-even mass difference of nuclei in the studied region.


The starting point of the method is the HF-BCS calculations of the ground state, where spherical symmetry is assumed for the ground states. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis.

Realization of QRPA

The residual interaction in the particle-hole channel $V_{\text{res}}^{\text{ph}}$ and in the particle-particle channel $V_{\text{res}}^{\text{pp}}$ can be obtained as the second derivative of the energy density functional $\mathcal{H}$ with respect to the particle density $\rho$ and the pair density $\tilde{\rho}$, respectively.

$$V_{\text{res}}^{\text{ph}} \sim \frac{\delta^2 \mathcal{H}}{\delta \rho_1 \delta \rho_2} \quad V_{\text{res}}^{\text{pp}} \sim \frac{\delta^2 \mathcal{H}}{\delta \tilde{\rho}_1 \delta \tilde{\rho}_2}.$$  

*G. T. Bertsch and S. F. Tsai, Phys. Rep. 18, 125 (1975).*

We simplify $V_{\text{res}}$ by approximating it by its Landau-Migdal form

$$V_{\text{res}}(\vec{k}_1, \vec{k}_2) = N_0^{-1} \sum_{l=0} F_l + G_l \sigma_1 \cdot \sigma_2 + (F'_l + G'_l \sigma_1 \cdot \sigma_2) \tau_1 \cdot \tau_2 \right] P_l \left( \frac{k_1, k_2}{k_F^2} \right),$$

where $\tau_i$ is the isospin operator, and $N_0 = 2k_F m^* / \pi^2 \hbar^2$ with $k_F$ and $m^*$ standing for the Fermi momentum and nucleon effective mass.

Realization of QRPA

Moreover we keep only Landau parameters $F_0$ and $F'_0$. Thus, we can write the residual interaction in the following form:

$$V_{\text{res}}^{(a)}(\vec{r}_1, \vec{r}_2) = N_0^{-1} \left[ F_0^{(a)}(r_1) + F'_0^{(a)}(r_1)(\tau_1 \cdot \tau_2) \right] \delta(\vec{r}_1 - \vec{r}_2),$$

where $a = \{\text{ph}, \text{pp}\}$ is the channel index.

The expressions for $F_0$ and $F'_0$ in terms of the Skyrme force parameters can write in the following form:

$$F_0^{\text{ph}} = N_0 \left\{ \frac{3}{4} t_0 + \frac{1}{16} t_3 \rho^\alpha (\alpha + 1)(\alpha + 2) + \frac{1}{8} k_F^2 [3 t_1 + (5 + 4 x_2) t_2] \right\},$$

$$F'_0^{\text{ph}} = - N_0 \left\{ \frac{1}{4} t_0 (1 + 2 x_0) + \frac{1}{24} t_3 \rho^\alpha (1 + 2 x_3) + \frac{1}{8} k_F^2 [t_1 (1 + 2 x_1) - t_2 (1 + 2 x_2)] \right\},$$

$$F_0^{\text{pp}}(r) = \frac{1}{4} N_0 V_0 \left( 1 - \frac{\rho(r)}{\rho_c} \right),$$

$$F'_0^{\text{pp}}(r) = F_0^{\text{pp}}(r).$$


Realization of QRPA

We introduce the phonon creation operators

\[ Q^+_{\lambda\mu i} = \frac{1}{2} \sum_{jj'} \left( X_{jj'}^\lambda A^+(jj'; \lambda \mu) - (-1)^{\lambda - \mu} Y_{jj'}^\lambda A(jj'; \lambda - \mu) \right), \]

\[ A^+(jj'; \lambda \mu) = \sum_{mm'} C_{jmjm'}^{\lambda\mu} \alpha_{jm}^+ \alpha_{jm'}^+. \]

The index \( \lambda \) denotes total angular momentum and \( \mu \) is its \( z \)-projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum \( |0\rangle \) and one-phonon excited states are \( Q^+_{\lambda\mu i}|0\rangle \) with the normalization condition

\[ \langle 0|[Q_{\lambda\mu i}, Q^+_{\lambda\mu i'}]|0\rangle = \delta_{ii'}. \]

Making use of the linearized equation-of-motion approach one can get the QRPA equations

\[ \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}. \]

Solutions of this set of linear equations yield the one-phonon energies \( \omega \) and the amplitudes \( X, Y \) of the excited states.

\[ P. \text{Ring and P. Schuck}, \text{The Nuclear Many Body Problem} \ (\text{Springer, Berlin 1980}). \]
Phonon-phonon coupling (PPC)

To take into account the effects of the phonon-phonon coupling (PPC) in the simplest case one can write the wave functions of excited states as

\[ \Psi_\nu (JM) = \left[ \sum_i R_i (J\nu) Q_{JM i}^+ + \sum_{\lambda_1 i_1 \lambda_2 i_2} P_{\lambda_2 i_2}^{\lambda_1 i_1} (J\nu) \left[ Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+ \right] JM \right] |0\rangle \]

with the normalization condition

\[ \sum_i R_i^2 (J\nu) + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} \left[ P_{\lambda_2 i_2}^{\lambda_1 i_1} (J\nu) \right]^2 = 1. \]

Phonon-phonon coupling (PPC)

Using the variational principle in the form

\[ \delta \left( \langle \Psi_\nu(JM) | \mathcal{H} | \Psi_\nu(JM) \rangle - E_\nu [\langle \Psi_\nu(JM) | \Psi_\nu(JM) \rangle - 1] \right) = 0, \]

one obtains a set of linear equations for the unknown amplitudes \( R_i(J\nu) \) and \( P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu) \):

\[ (\omega_{JI} - E_\nu) R_i(J\nu) + \sum_{\lambda_1i_1 \lambda_2i_2} U_{\lambda_2i_2}^{\lambda_1i_1}(JI) P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu) = 0; \]

\[ \sum_i U_{\lambda_2i_2}^{\lambda_1i_1}(JI) R_i(J\nu) + 2(\omega_{\lambda_1i_1} + \omega_{\lambda_2i_2} - E_\nu) P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu) = 0. \]

\( U_{\lambda_2i_2}^{\lambda_1i_1}(JI) \) is the matrix element coupling one- and two-phonon configurations:

\[ U_{\lambda_2i_2}^{\lambda_1i_1}(JI) = \langle 0 | Q_{JI} \mathcal{H} [Q_{\lambda_1i_1}^+ Q_{\lambda_2i_2}^+ ]_J | 0 \rangle. \]

These equations have the same form as the QPM equations, but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme forces.


Phonon-phonon coupling (PPC)

Distribution of coupling matrix elements $U_{\lambda_1 i_1}^{\lambda_2 i_2}(J_i)$ between the one- and two-phonon configurations in the PPC calculation of the GDR strength function for $^{132}$Sn

RESULTS AND DISCUSSION
Details of calculations

We use the Skyrme interactions SLy5 and SLy5+T. The SLy5+T involve the tensor terms added without refitting the parameters of the central interaction (the tensor interaction parameters are $\alpha_T = -170$ MeV·fm$^5$ and $\beta_T = 100$ MeV·fm$^5$). The pairing strength $V_0 = -270$ MeV·fm$^3$ is fitted to reproduce the experimental neutron pairing energies near $^{48}$Ca.


The photonuclear cross section for $^{48}$Ca  

**SLy5 vs SLy5+T**

The general shape of the GDR obtained in the PPC are rather close to those observed in experiments.

<table>
<thead>
<tr>
<th></th>
<th>$\bar{E}$</th>
<th>$\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLy5:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RPA</td>
<td>19.3</td>
<td>6.9</td>
</tr>
<tr>
<td>PPC</td>
<td>19.0</td>
<td>7.3</td>
</tr>
<tr>
<td>SLy5+T:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RPA</td>
<td>19.4</td>
<td>6.3</td>
</tr>
<tr>
<td>PPC</td>
<td>19.1</td>
<td>6.7</td>
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<tr>
<td>Expt.:</td>
<td>19.5</td>
<td>7.0</td>
</tr>
</tbody>
</table>

*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C95, 054312 (2017).*

Low-energy $1^-$ distributions of $^{48}$Ca

The dominant contribution in the wave function of the $1^-$ states comes from the two-phonon configurations ($>60\%$). These states originate from the fragmentation of the RPA states above 10 MeV.

<table>
<thead>
<tr>
<th></th>
<th>$\sum B(E1;\uparrow)$</th>
<th>$\sum EB(E1;\uparrow)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLy5:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RPA</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>PPC</td>
<td>0.06</td>
<td>0.50</td>
</tr>
<tr>
<td>Kamerdzhiev:</td>
<td>0.071</td>
<td>0.509</td>
</tr>
<tr>
<td>Egorova:</td>
<td>0.10</td>
<td>0.95</td>
</tr>
<tr>
<td>Expt.:</td>
<td>0.0687(75)</td>
<td>0.570(62)</td>
</tr>
</tbody>
</table>


The PDR fractions $f_{\text{PDR}}$ and $\sum B(E1)$ values

\[
f_{\text{PDR}} = \frac{\sum_{k} E_{1k} B(E1; 0^{+}_{gs} \rightarrow 1^{+}_{k}) \leq 10 \text{ MeV}}{14.8NZ/A e^{2}\text{fm}^{2}\text{MeV}}
\]

The strong increase of the summed $E1$ strength below 10 MeV [$\sum B(E1)$], with increasing neutron number from $^{48}\text{Ca}$ till $^{58}\text{Ca}$.


$^{48}\text{Ca}$ vs $^{50}\text{Ca}$

SLy5

\begin{align*}
\Sigma B(E1)(10^{-3} \text{e}^2\text{fm}^2) \quad &\text{vs} \quad E (\text{MeV}) \\
B(E1; 1^{-\text{gs}} \rightarrow 0^{-}) (\text{e}^2\text{fm}^2) \quad &\text{vs} \quad E_{2\text{qp}} (\text{MeV})
\end{align*}

Conclusions

Starting from the Skyrme mean-field calculations, the properties of the electric dipole strength in neutron-rich Ca isotopes are studied by taking into account the coupling between one- and two-phonons terms in the wave functions of excited states.

We have found that the strong increase of the summed $E1$ strength below 10 MeV [$\sum B(E1)$], with increasing neutron number from $^{48}$Ca till $^{58}$Ca. The dipole response for $^{52−58}$Ca is characterized by the fragmentation of the strength distribution and its spreading into the low-energy region.

In the general case, an investigation of the PDR requires to take into account complex configurations. Here the wave functions are considered up to two-phonon components. The model can be extended by enlarging the variational space for the $1^-$ states with the inclusion of the three-phonon configurations.
Acknowledgments

Many thanks for collaboration:

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## Properties of $2^+_1$, $3^-_1$, $4^+_1$ and $5^-_1$ phonons

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{1}^{\pi}$</th>
<th>Energy, MeV</th>
<th>$B(E\lambda; 0^+<em>gs \rightarrow \lambda</em>{1}^{\pi})$, $e^2b^\lambda$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Expt.</td>
<td>Theory</td>
<td>Expt.</td>
</tr>
<tr>
<td>$^{46}$Ca</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^+_1$</td>
<td>1.346</td>
<td>2.05</td>
<td>0.0127±0.0023</td>
</tr>
<tr>
<td>$3^-_1$</td>
<td>3.614</td>
<td>4.57</td>
<td>0.006±0.003</td>
</tr>
<tr>
<td>$4^+_1$</td>
<td>2.575</td>
<td>2.30</td>
<td></td>
</tr>
<tr>
<td>$5^-_1$</td>
<td>4.184</td>
<td>4.67</td>
<td></td>
</tr>
<tr>
<td>$^{48}$Ca</td>
<td></td>
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<tr>
<td>$2^+_1$</td>
<td>3.832</td>
<td>3.19</td>
<td>0.00968±0.00105</td>
</tr>
<tr>
<td>$3^-_1$</td>
<td>4.507</td>
<td>4.47</td>
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<tr>
<td>$4^+_1$</td>
<td>4.503</td>
<td>3.51</td>
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</tr>
<tr>
<td>$5^-_1$</td>
<td>5.729</td>
<td>4.52</td>
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</tr>
<tr>
<td>$^{50}$Ca</td>
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</tr>
<tr>
<td>$2^+_1$</td>
<td>1.027</td>
<td>1.50</td>
<td>0.00375±0.00010</td>
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<tr>
<td>$3^-_1$</td>
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<td>4.36</td>
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<tr>
<td>$4^+_1$</td>
<td>4.515</td>
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<tr>
<td>$5^-_1$</td>
<td>5.110</td>
<td>4.45</td>
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*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C95, 054312 (2017).*

[http://www.nndc.bnl.gov/ensdf/]
Properties of $2^+_1$ states in the Ca isotopes

- The $[2^+_1]_{QRPA}$ phonons of the even-even $^{46-58}$Ca isotopes exhibit pure neutron two-quasiparticle excitations ($> 72\%$).

- The crucial contribution in the wave function structure of the first $2^+$ state comes from the $[2^+_1]_{QRPA}$ ($> 89\%$) configuration.
