Topological geons with self-gravitating phantom scalar field

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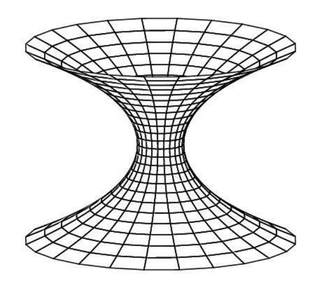
Topological geons

A topological geon is the quotient manifold $G = M/\mathbb{Z}_2$ where M is a static spherically symmetric wormhole having the reflectional symmetry with respect to its throat.

The $\mathbb{Z}_2$ action on M is defined in the spherical coordinates by $(t, r, \theta, \varphi) \to (t, -r, \pi - \theta, \varphi + \pi)$.

In what follows, we assume that the topological geon's spacetime is asymptotically flat.



References

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Action

$$\Sigma = \frac{1}{8\pi} \int \left(-\frac{1}{2} S + \varepsilon \langle d\phi, d\phi \rangle - 2V(\phi) \right) \sqrt{|g|}\, d^4 x \,,$$

where S — scalar curvature, $\varepsilon = \pm 1$, $V(\phi)$ is a self-interaction potential, and the angle brackets denote the scalar product with respect to the metric.

For topological geons $\varepsilon = -1$.

Energy condition $T_{ij} U^i U^j \geqslant 0$ does not hold for wormholes (see Morris M.S., Torn K.S. *Wormholes in space-time and their use for interstellar travels*// Am. J. Phys., 1988, V.56, p. 395-412)

Metric and the quadratures

$$ds^2 = A^2 dt^2 - \frac{dr^2}{A^2} - C^2\bigl(d\theta^2 + \sin^2\theta\, d\varphi^2\bigr) \tag{1}$$

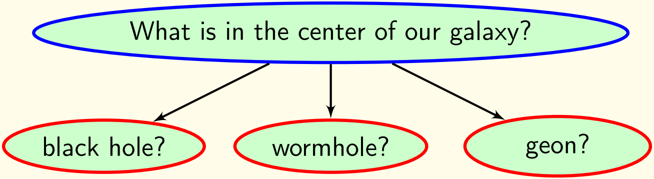
$$\phi' = \sqrt{-\,\varepsilon C''/C}\,, \quad A^2 = 2C^2\int\limits_r^\infty \frac{r-3m}{C^4}\,dr \tag{2}$$

$$\widetilde{V}(r) = \frac{1}{2C^2}\left(1 - 3C'^2 A^2 - CC'' A^2 + 2C'\,\frac{r-3m}{C}\right) \tag{3}$$

For topological geons we assume $\varepsilon = -1$, $m = 0$, C — even function, $C'' \geqslant 0$, $C = r + o(1)$, $r \to \infty$.

Physical interpretation

Topicality of current research is explained by one of the tasks of the project RadioAstron.



How can we distinguish a black hole from a symmetric wormhole or a geon?

	black hole	geon
Distance orbits	Yes	No $\left(A^2 \sim 1 - \frac{a}{r^4}, \quad r \to \infty\right)$
Angular velocity in ISCO[1]	$\omega \neq 0$	$\omega = 0$

[1] Innermost Stable Circular Orbit

Classification of topological geons

The motion of test particles are defined by the equation

$$\left(\frac{dr}{ds}\right)^2 = E^2 - V_{\mathsf{eff}}, \tag{4}$$

where $V_{\mathsf{eff}} = A^2\left(k + \frac{J^2}{C^2}\right)$, $k = 0, 1, -1$, E and J are constants (energy and angular momentum).

Classification

1. A^2 has a maximum at $r = 0$ and has no minima;
2. A^2 has a unique minimum at $r = 0$;
3. A^2 has a minimum at $r \neq 0$.

Instances

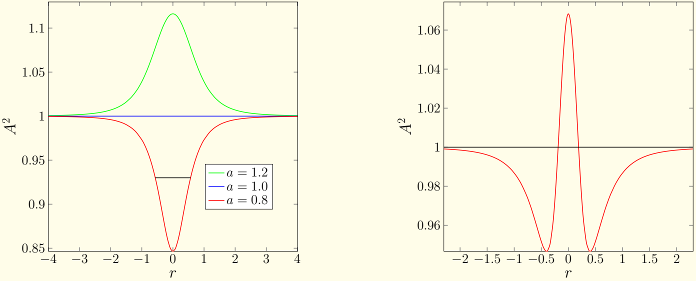


Figure: *Left*: Graphics of A^2 for $C = \left(r^4 + 2r^2 + a^4\right)^{1/4}$. Geon with $a = 1.2$ corresponds to the case 1 and has no any stable orbit. Geon with $a = 0.8$ corresponds to the case 2 and the motion of test particles is possible in the some neighborhood of topological threshold. $a = 1$ define Ellis solution $C = \sqrt{r^2 + a^2}$, $A^2 = 1$, $\phi = \arctan \dfrac{r}{a}$ with no one stable orbit. *Right*: Graphic of A^2 for $C = \left(r^6 + c^2 r^4 + b^4 r^2 + a^6\right)^{1/4}$ with $a = b = 0.5$, $c = 1$. This geon corresponds to the case 3.

Finite motion for the case 3 (I)

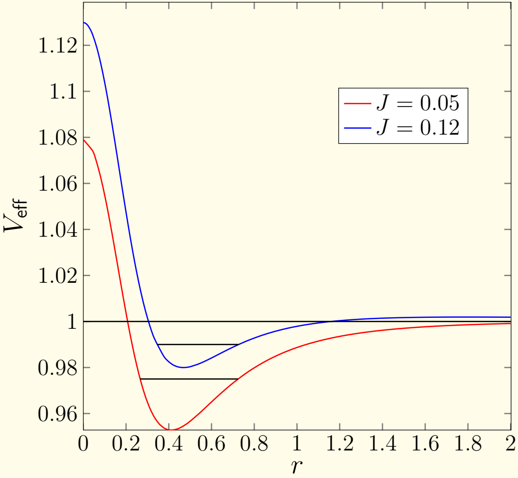


Figure: Graphics of V_{eff} for $C = \left(r^6 + c^2 r^4 + b^4 r^2 + a^6\right)^{1/4}$ with $a = b = 0.5$, $c = 1$.

Finite motion for the case 3 (II)

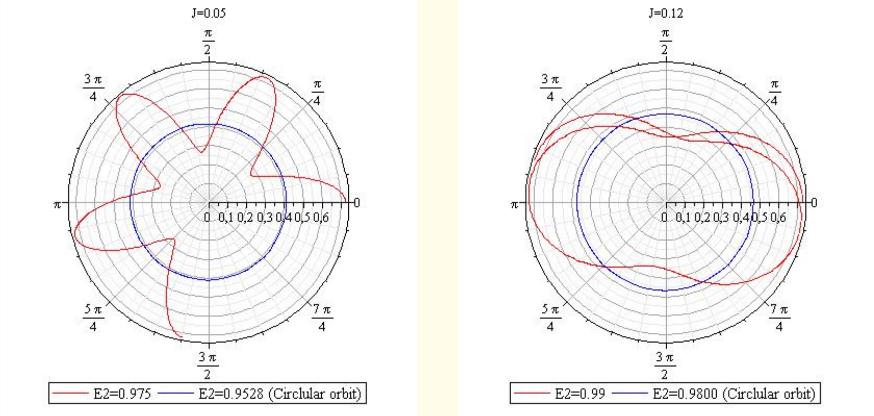


Figure: Orbits for geon with V_{eff} for $C = \left(r^6 + c^2 r^4 + b^4 r^2 + a^6\right)^{1/4}$ with $a = b = 0.5$, $c = 1$.