High temperature limit of the Standard Model due to gauge groups contraction

N. A. Gromov

Institute of Physics and Mathematics, Komi Science Center RAS,
E-mail: gromov@dm.komisc.ru

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The aim of my talk is to construct high temperature limit of the Standard Model and on this base describe evolution of the particles in the early Universe.

The Standard Model

- The modern knowledge of the particle world is concentrated in Standard Model (SM). This theory consist of two parts: Electroweak Model (EWM), which unified electromagnetic and weak interactions, and Quantum Chromodynamics (QCD), describing their strong interactions.
- The Electroweak Model is a gauge theory with the gauge group $SU(2) \times U(1)$, which act in the boson, lepton and quark sectors.
- QCD is $SU(3)$ gauge theory based on the local color degrees of freedom of quarks.

Elementary particles content of SM

Gauge bosons:

$$\gamma \text{ (photon)},$$

$$W^{\pm} \text{ (charged weak bosons)}, \quad Z^0 \text{ (neutral weak boson)},$$

$$A^k,\ k = 1, \ldots, 8 \text{ (gluons)}.$$

Special particle:

$$\chi \text{ (Higgs boson)}.$$

Leptons:

$$\left(\begin{array}{c} \nu_e \\ e \end{array}\right), \quad \left(\begin{array}{c} \nu_\mu \\ \mu \end{array}\right), \quad \left(\begin{array}{c} \nu_\tau \\ \tau \end{array}\right) \in \mathbb{C}_2.$$

Quarks:

$$\left(\begin{array}{c} u \\ d \end{array}\right), \quad \left(\begin{array}{c} c \\ s \end{array}\right), \quad \left(\begin{array}{c} t \\ b \end{array}\right) \in \mathbb{C}_2.$$

Electroweak Model

The Lagrangian of the model is given by the sum of the boson L_B, of the lepton L_L and of the quark L_Q Lagrangians:

$$L = L_B + L_L + L_Q.$$

It is invariant under the action of the gauge group $SU(2) \times U(1)$ in the 2-dim. complex space $\mathbb{C}_2$:

$$SU(2): \ \vec{z}\,' = G\vec{z},$$

$$\left(\begin{array}{c} z_1' \\ z_2' \end{array}\right) = \left(\begin{array}{cc} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{array}\right) \left(\begin{array}{c} z_1 \\ z_2 \end{array}\right), \quad |\alpha|^2 + |\beta|^2 = 1,$$

$$U(1): \ \vec{z}\,' = e^{i\omega}\vec{z}, \ \omega \in \mathbf{R}.$$

Boson sector $L_B = L_A + L_\phi$ involve two parts:
the gauge field Lagrangian

$$L_A = -\frac{1}{4}[(F^1_{\mu\nu})^2 + (F^2_{\mu\nu})^2 + (F^3_{\mu\nu})^2] - \frac{1}{4}(B_{\mu\nu})^2,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu], \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

and the matter field Lagrangian

$$L_\phi = \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi - \frac{\lambda}{4}\left(\phi^\dagger\phi - v^2\right)^2,$$

where $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \in \mathbb{C}_2$ are the matter fields.

Fermion sector is represented by **the lepton** L_L and **quark** L_Q Lagrangians.
For the first generation **the lepton** Lagrangian is taken in the form:

$$L_{L,e} = L_l^\dagger i\tilde{\tau}_\mu D_\mu L_l + e_r^\dagger i\tau_\mu D_\mu e_r - h_e[e_r^\dagger(\phi^\dagger L_l) + (L_l^\dagger\phi)e_r],$$

where $L_l = \left(\begin{array}{c} \nu_l \\ e_l \end{array}\right) \in \mathbb{C}_2$ is the $SU(2)$-doublet, e_r is the $SU(2)$-singlet, h_e is constant, $\tau_0 = \tilde{\tau}_0 = \mathbf{1}$, $\tilde{\tau}_k = -\tau_k$, τ_μ are Pauli matrices and e_r, e_l, ν_l are the two component Lorentz spinors. First and second terms in $L_{L,e}$ describe free movement of left and right fermions and their interactions with gauge fields. Last term corresponds to the electron mass.

The quark Lagrangian is constructed in a similar way

$$L_Q = Q_l^\dagger i\tilde{\tau}_\mu D_\mu Q_l + u_r^\dagger i\tau_\mu D_\mu u_r + d_r^\dagger i\tau_\mu D_\mu d_r -$$

$$-h_d[d_r^\dagger(\phi^\dagger Q_l) + (Q_l^\dagger \phi) d_r] - h_u[u_r^\dagger(\tilde{\phi}^\dagger Q_l) + (Q_l^\dagger \tilde{\phi}) u_r],$$

where left quark fields form the $SU(2)$-doublet $Q_l = \left(\begin{array}{c} u_l \\ d_l \end{array} \right) \in \mathbb{C}_2$, right quark fields u_r, d_r are the $SU(2)$-singlets, $\tilde{\phi}_i = \epsilon_{ik}\bar{\phi}_k, \epsilon_{00} = 1, \epsilon_{ii} = -1$ is the conjugate representation of $SU(2)$ group and h_u, h_d are constants. All fields u_l, d_l, u_r, d_r are two component Lorentz spinors.

The new gauge fields are introduced

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp iA_\mu^2\right), \quad Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(gA_\mu^3 - g'B_\mu\right),$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g'A_\mu^3 + gB_\mu\right).$$

They are expressed through the old fields

$$A_\mu(x) = -ig\sum_{k=1}^{3} T_k A_\mu^k(x), \quad B_\mu(x) = -ig'B_\mu(x).$$

Chromodynamics

- The QCD gauge group is $SU(3)$, acting in 3-dim. complex space $\mathbb{C}_3$ of color quark states

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \equiv \begin{pmatrix} q_R \\ q_G \\ q_B \end{pmatrix} \in \mathbb{C}_3,$$

  where $q(x)$ are quark fields $q = u, d, s, c, b, t$ and R (red), G (green), B (blue) are color degrees of freedom.
- The $SU(3)$ gauge bosons are called gluons.

- QCD Lagrangian is taken in the form

$$\mathcal{L} = \sum_q \bar{q}^i (i\gamma^\mu)(D_\mu)_{ij} q^j - \frac{1}{4} \sum_{\alpha=1}^{8} F^\alpha_{\mu\nu} F^{\mu\nu\,\alpha},$$

- where $D_\mu q$ are covariant derivatives of quark fields

$$D_\mu q = \left(\partial_\mu - i g_s \left(\frac{\lambda^\alpha}{2} \right) A^\alpha_\mu \right) q,$$

  g_s is the strong coupling constant, $t^a = \lambda^a/2$ are generators of $SU(3)$, λ^a — Gell-Mann matrices
- and gluon stress tensor has the form

$$F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + g_s f^{\alpha\beta\gamma} A^\beta_\mu A^\gamma_\nu.$$

Contraction of the Electroweak Model

- The infinite energy limit of EWM corresponds to the consistent rescaling of $SU(2)$ and $\mathbb{C}_2$

$$\left(\begin{array}{c} z_1' \\ \epsilon z_2' \end{array}\right) = \left(\begin{array}{cc} \alpha & \epsilon\beta \\ -\epsilon\bar{\beta} & \bar{\alpha} \end{array}\right)\left(\begin{array}{c} z_1 \\ \epsilon z_2 \end{array}\right), \quad |\alpha|^2 + \epsilon^2|\beta|^2 = 1,$$

  when contraction parameter tends to zero $\epsilon \to 0$.
- Substitution $\beta \to \epsilon\beta$ induces another ones for the gauge fields:

$$W_\mu^\pm \to \epsilon W_\mu^\pm, \ \ Z_\mu \to Z_\mu, \ \ A_\mu \to A_\mu.$$

  and substitution $z_2 \to \epsilon z_2$ induces the following transformation of lepton and quark fields:

$$e_l \to \epsilon e_l, \ \ d_l \to \epsilon d_l, \ \ \nu_l \to \nu_l, \ \ u_l \to u_l.$$

- The next reason is the special mechanism of spontaneous symmetry breaking, which is used to generate mass for the vector bosons. One of L_B ground states

$$\phi^{vac} = \left(\begin{array}{c} 0 \\ v \end{array} \right), \quad A_\mu^k = B_\mu = 0$$

is taken as vacuum and then small field excitations $v + \chi(x)$ with respect to this vacuum are regarded.

- So, Higgs boson field χ, constant v and particle masses m_p, which depend on v, are multiplied by contraction parameter:

$$\chi \to \epsilon\chi, \quad v \to \epsilon v, \quad m_p \to \epsilon m_p, \quad p = \chi, W, Z, e, u, d.$$

- After these transformations the Lagrangian can be represented in the form

$$L(\epsilon) = L_\infty - \epsilon\, m_u(u_r^\dagger u_l + u_l^\dagger u_r) +$$

$$+\epsilon^2 L_2 + \epsilon^3 L_3 + \epsilon^4 L_4.$$

  For $\epsilon \to 0$ the terms with the higher powers of ϵ contribute less then the terms with lower powers.
- So the Electroweak Model demonstrates five stages of behaviour in the infinite energy limit, which are distinguished by the powers of the contraction parameter.

QCD with contracted gauge group

The contracted group $SU(3;\epsilon)$ is defined by the action

$$q'(\epsilon) = \begin{pmatrix} q'_1 \\ \epsilon q'_2 \\ \epsilon^2 q'_3 \end{pmatrix} = \begin{pmatrix} u_{11} & \epsilon u_{12} & \epsilon^2 u_{13} \\ \epsilon u_{21} & u_{22} & \epsilon u_{23} \\ \epsilon^2 u_{31} & \epsilon u_{32} & u_{33} \end{pmatrix} \begin{pmatrix} q_1 \\ \epsilon q_2 \\ \epsilon^2 q_3 \end{pmatrix} = U(\epsilon) q(\epsilon)$$

on the color space $\mathbb{C}_3(\epsilon)$.

The quark and gluon fields are transformed as follows:

$$q_1 \to q_1, \quad q_2 \to \epsilon q_2, \quad q_3 \to \epsilon^2 q_3,$$

$$A_\mu^{GR} \to \epsilon A_\mu^{GR}, \quad A_\mu^{BG} \to \epsilon A_\mu^{BG}, \quad A_\mu^{BR} \to \epsilon^2 A_\mu^{BR},$$

and diagonal gauge fields are not changed

$$A_\mu^{RR} \to A_\mu^{RR}, \quad A_\mu^{GG} \to A_\mu^{GG}, \quad A_\mu^{BB} \to A_\mu^{BB}.$$

- **With this substitution, we obtain the quark part of QCD Lagrangian in the form**

$$\mathcal{L}_q(\kappa) = \sum_q i\bar{q}_1\gamma^\mu\partial_\mu q_1 + \frac{g_s}{2}\left|q_1\right|^2\gamma^\mu A_\mu^{RR} +$$

$$+\epsilon^2\left\{i\bar{q}_2\gamma^\mu\partial_\mu q_2 + \frac{g_s}{2}\left(\left|q_2\right|^2\gamma^\mu A_\mu^{GG} + q_1\bar{q}_2\gamma^\mu A_\mu^{GR} + \bar{q}_1 q_2\gamma^\mu \bar{A}_\mu^{GR}\right)\right\}+$$

$$+\epsilon^4\left[i\bar{q}_3\gamma^\mu\partial_\mu q_3 + \frac{g_s}{2}\left(\left|q_3\right|^2\gamma^\mu A_\mu^{BB} + q_1\bar{q}_3\gamma^\mu A_\mu^{BR} + \bar{q}_1 q_3\gamma^\mu \bar{A}_\mu^{BR} + \right.\right.$$

$$\left.\left. + q_2\bar{q}_3\gamma^\mu A_\mu^{BG} + \bar{q}_2 q_3\gamma^\mu \bar{A}_\mu^{BG}\right)\right] = L_q^\infty + \epsilon^2 L_q^{(2)} + \epsilon^4 L_q^{(4)}.$$

- **Gluon part** $L_{gl} = -\frac{1}{4}F_{\mu\nu}^{\alpha}F^{\mu\nu\,\alpha}$ **of Lagrangian is very cumbersome, therefore we omit its general form.**

- The contracted QCD Lagrangian has the form

$$\mathcal{L}(\epsilon) = L^{\infty} + \epsilon^2 L^{(2)} + \epsilon^4 L^{(4)} + \epsilon^6 L^{(6)} + \epsilon^8 L^{(8)},$$

  with the explicit expressions for each $L^{(k)}$.
- The contraction parameter is monotonous function of the average energy E (or temperature T) with the property $\epsilon(E) \to 0$ for $E \to \infty$.

Very higher energies (temperatures) can exist in the early Universe after inflation and reheating on the first stages of the Hot Big Bang ($1eV = 10^4 K$).

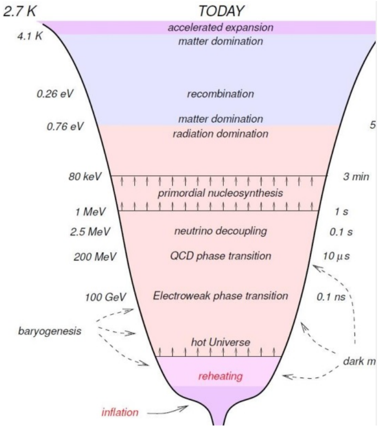

Рис.: **History of the Universe (V. Rubakov, D. Gorbunov, INR RAS)**

Estimation of boundary values

- To estimate the absolute dates of different stages of SM development we use the fact that the electroweak epoch starts at the temperature $T_4 = 100\, GeV \;\; (1\, GeV = 10^{13} K)$ and the QCD epoch begins at $T_8 = 0,2\, GeV$,
- i.e. we assume that complete reconstruction of the EWM, whose Lagrangian has minimal terms $\approx \epsilon^4$, and QCD with minimal terms $\approx \epsilon^8$, take place at these temperatures.

- Let us denote by Δ cutoff level for $\epsilon^k, \; k = 1, 2, 4, 6, 8$, i.e. for $\epsilon^k < \Delta$ all the terms proportionate to ϵ^k are negligible quantities in Lagrangian.
- At last we suppose that the contraction parameter inversely depends on temperature

$$\epsilon(T) = \frac{A}{T}, \quad A = const. \tag{1}$$

- From the equation for QCD $\epsilon^8(T_8) = A^8 T_8^{-8} = \Delta$ we obtain $A = T_8 \Delta^{1/8} = 0,2 \Delta^{1/8}\, GeV$.
  From the similar equation for EWM we obtain the cutoff level $\Delta = (T_8 E_4^{-1})^8 = (0,2 \cdot 10^{-2})^8 \approx 10^{-22}$.

- From the equation for k-th power $\epsilon^k(T_k) = A^k T_k^{-k} = \Delta$ we have

$$T_k = T_8 \Delta^{\frac{k-8}{8k}} \approx 10^{\frac{88-15k}{4k}} \, GeV$$

and easily find the boundary values (GeV):

$$T_1 = 10^{18},\ T_2 = 10^7,\ T_3 = 10^3,\ T_4 = 10^2,\ T_6 = 1,\ T_8 = 2{\cdot}10^{-1}$$

- The estimation for "infinity" temperature $T_1 \approx 10^{18}\, GeV$ is comparable with Planck energy $\approx 10^{19}\, GeV$, where the gravitation effects are important.
- So the developed evolution of the elementary particles does not exceed the range of the problems described by electroweak and strong interactions.

Evolution of elementary particles

$$(T > 10^{18}\, GeV).$$

- At the infinite temperature ($\epsilon = 0$) the EWM Lagrangian is as follows

$$L_{\infty} = -\frac{1}{4}\mathcal{Z}_{\mu\nu}^2 - \frac{1}{4}\mathcal{F}_{\mu\nu}^2 + \nu_l^{\dagger} i\tilde{\tau}_{\mu}\partial_{\mu}\nu_l + u_l^{\dagger} i\tilde{\tau}_{\mu}\partial_{\mu}u_l +$$

$$+ e_r^{\dagger} i\tau_{\mu}\partial_{\mu}e_r + d_r^{\dagger} i\tau_{\mu}\partial_{\mu}d_r + u_r^{\dagger} i\tau_{\mu}\partial_{\mu}u_r + L_{\infty}^{int}(A_{\mu}, Z_{\mu}).$$

- So the Electroweak Model includes only massless particles: photons A_{μ} and neutral bosons Z_{μ}, left quarks u_l and neutrinos ν_l, right electrons e_r and quarks u_r, d_r.
- The electroweak interactions become long-range because they are mediated by the massless Z-bosons and photons.

- From the explicit form of the interaction part

$$L_{\infty}^{int}(A_\mu, Z_\mu) = \frac{g}{2\cos\theta_w}\nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \frac{2e}{3} u_l^\dagger \tilde{\tau}_\mu A_\mu u_l +$$

$$+\frac{g}{\cos\theta_w}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w\right) u_l^\dagger \tilde{\tau}_\mu Z_\mu u_l + g'\sin\theta_w e_r^\dagger \tau_\mu Z_\mu e_r -$$

$$-g'\cos\theta_w e_r^\dagger \tau_\mu A_\mu e_r - \frac{1}{3} g'\cos\theta_w d_r^\dagger \tau_\mu A_\mu d_r + \frac{1}{3} g'\sin\theta_w d_r^\dagger \tau_\mu Z_\mu d_r +$$

$$+\frac{2}{3} g'\cos\theta_w u_r^\dagger \tau_\mu A_\mu u_r - \frac{2}{3} g'\sin\theta_w u_r^\dagger \tau_\mu Z_\mu u_r$$

  it follows that there are no interactions between particles of different kind, for example neutrinos interact only with each other by neutral currents.
- It looks like some stratification of the Electroweak Model with only one sort of particles in each stratum.

- From the limit QCD Lagrangian

$$\mathcal{L}_\infty = L_q^\infty + L_{gl}^\infty = \sum_q i\bar{q}_1\gamma^\mu\partial_\mu q_1 + \frac{g_s}{2}\left|q_1\right|^2\gamma^\mu A_\mu^{RR} -$$

$$-\frac{1}{4}\left(F_{\mu\nu}^{RR}\right)^2 - \frac{1}{4}\left(F_{\mu\nu}^{GG}\right)^2 - \frac{1}{4}F_{\mu\nu}^{RR}F_{\mu\nu}^{GG}.$$

  it follows that only dynamic terms for the first color components of massless quarks survive under infinite temperature, which means that quarks are monochromatic.
- The terms also survive, which describe the interactions of these components with R-gluons. So the stratification is conserved in the QCD sector.

Evolution of elementary particles

$$(\approx \epsilon, \quad 10^{18}\, GeV \geq T > 10^{7}\, GeV).$$

- **The mass term of u-quark in the complete Lagrangian $L(\epsilon)$ is proportional to ϵ**

$$\epsilon\, m_u(u_r^\dagger u_l + u_l^\dagger u_r).$$

The same is held for c- and t-quark. So u-, c- and t-quark first restores its mass in the evolution of the Universe.

Evolution of elementary particles

$$(\approx \epsilon^2, \quad 10^7\,GeV \geq T > 10^3\,GeV).$$

- The mass terms of electron and d-quark are multiplied by ϵ^2

$$\epsilon^2 \left[m_e(e_r^\dagger e_l + e_l^\dagger e_r) + m_d(d_r^\dagger d_l + d_l^\dagger d_r)\right],$$

  The same is true for μ- and τ-lepton, for s- and b-quark. These particles become massive in the second stage.
- The quarks obtain the second color degree of freedom.
- The main part of electroweak and color interactions are restored in this epoch.

Evolution of elementary particles

$$(\approx \epsilon^3, \quad 10^3\, GeV \geq T > 10^2\, GeV).$$

There is one term in Lagrangian $L_3 = gW_\mu^+ W_\mu^- \chi$ proportionate to ϵ^3, which describe Higgs boson interaction with charged W-bosons.

$$(\approx \epsilon^4, \quad 10^2\, GeV \geq T > 1\, GeV).$$

- Higgs boson χ and charged W-boson last restore their masses after all other particles of SM.
- The final reconstruction of the EWM takes place in this epoch.
- The quarks obtain the third color degree of freedom.

Evolution of elementary particles

$$(\approx \epsilon^6, \quad 1\,GeV \geq T > 2 \cdot 10^{-1}\,GeV).$$

Next part of color interactions is restored.

$$(\approx \epsilon^8, \quad T \leq 2 \cdot 10^{-1}\,GeV).$$

- The color interactions are completely valid.
- The QCD is fully reconstructed .
- Start the time of the Standard Model.

Conclusion

On the base of the modern knowledge of the particle world, which is concentrated in SM, the hypothesis of the particles evolution in the early Universe is offered,

The exact Lagrangians for any stage of evolution takes away the hierarchy problem of the SM.

- more details:

  N.G., Elementary particles in the early Universe, Journal of Cosmology and Astroparticle Physics 03(2016) 053.

- Thank you for attention.