On multidimensional analog of Melvin's solution for the Lie algebra E_6

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Overview

- We consider a multidimensional analog of Melvin's solution for an arbitrary simple Lie algebra $\mathcal{G}$
  - The gravitational model contains n 2-forms and $l \geq n$ scalar fields, where n is the rank of $\mathcal{G}$.
  - The solution is governed by a set of n functions obeying n ordinary differential equations of second order with certain boundary conditions. It was conjectured earlier that these functions should be polynomials [Ivashchuk'2002].
- The polynomials corresponding to the Lie algebra E_6 are obtained and some their properties (symmetry, duality) are analyzed.
- The asymptotic behavior of the solution for large values of a radial variable is found.

The model

We consider a D-dimensional model governed by the action

$$S = \int d^D x \sqrt{|g|} \left\{ R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \frac{1}{2} \sum_{s=1}^{n} \mathrm{e}^{2\lambda_s(\varphi)} (F^s)^2 \right\}$$

- $g = g_{MN}(x) dx^M \otimes dx^N$ is a metric; $|g| = |\det(g_{MN})|$
- $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector of l scalar fields
- $(h_{\alpha\beta})$ is a constant symmetric non-degenerate $l \times l$ matrix
- $F^s = dA^s = \frac{1}{2} F^s_{MN} dz^M \wedge dz^N$ is a 2-form, $s = 1, ..., n$;
  $(F^s)^2 = F^s_{M_1 M_2} F^s_{N_1 N_2} g^{M_1 N_1} g^{M_2 N_2}$
- λ_s is a 1-form on $\mathbb{R}^l$: $\lambda_s(\varphi) = \lambda_{s\alpha} \varphi^\alpha$, $s = 1, ..., n$; $\alpha = 1, \ldots, l$.

The solution

Consider a family of exact solutions depending on one variable ρ and defined on the manifold $M = (0, +\infty) \times M_1 \times M_2$.

- M_1 is a one-dimensional manifold (say S^1 or $\mathbb{R}$);
- M_2 is a $(D-2)$-dimensional Ricci-flat manifold.

The solution reads:

$$g = \left(\prod_{s=1}^{n} H_s^{2h_s/(D-2)} \right) \left\{ w d\rho \otimes d\rho + \left(\prod_{s=1}^{n} H_s^{-2h_s} \right) \rho^2 d\phi \otimes d\phi + g^2 \right\}$$

$$F^s = -Q_s \left(\prod_{s'=1}^{n} H_{s'}^{-A_{ss'}} \right) \rho d\rho \wedge d\phi; \quad \exp(\varphi^\alpha) = \prod_{s=1}^{n} H_s^{h_s \lambda_s^\alpha}$$

where $s = 1, ..., n$; $w = \pm 1$;
$g^1 = d\phi \otimes d\phi$ is a metric on M_1; g^2 is a Ricci-flat metric on M_2.

The solution

The functions $H_s(z) > 0$, $z = \rho^2$, obey the equations

$$\boxed{\frac{d}{dz}\left(\frac{z}{H_s}\frac{d}{dz}H_s\right) = P_s \prod_{s'=1}^{n} H_{s'}^{-A_{ss'}}}, \qquad H_s(+0) = 1$$

- $P_s = (1/4)K_s Q_s^2$
- $h_s = K_s^{-1}$
- $K_s = B_{ss} > 0$
- $B_{ss'} \equiv 1 + \frac{1}{2-D} + \lambda_{s\alpha}\lambda_{s'\beta}h^{\alpha\beta}, \quad s, s' = 1, ..., n$
- $(h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}; \quad \lambda_s^{\alpha} = h^{\alpha\beta}\lambda_{s\beta}$
- $A_{ss'} = 2B_{ss'}/B_{s's'}$ is the **Cartan matrix** for a simple Lie algebra $\mathcal{G}$ of rank n.

Special cases

- The solution under consideration is as a special case of the **fluxbrane** (for $w = +1$, $M_1 = S^1$) and S**-brane** ($w = -1$) solutions.
- If $w = +1$ and the Ricci-flat metric g_2 has a pseudo-Euclidean signature, we get a multidimensional generalization of **Melvin's solution** [Melvin'1964].
  - Melvin's solution (without scalar field) corresponds to $D = 4$, $n = 1$, $M_1 = S^1$ $(0 < \phi < 2\pi)$, $M_2 = \mathbb{R}^2$, $g_2 = -dt \otimes dt + d\xi \otimes d\xi$ and $\mathcal{G} = A_1$.
- For $w = -1$ and g_2 of Euclidean signature we obtain a **cosmological solution** with a horizon (as $\rho = +0$) if $M_1 = \mathbb{R}$ $(-\infty < \phi < +\infty)$

The Polynomial Conjecture

According to a conjecture [Ivashchuk'2002], the solutions governed by the Cartan matrix $(A_{ss'})$ are **polynomials** (the so-called fluxbrane polynomials):

$$\boxed{H_s(z) = 1 + \sum_{k=1}^{n_s} P_s^{(k)} z^k}$$

- $P_s^{(k)}$ are constants ($P_s^{(1)} \equiv P_s$). Here $P_s^{(n_s)} \neq 0$.
- $n_s = 2 \sum_{s'=1}^{n} A^{ss'}$ – components of a twice dual Weyl vector in the basis of simple co-roots
- $(A^{ss'}) \equiv (A_{ss'})^{-1}$ – the inverse Cartan matrix.

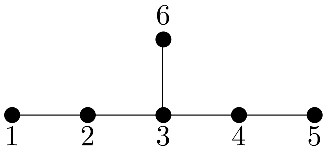
In previous works this conjecture was verified for some special cases of the classic Lie algebra series. Here we verify this conjecture for the case of the exceptional Lie algebra E_6.

We deal with the solution for $n = l = 6$, $w = +1$ and $M_1 = S^1$. We put here $h_{\alpha\beta} = \delta_{\alpha\beta}$ and denote $(\lambda_{sa}) = (\lambda_s^a) = \vec{\lambda}_s$, $s = 1, \dots, 6$.

- The Cartan matrix:

$$A = (A_{ss'}) = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 2 \end{pmatrix}$$

- The Dynkin diagram for the Lie algebra E_6:



Case of the Lie algebra E_6

- The inverse Cartan matrix:

$$A^{-1} = (A^{ss'}) = \begin{pmatrix} \frac{4}{3} & \frac{5}{3} & 2 & \frac{4}{3} & \frac{2}{3} & 1 \\ \frac{5}{3} & \frac{10}{3} & 4 & \frac{8}{3} & \frac{4}{3} & 2 \\ 2 & 4 & 6 & 4 & 2 & 3 \\ \frac{4}{3} & \frac{8}{3} & 4 & \frac{10}{3} & \frac{5}{3} & 2 \\ \frac{2}{3} & \frac{4}{3} & 2 & \frac{5}{3} & \frac{4}{3} & 1 \\ 1 & 2 & 3 & 2 & 1 & 2 \end{pmatrix}$$

- $(n_1, n_2, n_3, n_4, n_5, n_6) = (16, 30, 42, 30, 16, 22)$
- We parametrize the polynomials by using parameters B_s instead of P_s for simplicity:

$$P_s = n_s B_s, \quad s = 1, \ldots, 6$$

.

The fluxbrane polynomials $H_s(z, B_i)$ for E_6

$$H_1 = 1 + 16B_1 z + 120B_1B_2z^2 + \dots + 120B_1^2B_2^3B_3^4B_4^2B_5B_6^2z^{14} + 16B_1^2B_2^3B_3^4B_4^3B_5B_6^2z^{15} + B_1^2B_2^3B_3^4B_4^3B_5^2B_6^2z^{16},$$

$$H_2 = 1+30B_2z+(120B_1B_2+315B_3B_2)z^2 \dots +(120B_1^3B_2^6B_4^5B_5^2B_6^4B_3^8+315B_1^3B_2^6B_4^5B_5^3B_6^4B_3^7)z^{28} + 30B_1^3B_2^6B_3^8B_4^5B_5^3B_6^4z^{29} + B_1^3B_2^6B_3^8B_4^6B_5^3B_6^4z^{30},$$

$$H_3 = 1 + 42B_3z + (315B_2B_3 + 315B_4B_3 + 231B_6B_3)z^2 \dots +(315B_1^4B_2^7B_4^8B_5^4B_6^6B_3^{11}+315B_1^4B_2^8B_4^7B_5^4B_6^6B_3^{11}+231B_1^4B_2^8B_4^8B_5^4B_6^5B_3^{11})z^{40} + 42B_1^4B_2^8B_3^{11}B_4^8B_5^4B_6^6z^{41} + B_1^4B_2^8B_3^{12}B_4^8B_5^4B_6^6z^{42},$$

$$H_4 = 1+30B_4z+(315B_3B_4+120B_5B_4)z^2 \dots +(120B_1^2B_2^5B_4^6B_5^3B_6^4B_3^8+315B_1^3B_2^5B_4^6B_5^3B_6^4B_3^7)z^{28} + 30B_1^3B_2^5B_3^8B_4^6B_5^3B_6^4z^{29} + B_1^3B_2^6B_3^8B_4^6B_5^3B_6^4z^{30},$$

$$H_5 = 1 + 16B_5z + 120B_4B_5z^2 + \dots + 120B_1B_2^2B_3^4B_4^3B_5^2B_6^2z^{14} + 16B_1B_2^3B_3^4B_4^3B_5^2B_6^2z^{15} + B_1^2B_2^3B_3^4B_4^3B_5^2B_6^2z^{16},$$

$$H_6 = 1 + 22B_6z + 231B_3B_6z^2 + \dots + 231B_1^2B_2^4B_3^5B_4^4B_5^2B_6^3z^{20} + 22B_1^2B_2^4B_3^6B_4^4B_5^2B_6^3z^{21} + B_1^2B_2^4B_3^6B_4^4B_5^2B_6^4z^{22}.$$

The Symmetry and the Duality

- Consider a permutation $\sigma : (1,2,3,4,5,6) \mapsto (5,4,3,2,1,6)$ of the vertices of the E_6 Dynkin diagram as a generator of a symmetry subgroup $G = \{\sigma, \mathrm{id}\} \cong \mathbb{Z}_2$ of the diagram.
- Define the dual ordered set $\boxed{\hat{B}_i \equiv B_{\sigma(i)}}$, $i = 1, \dots, 6$
- The asymptotical behaviour of the polynomials as $z \to \infty$:

$$\boxed{H_s = H_s(z, (B_i)) \sim \left(\prod_{l=1}^{6} (B_l)^{\nu^{sl}} \right) z^{n_s} \equiv H_s^{as}(z, (B_i))}$$

- The matrix $\nu = A^{-1}(I + P)$; $\quad \sum_{s=1}^{6} \nu^{sl} = n_l$

$$(\nu^{sl}) = \begin{pmatrix} 2 & 3 & 4 & 3 & 2 & 2 \\ 3 & 6 & 8 & 6 & 3 & 4 \\ 4 & 8 & 12 & 8 & 4 & 6 \\ 3 & 6 & 8 & 6 & 3 & 4 \\ 2 & 3 & 4 & 3 & 2 & 2 \\ 2 & 4 & 6 & 4 & 2 & 4 \end{pmatrix}$$

- $P = (P^i_j) = (\delta^i_{\sigma(j)})$ – the permutation matrix

The Symmetry and the Duality

Then the following properties hold true:

Proposition 1 (the symmetry relations)

For all B_i and z

$$H_{\sigma(s)}(z,(B_i)) = H_s(z,(\hat{B}_i)), \quad s = 1,\ldots,6$$

Proposition 2 (the duality relations)

For all $B_i \neq 0$ and $z \neq 0$

$$H_s(z,(B_i)) = H_s^{as}(z,(B_i))H_s(z^{-1},(\hat{B}_i^{-1})), \quad \mathrm{s} = 1,\ \ldots,\ 6.$$

An exact solution for the Lie algebra E_6

The manifold: $M = (0, +\infty) \times M_1 \times M_2$.

- $M_1 = S^1$ $(0 < \phi < 2\pi)$;
- M_2 is a $(D-2)$-dimensional Ricci-flat manifold.

The solution:

$$g = \Big(\prod_{s=1}^{6} H_s^{2h/(D-2)}\Big)\Big\{d\rho \otimes d\rho + \Big(\prod_{s=1}^{6} H_s^{-2h}\Big)\rho^2 d\phi \otimes d\phi + g^2\Big\}$$

$$F^s = \mathcal{B}^s \rho d\rho \wedge d\phi; \quad \exp(\varphi^a) = \prod_{s=1}^{6} H_s^{h\lambda_s^a}; \qquad a, s = 1, \dots, 6$$

Here $\mathcal{B}^s = -Q_s \left(\prod_{l=1}^{6} H_l^{-A_{sl}}\right)$, $\quad h_s = h = K^{-1}$,

$$K = K_s = \frac{D-3}{D-2} + \vec{\lambda}_s^2, \quad \vec{\lambda}_s \vec{\lambda}_{s'} = \frac{1}{2} K A_{ss'} - \frac{D-3}{D-2}$$

The fluxes

- Consider 2-dimensional manifold $M_* = (0, +\infty) \times S^1$
- The flux integrals:

$$\Phi^s = \int\limits_{M_*} F^s = 2\pi \int\limits_0^{+\infty} d\rho \rho \mathcal{B}^s = -2\pi \int\limits_0^{+\infty} d\rho\rho\, Q_s \prod_{l=1}^{6} H_l^{-A_{sl}}$$

$$= -\frac{1}{2} Q_s P_s^{-1} \lim_{z \to +\infty} \left(\frac{z}{H_s} \frac{d}{dz} H_s \right) = -\frac{1}{2} n_s Q_s P_s^{-1}$$

$$= \boxed{-4\pi n_s Q_s^{-1} h}, \qquad h = K^{-1}$$

- Any flux Φ^s depends only upon n_s and the integration constant Q_s, which for $D = 4$ and $g^2 = -dt \otimes dt + dx \otimes dx$ is coinciding up to a sign with the value of the x-component of the magnetic field on the axis of symmetry

Asymptotic behaviour

The asymptotic relations for the solution for $\rho \to +\infty$ read

$$g_{as} = \left(\prod_{l=1}^{6} B_l^{n_l}\right)^{\frac{2h}{D-2}} \rho^{2A} \left\{ d\rho \otimes d\rho + \left(\prod_{s=1}^{6} B_s^{n_s}\right)^{-2h} \rho^{2-2A(D-2)} d\phi \otimes d\phi + g^2 \right\}$$

$$\varphi_{as}^a = h \sum_{s=1}^{6} \lambda_s^a \left(\sum_{l=1}^{6} \nu^{sl} \ln B_l + 2 n_s \ln \rho\right)$$

$$F_{as}^s = -Q_s B_s^{-1} B_{\sigma(s)}^{-1} \rho^{-3} d\rho \wedge d\phi, \qquad a, s = 1, \dots, 6$$

where

$$A = (2h/(D-2)) \sum_{s=1}^{6} n_s = 312h/(D-2).$$

Conclusions

- We have obtained a multidimensional generalization of Melvin's solution for the Lie algebra E_6. The solution is governed by a set of six fluxbrane polynomials $H_s(z)$, $s = 1, \dots, 6$.
- The polynomials $H_s(z)$ depend also on parameters Q_s, which are coinciding for $D = 4$ (up to a sign) with the values of colored magnetic fields on the axis of symmetry.
- The symmetry and duality identities for polynomials were verified. The duality identities may be used in deriving $1/\rho$-expansion for solutions at large distances ρ.
- The asymptotic relations for E_6-polynomials at large z are governed by integer-valued matrix $\nu = A^{-1}(I + P)$. The matrix P corresponds to a permutation $\sigma \in S_6$ which is the generator of the Z_2-group of symmetry of the Dynkin diagram.
- $2d$ flux integrals Φ^s, $s = 1, \dots, 6$ are calculated. Any flux Φ^s depends only upon one parameter Q_s.

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Appendix: the full set of E_6 polynomials I

$$
\boxed{H_1} = B_1^2 B_2^3 B_3^4 B_4^3 B_5^2 B_6^2 \mathbf{Z}^{16} + 16 B_1^2 B_2^3 B_3^4 B_4^3 B_5 B_6^2 \mathbf{Z}^{15} + 120 B_1^2 B_2^3 B_3^4 B_4^2 B_5 B_6^2 \mathbf{Z}^{14} +
560 B_1^2 B_2^3 B_3^3 B_4^2 B_5 B_6^2 \mathbf{Z}^{13} + (1050 B_1^2 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 770 B_1^2 B_2^3 B_4^2 B_5 B_6 B_3^3)\mathbf{Z}^{12} +
(672 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 3696 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^3)\mathbf{Z}^{11} + (3696 B_1 B_2^2 B_4^2 B_5 B_6 B_3^3 +
4312 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^2)\mathbf{Z}^{10} + (8800 B_1 B_2^2 B_4^2 B_5 B_6 B_3^2 + 2640 B_1^2 B_2^2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^{9} + (660 B_1^2 B_2^2 B_4 B_6 B_3^2 +
4125 B_1 B_2 B_4^2 B_5 B_6 B_3^2 + 8085 B_1 B_2^2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^{8} + (2640 B_1 B_2^2 B_4 B_6 B_3^2 + 8800 B_1 B_2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^{7} +
(4312 B_1 B_2 B_4 B_6 B_3^2 + 3696 B_1 B_2 B_4 B_5 B_6 B_3)\mathbf{Z}^{6} + (672 B_1 B_2 B_3 B_4 B_5 + 3696 B_1 B_2 B_3 B_4 B_6)\mathbf{Z}^{5} +
(1050 B_1 B_2 B_3 B_4 + 770 B_1 B_2 B_3 B_6)\mathbf{Z}^{4} + 560 B_1 B_2 B_3 \mathbf{Z}^{3} + 120 B_1 B_2 \mathbf{Z}^{2} + 16 B_1 \mathbf{Z} + 1
$$

$$
\boxed{H_2} = B_1^3 B_2^6 B_3^8 B_4^6 B_5^3 B_6^4 \mathbf{Z}^{30} + 30 B_1^3 B_2^6 B_3^8 B_4^5 B_5^3 B_6^4 \mathbf{Z}^{29} + (120 B_1^3 B_2^6 B_4^5 B_5^2 B_6^4 B_3^8 +
315 B_1^3 B_2^6 B_4^5 B_5^3 B_6^4 B_3^7)\mathbf{Z}^{28} + (1050 B_1^3 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7 + 2240 B_1^3 B_2^6 B_4^5 B_5^2 B_6^4 B_3^7 +
770 B_1^3 B_2^6 B_4^5 B_5^3 B_6^3 B_3^7)\mathbf{Z}^{27} + (1050 B_1^2 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7 + 9450 B_1^3 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7 + 4200 B_1^3 B_2^6 B_4^4 B_5^2 B_6^4 B_3^7 +
5775 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7 + 6930 B_1^3 B_2^6 B_4^5 B_5^2 B_6^3 B_3^7)\mathbf{Z}^{26} + (10752 B_1^2 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7 +
31500 B_1^3 B_2^5 B_4^4 B_5^2 B_6^4 B_3^7 + 8316 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7 + 59136 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7 + 23100 B_1^3 B_2^6 B_4^4 B_5^2 B_6^3 B_3^7 +
9702 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6)\mathbf{Z}^{25} + (45360 B_1^2 B_2^5 B_4^4 B_5^2 B_6^4 B_3^7 + 92400 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7 +
249480 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7 + 36750 B_1^3 B_2^5 B_4^4 B_5^2 B_6^4 B_3^6 + 26950 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6 + 8085 B_1^3 B_2^5 B_4^4 B_5^3 B_6^3 B_3^6 +
107800 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6 + 26950 B_1^3 B_2^6 B_4^4 B_5^2 B_6^3 B_3^6)\mathbf{Z}^{24} + (443520 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7 +
94080 B_1^2 B_2^5 B_4^4 B_5^2 B_6^4 B_3^6 + 16500 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6 + 32340 B_1^2 B_2^5 B_4^4 B_5^3 B_6^3 B_3^6 + 316800 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6 +
1132560 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6)\mathbf{Z}^{23} + (44100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6 + 44550 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6 +
202125 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 3256110 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6 + 242550 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 +
495000 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^6 + 32340 B_1^3 B_2^5 B_4^4 B_5 B_6^3 B_3^6 + 177870 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^6 +
1358280 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^5)\mathbf{Z}^{22} + (2674100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 + 2182950 B_1^2 B_2^5 B_4^3 B_5^2 B_6^3 B_3^6 +
168960 B_1^2 B_2^5 B_4^4 B_5 B_6^3 B_3^6 + 178200 B_1^3 B_2^5 B_4^3 B_5 B_6^3 B_3^6 + 711480 B_1^2 B_2^5 B_4^4 B_5^2 B_6^2 B_3^6 +
23100 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^5 + 4928000 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^5 + 1131900 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5 +
1478400 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^5 + 830060 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^5)\mathbf{Z}^{21} + (155232 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 +
$$

Appendix: the full set of E_6 polynomials II

$$3234000B_1^2B_2^4B_4^3B_5^2B_6^3B_3^6 + 349272B_1^2B_2^4B_4^4B_5B_6^3B_3^6 + 970200B_1^2B_2^5B_4^3B_5B_6^3B_3^6 +$$
$$853776B_1^2B_2^4B_4^4B_5^2B_6^2B_3^6 + 9315306B_1^2B_2^4B_4^4B_5^2B_6^3B_3^5 + 7074375B_1^2B_2^5B_4^3B_5^2B_6^3B_3^5 +$$
$$1559250B_1^3B_2^4B_4^3B_5^2B_6^3B_3^5 + 577500B_1^3B_2^5B_4^3B_5B_6^3B_3^5 + 5082B_1^2B_2^4B_4^4B_5^3B_6^2B_3^5 +$$
$$3811500B_1^2B_2^5B_4^4B_5^2B_6^2B_3^5 + 996072B_1^3B_2^4B_4^4B_5^2B_6^2B_3^5 + 1143450B_1^3B_2^5B_4^3B_5^2B_6^2B_3^5)\mathbf{Z}^{20} +$$
$$(2069760B_1^2B_2^4B_4^3B_5B_6^3B_3^6 + 1478400B_1B_2^4B_4^4B_5^2B_6^3B_3^5 + 4331250B_1^2B_2^3B_4^4B_5^2B_6^3B_3^5 +$$
$$21801780B_1^2B_2^4B_4^3B_5^2B_6^3B_3^5 + 369600B_1^2B_2^4B_4^4B_5B_6^3B_3^5 + 3326400B_1^2B_2^5B_4^3B_5B_6^3B_3^5 +$$
$$693000B_1^3B_2^4B_4^3B_5B_6^3B_3^5 + 11384100B_1^2B_2^4B_4^4B_5^2B_6^2B_3^5 + 6225450B_1^2B_2^5B_4^3B_5^2B_6^2B_3^5 +$$
$$2439360B_1^3B_2^4B_4^3B_5^2B_6^2B_3^5 + 508200B_1^3B_2^5B_4^3B_5B_6^2B_3^5)\mathbf{Z}^{19} + (1559250B_1B_2^3B_4^4B_5^2B_6^3B_3^5 +$$
$$3056130B_1B_2^4B_4^3B_5^2B_6^3B_3^5 + 14437500B_1^2B_2^3B_4^3B_5^2B_6^3B_3^5 + 14314300B_1^2B_2^4B_4^3B_5B_6^3B_3^5 +$$
$$2032800B_1B_2^4B_4^4B_5^2B_6^2B_3^5 + 8575875B_1^2B_2^3B_4^4B_5^2B_6^2B_3^5 + 28420210B_1^2B_2^4B_4^3B_5^2B_6^2B_3^5 +$$
$$127050B_1^2B_2^4B_4^4B_5B_6^2B_3^5 + 3176250B_1^2B_2^5B_4^3B_5B_6^2B_3^5 + 1372140B_1^3B_2^4B_4^3B_5B_6^2B_3^5 +$$
$$6338640B_1^2B_2^4B_4^3B_5^2B_6^3B_3^4 + 2371600B_1^2B_2^4B_4^4B_5^2B_6^2B_3^4 + 711480B_1^3B_2^4B_4^3B_5^2B_6^2B_3^4)\mathbf{Z}^{18} +$$
$$(5913600B_1B_2^3B_4^3B_5^2B_6^3B_3^5 + 1774080B_1B_2^4B_4^3B_5B_6^3B_3^5 + 10187100B_1^2B_2^3B_4^3B_5B_6^3B_3^5 +$$
$$577500B_1^2B_2^4B_4^2B_5B_6^3B_3^5 + 3811500B_1B_2^3B_4^4B_5^2B_6^2B_3^5 + 7470540B_1B_2^4B_4^3B_5^2B_6^2B_3^5 +$$
$$32524800B_1^2B_2^3B_4^3B_5^2B_6^2B_3^5 + 18705960B_1^2B_2^4B_4^3B_5B_6^2B_3^5 + 8731800B_1^2B_2^3B_4^3B_5^2B_6^3B_3^4 +$$
$$8279040B_1^2B_2^4B_4^3B_5B_6^3B_3^4 + 4446750B_1^2B_2^3B_4^4B_5^2B_6^2B_3^4 + 16625700B_1^2B_2^4B_4^3B_5^2B_6^2B_3^4 +$$
$$711480B_1^3B_2^4B_4^3B_5B_6^2B_3^4)\mathbf{Z}^{17} + (4527600B_1B_2^3B_4^3B_5B_6^3B_3^5 + 18295200B_1B_2^3B_4^3B_5^2B_6^2B_3^5 +$$
$$5488560B_1B_2^4B_4^3B_5B_6^2B_3^5 + 24901800B_1^2B_2^3B_4^3B_5B_6^2B_3^5 + 508200B_1^2B_2^4B_4^2B_5B_6^2B_3^5 +$$
$$3880800B_1B_2^3B_4^3B_5^2B_6^3B_3^4 + 11884950B_1^2B_2^3B_4^3B_5B_6^3B_3^4 + 2182950B_1^2B_2^4B_4^2B_5B_6^3B_3^4 +$$
$$2268750B_1B_2^3B_4^4B_5^2B_6^2B_3^4 + 4446750B_1B_2^4B_4^3B_5^2B_6^2B_3^4 + 45530550B_1^2B_2^3B_4^3B_5^2B_6^2B_3^4 +$$
$$1334025B_1^2B_2^4B_4^2B_5^2B_6^2B_3^4 + 18478980B_1^2B_2^4B_4^3B_5B_6^2B_3^4 + 108900B_1^3B_2^4B_4^2B_5B_6^2B_3^4 +$$
$$1584660B_1^2B_2^4B_4^3B_5^2B_6B_3^4)\mathbf{Z}^{16} + (15937152B_1B_2^3B_4^3B_5B_6^2B_3^5 + 5588352B_1B_2^3B_4^3B_5B_6^3B_3^4 +$$
$$3234000B_1^2B_2^3B_4^2B_5B_6^3B_3^4 + 34036496B_1B_2^3B_4^3B_5^2B_6^2B_3^4 + 5808000B_1^2B_2^3B_4^2B_5^2B_6^2B_3^4 +$$
$$5808000B_1B_2^4B_4^3B_5B_6^2B_3^4 + 53742416B_1^2B_2^3B_4^3B_5B_6^2B_3^4 + 6203600B_1^2B_2^4B_4^2B_5B_6^2B_3^4 +$$
$$5588352B_1^2B_2^3B_4^3B_5^2B_6B_3^4 + 3234000B_1^2B_2^4B_4^3B_5B_6B_3^4 + 15937152B_1^2B_2^3B_4^3B_5^2B_6^2B_3^3)\mathbf{Z}^{15} +$$

$$
\begin{aligned}
&(108900B_1^2B_3^4B_4^2B_6^2B_2^4 + 1334025B_1B_3^4B_4^2B_5B_6^2B_2^4 + 508200B_1^2B_3^3B_4^2B_5B_6^2B_2^4 + \\
&2182950B_1^2B_3^4B_4^2B_5B_6B_2^4 + 1584660B_1B_3^4B_4^2B_5B_6^3B_2^3 + 18295200B_1B_3^3B_4^3B_5^2B_6^2B_2^3 + \\
&4446750B_1B_3^4B_4^2B_5^2B_6^2B_2^3 + 5488560B_1^2B_3^3B_4^2B_5^2B_6^2B_2^3 + 45530550B_1B_3^4B_4^3B_5B_6^2B_2^3 + \\
&24901800B_1^2B_3^3B_4^3B_5B_6^2B_2^3 + 18478980B_1^2B_3^4B_4^2B_5B_6^2B_2^3 + 3880800B_1B_3^4B_4^3B_5^2B_6B_2^3 + \\
&4527600B_1^2B_3^3B_4^3B_5^2B_6B_2^3 + 11884950B_1^2B_3^4B_4^3B_5B_6B_2^3 + 2268750B_1B_3^4B_4^3B_5^2B_6^2B_2^2)\mathbf{Z}^{14} + \\
&(577500B_1^2B_3^3B_4^2B_5B_6B_2^4 + 711480B_1^2B_3^4B_4^2B_6^2B_2^3 + 7470540B_1B_3^3B_4^2B_5^2B_6^2B_2^3 + \\
&32524800B_1B_3^3B_4^3B_5B_6^2B_2^3 + 16625700B_1B_3^4B_4^2B_5B_6^2B_2^3 + 18705960B_1^2B_3^3B_4^2B_5B_6^2B_2^3 + \\
&5913600B_1B_3^3B_4^3B_5^2B_6B_2^3 + 1774080B_1^2B_3^3B_4^2B_5^2B_6B_2^3 + 8731800B_1B_3^4B_4^3B_5B_6B_2^3 + \\
&10187100B_1^2B_3^3B_4^3B_5B_6B_2^3 + 8279040B_1^2B_3^4B_4^2B_5B_6B_2^3 + 3811500B_1B_3^3B_4^3B_5^2B_6^2B_2^2 + \\
&4446750B_1B_3^4B_4^3B_5B_6^2B_2^2)\mathbf{Z}^{13} + (711480B_1B_2^3B_4^2B_6^2B_3^4 + 2371600B_1B_2^2B_4^2B_5B_6^2B_3^4 + \\
&6338640B_1B_2^3B_4^2B_5B_6B_3^4 + 1372140B_1^2B_2^3B_4^2B_6^2B_3^3 + 2032800B_1B_2^2B_4^2B_5^2B_6^2B_3^3 + \\
&8575875B_1B_2^2B_4^3B_5B_6^2B_3^3 + 28420210B_1B_2^3B_4^2B_5B_6^2B_3^3 + 127050B_1^2B_2^2B_4^2B_5B_6^2B_3^3 + \\
&3176250B_1^2B_2^3B_4B_5B_6^2B_3^3 + 1559250B_1B_2^2B_4^3B_5^2B_6B_3^3 + 3056130B_1B_2^3B_4^2B_5^2B_6B_3^3 + \\
&14437500B_1B_2^3B_4^3B_5B_6B_3^3 + 14314300B_1^2B_2^3B_4^2B_5B_6B_3^3)\mathbf{Z}^{12} + (2439360B_1B_3^3B_4^2B_6^2B_2^3 + \\
&508200B_1^2B_3^3B_4B_6^2B_2^3 + 6225450B_1B_3^3B_4B_5B_6^2B_2^3 + 693000B_1^2B_3^3B_4^2B_6B_2^3 + 21801780B_1B_3^3B_4^2B_5B_6B_2^3 + \\
&2069760B_1^2B_3^2B_4^2B_5B_6B_2^3 + 3326400B_1^2B_3^3B_4B_5B_6B_2^3 + 11384100B_1B_3^3B_4^2B_5B_6^2B_2^2 + \\
&1478400B_1B_3^3B_4^2B_5^2B_6B_2^2 + 4331250B_1B_3^3B_4^3B_5B_6B_2^2 + 369600B_1^2B_3^3B_4^2B_5B_6B_2^2)\mathbf{Z}^{11} + \\
&(1143450B_1B_3^3B_4B_6^2B_2^3 + 1559250B_1B_3^3B_4^2B_6B_2^3 + 577500B_1^2B_3^3B_4B_6B_2^3 + 3234000B_1B_3^2B_4^2B_5B_6B_2^3 + \\
&7074375B_1B_3^3B_4B_5B_6B_2^3 + 970200B_1^2B_3^2B_4B_5B_6B_2^3 + 996072B_1B_3^3B_4^2B_6^2B_2^2 + 5082B_3^3B_4^2B_5B_6^2B_2^2 + \\
&853776B_1B_3^2B_4^2B_5B_6^2B_2^2 + 3811500B_1B_3^3B_4B_5B_6^2B_2^2 + 155232B_1B_3^2B_4^2B_5^2B_6B_2^2 + \\
&9315306B_1B_3^3B_4^2B_5B_6B_2^2 + 349272B_1^2B_3^2B_4^2B_5B_6B_2^2)\mathbf{Z}^{10} + (1478400B_1B_3^3B_4B_6B_2^3 + \\
&178200B_1^2B_3^2B_4B_6B_2^3 + 2182950B_1B_3^2B_4B_5B_6B_2^3 + 830060B_1B_3^3B_4B_6^2B_2^2 + 711480B_1B_3^2B_4B_5B_6^2B_2^2 + \\
&1131900B_1B_3^3B_4^2B_6B_2^2 + 23100B_3^3B_4^2B_5B_6B_2^2 + 2674100B_1B_3^2B_4^2B_5B_6B_2^2 + 4928000B_1B_3^3B_4B_5B_6B_2^2 + \\
&168960B_1^2B_3^2B_4B_5B_6B_2^2)\mathbf{Z}^9 + (495000B_1B_3^2B_4B_6B_2^3 + 177870B_1B_3^2B_4B_6^2B_2^2 + 44100B_1B_3^2B_4^2B_5B_2^2 + \\
&242550B_1B_3^2B_4^2B_6B_2^2 + 1358280B_1B_3^3B_4B_6B_2^2 + 32340B_1^2B_3^2B_4B_6B_2^2 + 44550B_3^2B_4^2B_5B_6B_2^2 +
\end{aligned}
$$

Appendix: the full set of E_6 polynomials IV

$3256110 B_1 B_3^2 B_4 B_5 B_6 B_2^2 + 202125 B_1 B_3^2 B_4^2 B_5 B_6 B_2)\mathbf{Z}^8 + (94080 B_1 B_3^2 B_4 B_5 B_2^2 +$
$1132560 B_1 B_3^2 B_4 B_6 B_2^2 + 32340 B_3^2 B_4 B_5 B_6 B_2^2 + 443520 B_1 B_3 B_4 B_5 B_6 B_2^2 + 16500 B_3^2 B_4^2 B_5 B_6 B_2 +$
$316800 B_1 B_3^2 B_4 B_5 B_6 B_2)\mathbf{Z}^7 + (36750 B_1 B_3^2 B_4 B_2^2 + 45360 B_1 B_3 B_4 B_5 B_2^2 + 26950 B_1 B_3^2 B_6 B_2^2 +$
$8085 B_3^2 B_4 B_6 B_2^2 + 249480 B_1 B_3 B_4 B_6 B_2^2 + 107800 B_1 B_3^2 B_4 B_6 B_2 + 26950 B_3^2 B_4 B_5 B_6 B_2 +$
$92400 B_1 B_3 B_4 B_5 B_6 B_2)\mathbf{Z}^6 + (31500 B_1 B_3 B_4 B_2^2 + 23100 B_1 B_3 B_6 B_2^2 + 10752 B_1 B_3 B_4 B_5 B_2 +$
$9702 B_3^2 B_4 B_6 B_2 + 59136 B_1 B_3 B_4 B_6 B_2 + 8316 B_3 B_4 B_5 B_6 B_2)\mathbf{Z}^5 + (4200 B_1 B_3 B_2^2 + 9450 B_1 B_3 B_4 B_2 +$
$1050 B_3 B_4 B_5 B_2 + 6930 B_1 B_3 B_6 B_2 + 5775 B_3 B_4 B_6 B_2)\mathbf{Z}^4 + (2240 B_1 B_2 B_3 + 1050 B_2 B_4 B_3 +$
$770 B_2 B_6 B_3)\mathbf{Z}^3 + (120 B_1 B_2 + 315 B_3 B_2)\mathbf{Z}^2 + 30 B_2 \mathbf{Z} + 1$

$\boxed{H_3} =$
$B_1^4 B_2^8 B_3^{12} B_4^8 B_5^4 B_6^6 \mathbf{Z}^{42} + 42 B_1^4 B_2^8 B_3^{11} B_4^8 B_5^4 B_6^6 \mathbf{Z}^{41} + (315 B_1^4 B_2^7 B_4^8 B_5^4 B_6^6 B_3^{11} + 315 B_1^4 B_2^8 B_4^7 B_5^4 B_6^6 B_3^{11} +$
$231 B_1^4 B_2^8 B_4^8 B_5^4 B_6^5 B_3^{11})\mathbf{Z}^{40} + (560 B_1^3 B_2^7 B_4^8 B_5^4 B_6^6 B_3^{11} + 4200 B_1^4 B_2^7 B_4^7 B_5^4 B_6^6 B_3^{11} +$
$560 B_1^4 B_2^8 B_4^7 B_5^3 B_6^6 B_3^{11} + 3080 B_1^4 B_2^7 B_4^8 B_5^4 B_6^5 B_3^{11} + 3080 B_1^4 B_2^8 B_4^7 B_5^4 B_6^5 B_3^{11})\mathbf{Z}^{39} +$
$(9450 B_1^3 B_2^7 B_4^7 B_5^4 B_6^6 B_3^{11} + 9450 B_1^4 B_2^7 B_4^7 B_5^3 B_6^6 B_3^{11} + 6930 B_1^3 B_2^7 B_4^8 B_5^4 B_6^5 B_3^{11} + 51975 B_1^4 B_2^7 B_4^7 B_5^4 B_6^5 B_3^{11} +$
$6930 B_1^4 B_2^8 B_4^7 B_5^3 B_6^5 B_3^{11} + 11025 B_1^4 B_2^7 B_4^7 B_5^4 B_6^6 B_3^{10} + 8085 B_1^4 B_2^7 B_4^8 B_5^4 B_6^5 B_3^{10} +$
$8085 B_1^4 B_2^8 B_4^7 B_5^4 B_6^5 B_3^{10})\mathbf{Z}^{38} + (24192 B_1^3 B_2^7 B_4^7 B_5^3 B_6^6 B_3^{11} + 133056 B_1^3 B_2^7 B_4^7 B_5^4 B_6^5 B_3^{11} +$
$133056 B_1^4 B_2^7 B_4^7 B_5^3 B_6^5 B_3^{11} + 44100 B_1^3 B_2^7 B_4^7 B_5^4 B_6^6 B_3^{10} + 44100 B_1^4 B_2^7 B_4^7 B_5^3 B_6^6 B_3^{10} +$
$32340 B_1^3 B_2^7 B_4^8 B_5^4 B_6^5 B_3^{10} + 407484 B_1^4 B_2^7 B_4^7 B_5^4 B_6^5 B_3^{10} + 32340 B_1^4 B_2^8 B_4^7 B_5^3 B_6^5 B_3^{10})\mathbf{Z}^{37} +$
$(369600 B_1^3 B_2^7 B_4^7 B_5^3 B_6^5 B_3^{11} + 36750 B_1^3 B_2^6 B_4^7 B_5^4 B_6^6 B_3^{10} + 200704 B_1^3 B_2^7 B_4^7 B_5^3 B_6^6 B_3^{10} +$
$36750 B_1^4 B_2^7 B_4^6 B_5^3 B_6^6 B_3^{10} + 26950 B_1^3 B_2^6 B_4^8 B_5^4 B_6^5 B_3^{10} + 1539384 B_1^3 B_2^7 B_4^7 B_5^4 B_6^5 B_3^{10} +$
$202125 B_1^4 B_2^6 B_4^7 B_5^4 B_6^5 B_3^{10} + 202125 B_1^4 B_2^7 B_4^6 B_5^4 B_6^5 B_3^{10} + 1539384 B_1^4 B_2^7 B_4^7 B_5^3 B_6^5 B_3^{10} +$
$26950 B_1^4 B_2^8 B_4^6 B_5^3 B_6^5 B_3^{10} + 148225 B_1^4 B_2^7 B_4^7 B_5^4 B_6^4 B_3^{10} + 916839 B_1^4 B_2^7 B_4^7 B_5^4 B_6^5 B_3^9)\mathbf{Z}^{36} +$
$(211680 B_1^3 B_2^6 B_4^7 B_5^3 B_6^6 B_3^{10} + 211680 B_1^3 B_2^7 B_4^6 B_5^3 B_6^6 B_3^{10} + 1853280 B_1^3 B_2^6 B_4^7 B_5^4 B_6^5 B_3^{10} +$
$1164240 B_1^3 B_2^7 B_4^6 B_5^4 B_6^5 B_3^{10} + 6044544 B_1^3 B_2^7 B_4^7 B_5^3 B_6^5 B_3^{10} + 1164240 B_1^4 B_2^6 B_4^7 B_5^3 B_6^5 B_3^{10} +$
$1853280 B_1^4 B_2^7 B_4^6 B_5^3 B_6^5 B_3^{10} + 853776 B_1^3 B_2^7 B_4^7 B_5^4 B_6^4 B_3^{10} + 853776 B_1^4 B_2^7 B_4^7 B_5^3 B_6^4 B_3^{10} +$

Appendix: the full set of E_6 polynomials V

$$
\begin{aligned}
&4527600 B_1^3 B_2^7 B_4^7 B_5^4 B_6^5 B_3^9 + 1358280 B_1^4 B_2^6 B_4^7 B_5^4 B_6^5 B_3^9 + 1358280 B_1^4 B_2^7 B_4^6 B_5^4 B_6^5 B_3^9 + \\
&4527600 B_1^4 B_2^7 B_4^7 B_5^3 B_6^5 B_3^9 + 996072 B_1^4 B_2^7 B_4^7 B_5^4 B_6^4 B_3^9)\mathbf{Z}^{35} + (396900 B_1^3 B_2^6 B_4^6 B_5^3 B_6^6 B_3^{10} + \\
&291060 B_1^2 B_2^6 B_4^7 B_5^4 B_6^5 B_3^{10} + 2182950 B_1^3 B_2^6 B_4^6 B_5^4 B_6^5 B_3^{10} + 9168390 B_1^3 B_2^6 B_4^7 B_5^3 B_6^5 B_3^{10} + \\
&9168390 B_1^3 B_2^7 B_4^6 B_5^3 B_6^5 B_3^{10} + 2182950 B_1^4 B_2^6 B_4^6 B_5^3 B_6^5 B_3^{10} + 291060 B_1^4 B_2^7 B_4^6 B_5^2 B_6^5 B_3^{10} + \\
&1600830 B_1^3 B_2^6 B_4^7 B_5^4 B_6^4 B_3^{10} + 5336100 B_1^3 B_2^7 B_4^7 B_5^3 B_6^4 B_3^{10} + 1600830 B_1^4 B_2^7 B_4^6 B_5^3 B_6^4 B_3^{10} + \\
&13222440 B_1^3 B_2^6 B_4^7 B_5^4 B_6^5 B_3^9 + 8489250 B_1^3 B_2^7 B_4^6 B_5^4 B_6^5 B_3^9 + 2546775 B_1^4 B_2^6 B_4^6 B_5^4 B_6^5 B_3^9 + \\
&23654400 B_1^3 B_2^7 B_4^7 B_5^3 B_6^5 B_3^9 + 8489250 B_1^4 B_2^6 B_4^7 B_5^3 B_6^5 B_3^9 + 13222440 B_1^4 B_2^7 B_4^6 B_5^3 B_6^5 B_3^9 + \\
&6225450 B_1^3 B_2^7 B_4^7 B_5^4 B_6^4 B_3^9 + 1867635 B_1^4 B_2^6 B_4^7 B_5^4 B_6^4 B_3^9 + 1867635 B_1^4 B_2^7 B_4^6 B_5^4 B_6^4 B_3^9 + \\
&6225450 B_1^4 B_2^7 B_4^7 B_5^3 B_6^4 B_3^9)\mathbf{Z}^{34} + (2069760 B_1^2 B_2^6 B_4^7 B_5^3 B_6^5 B_3^{10} + 24147200 B_1^3 B_2^6 B_4^6 B_5^3 B_6^5 B_3^{10} + \\
&2069760 B_1^3 B_2^7 B_4^6 B_5^2 B_6^5 B_3^{10} + 11383680 B_1^3 B_2^6 B_4^7 B_5^3 B_6^4 B_3^{10} + 11383680 B_1^3 B_2^7 B_4^6 B_5^3 B_6^4 B_3^{10} + \\
&205800 B_1^3 B_2^6 B_4^6 B_5^3 B_6^6 B_3^9 + 3773000 B_1^2 B_2^6 B_4^7 B_5^4 B_6^5 B_3^9 + 9240000 B_1^3 B_2^5 B_4^7 B_5^4 B_6^5 B_3^9 + \\
&37560600 B_1^3 B_2^6 B_4^6 B_5^4 B_6^5 B_3^9 + 82222140 B_1^3 B_2^6 B_4^7 B_5^3 B_6^5 B_3^9 + 82222140 B_1^3 B_2^7 B_4^6 B_5^3 B_6^5 B_3^9 + \\
&37560600 B_1^4 B_2^6 B_4^6 B_5^3 B_6^5 B_3^9 + 9240000 B_1^4 B_2^7 B_4^5 B_5^3 B_6^5 B_3^9 + 3773000 B_1^4 B_2^7 B_4^6 B_5^2 B_6^5 B_3^9 + \\
&27544440 B_1^3 B_2^6 B_4^7 B_5^4 B_6^4 B_3^9 + 13280960 B_1^3 B_2^7 B_4^6 B_5^4 B_6^4 B_3^9 + 6225450 B_1^4 B_2^6 B_4^6 B_5^4 B_6^4 B_3^9 + \\
&41164200 B_1^3 B_2^7 B_4^7 B_5^3 B_6^4 B_3^9 + 13280960 B_1^4 B_2^6 B_4^7 B_5^3 B_6^4 B_3^9 + 27544440 B_1^4 B_2^7 B_4^6 B_5^3 B_6^4 B_3^9)\mathbf{Z}^{33} + \\
&(5588352 B_1^2 B_2^6 B_4^6 B_5^3 B_6^5 B_3^{10} + 5588352 B_1^3 B_2^6 B_4^6 B_5^2 B_6^5 B_3^{10} + 30735936 B_1^3 B_2^6 B_4^6 B_5^3 B_6^4 B_3^{10} + \\
&5197500 B_1^2 B_2^5 B_4^7 B_5^4 B_6^5 B_3^9 + 10187100 B_1^2 B_2^6 B_4^6 B_5^4 B_6^5 B_3^9 + 38981250 B_1^3 B_2^5 B_4^6 B_5^4 B_6^5 B_3^9 + \\
&28385280 B_1^2 B_2^6 B_4^7 B_5^3 B_6^5 B_3^9 + 63669375 B_1^3 B_2^5 B_4^7 B_5^3 B_6^5 B_3^9 + 440527626 B_1^3 B_2^6 B_4^6 B_5^3 B_6^5 B_3^9 + \\
&63669375 B_1^3 B_2^7 B_4^5 B_5^3 B_6^5 B_3^9 + 38981250 B_1^4 B_2^6 B_4^5 B_5^3 B_6^5 B_3^9 + 28385280 B_1^3 B_2^7 B_4^6 B_5^2 B_6^5 B_3^9 + \\
&10187100 B_1^4 B_2^6 B_4^6 B_5^2 B_6^5 B_3^9 + 5197500 B_1^4 B_2^7 B_4^5 B_5^2 B_6^5 B_3^9 + 7470540 B_1^2 B_2^6 B_4^7 B_5^4 B_6^4 B_3^9 + \\
&28586250 B_1^3 B_2^5 B_4^7 B_5^4 B_6^4 B_3^9 + 87268104 B_1^3 B_2^6 B_4^6 B_5^4 B_6^4 B_3^9 + 202848030 B_1^3 B_2^6 B_4^7 B_5^3 B_6^4 B_3^9 + \\
&202848030 B_1^3 B_2^7 B_4^6 B_5^3 B_6^4 B_3^9 + 87268104 B_1^4 B_2^6 B_4^6 B_5^3 B_6^4 B_3^9 + 28586250 B_1^4 B_2^7 B_4^5 B_5^3 B_6^4 B_3^9 + \\
&7470540 B_1^4 B_2^7 B_4^6 B_5^2 B_6^4 B_3^9 + 11884950 B_1^3 B_2^6 B_4^6 B_5^4 B_6^5 B_3^8 + 11884950 B_1^4 B_2^6 B_4^6 B_5^3 B_6^5 B_3^8 + \\
&8715630 B_1^3 B_2^6 B_4^7 B_5^4 B_6^4 B_3^8 + 2614689 B_1^4 B_2^6 B_4^6 B_5^4 B_6^4 B_3^8 + 8715630 B_1^4 B_2^7 B_4^6 B_5^3 B_6^4 B_3^8)\mathbf{Z}^{32} + \\
&(24948000 B_1^2 B_2^5 B_4^6 B_5^4 B_6^5 B_3^9 + 40748400 B_1^2 B_2^5 B_4^7 B_5^3 B_6^5 B_3^9 + 177031008 B_1^2 B_2^6 B_4^6 B_5^3 B_6^5 B_3^9 +
\end{aligned}
$$

Appendix: the full set of E_6 polynomials VI

$$467082000B_1^3B_2^5B_4^6B_5^3B_6^5B_3^9 + 467082000B_1^3B_2^6B_4^5B_5^3B_6^5B_3^9 + 177031008B_1^3B_2^6B_4^6B_5^2B_6^5B_3^9 +$$
$$40748400B_1^3B_2^7B_4^5B_5^2B_6^5B_3^9 + 24948000B_1^4B_2^6B_4^5B_5^2B_6^5B_3^9 + 18295200B_1^2B_2^5B_4^7B_5^4B_6^4B_3^9 +$$
$$35858592B_1^2B_2^6B_4^6B_5^4B_6^4B_3^9 + 137214000B_1^3B_2^5B_4^6B_5^4B_6^4B_3^9 + 60984000B_1^2B_2^6B_4^7B_5^3B_6^4B_3^9 +$$
$$224116200B_1^3B_2^5B_4^7B_5^3B_6^4B_3^9 + 1339753968B_1^3B_2^6B_4^6B_5^3B_6^4B_3^9 + 224116200B_1^3B_2^7B_4^5B_5^3B_6^4B_3^9 +$$
$$137214000B_1^4B_2^6B_4^5B_5^3B_6^4B_3^9 + 60984000B_1^3B_2^7B_4^6B_5^2B_6^4B_3^9 + 35858592B_1^4B_2^6B_4^6B_5^2B_6^4B_3^9 +$$
$$18295200B_1^4B_2^7B_4^5B_5^2B_6^4B_3^9 + 29106000B_1^3B_2^5B_4^6B_5^4B_6^5B_3^8 + 191866752B_1^3B_2^6B_4^6B_5^3B_6^5B_3^8 +$$
$$29106000B_1^4B_2^6B_4^5B_5^3B_6^5B_3^8 + 21344400B_1^3B_2^5B_4^7B_5^4B_6^4B_3^8 + 66594528B_1^3B_2^6B_4^6B_5^4B_6^4B_3^8 +$$
$$71148000B_1^3B_2^6B_4^7B_5^3B_6^4B_3^8 + 71148000B_1^3B_2^7B_4^6B_5^3B_6^4B_3^8 + 66594528B_1^4B_2^6B_4^6B_5^3B_6^4B_3^8 +$$
$$21344400B_1^4B_2^7B_4^5B_5^3B_6^4B_3^8)\mathbf{Z}^{31} + (353089660B_1^2B_2^5B_4^6B_5^3B_6^5B_3^9 + 247546530B_1^2B_2^6B_4^5B_5^3B_6^5B_3^9 +$$
$$707437500B_1^3B_2^5B_4^5B_5^3B_6^5B_3^9 + 60555264B_1^2B_2^6B_4^6B_5^2B_6^5B_3^9 + 247546530B_1^3B_2^5B_4^6B_5^2B_6^5B_3^9 +$$
$$353089660B_1^3B_2^6B_4^5B_5^2B_6^5B_3^9 + 111143340B_1^2B_2^5B_4^6B_5^4B_6^4B_3^9 + 155636250B_1^2B_2^5B_4^7B_5^3B_6^4B_3^9 +$$
$$542666124B_1^2B_2^6B_4^6B_5^3B_6^4B_3^9 + 1941993130B_1^3B_2^5B_4^6B_5^3B_6^4B_3^9 + 1941993130B_1^3B_2^6B_4^5B_5^3B_6^4B_3^9 +$$
$$542666124B_1^3B_2^6B_4^6B_5^2B_6^4B_3^9 + 155636250B_1^3B_2^7B_4^5B_5^2B_6^4B_3^9 + 111143340B_1^4B_2^6B_4^5B_5^2B_6^4B_3^9 +$$
$$5478396B_1^3B_2^6B_4^6B_5^3B_6^3B_3^9 + 20212500B_1^2B_2^5B_4^6B_5^4B_6^5B_3^8 + 110387200B_1^2B_2^6B_4^6B_5^3B_6^5B_3^8 +$$
$$388031490B_1^3B_2^5B_4^6B_5^3B_6^5B_3^8 + 388031490B_1^3B_2^6B_4^5B_5^3B_6^5B_3^8 + 110387200B_1^3B_2^6B_4^6B_5^2B_6^5B_3^8 +$$
$$20212500B_1^4B_2^6B_4^5B_5^2B_6^5B_3^8 + 14822500B_1^2B_2^5B_4^7B_5^4B_6^4B_3^8 + 29052100B_1^2B_2^6B_4^6B_5^4B_6^4B_3^8 +$$
$$253998360B_1^3B_2^5B_4^6B_5^4B_6^4B_3^8 + 8715630B_1^3B_2^6B_4^5B_5^4B_6^4B_3^8 + 181575625B_1^3B_2^5B_4^7B_5^3B_6^4B_3^8 +$$
$$1554121926B_1^3B_2^6B_4^6B_5^3B_6^4B_3^8 + 8715630B_1^4B_2^5B_4^6B_5^3B_6^4B_3^8 + 181575625B_1^3B_2^7B_4^5B_5^3B_6^4B_3^8 +$$
$$253998360B_1^4B_2^6B_4^5B_5^3B_6^4B_3^8 + 29052100B_1^4B_2^6B_4^6B_5^2B_6^4B_3^8 + 14822500B_1^4B_2^7B_4^5B_5^2B_6^4B_3^8 +$$
$$6391462B_1^3B_2^6B_4^6B_5^4B_6^3B_3^8 + 6391462B_1^4B_2^6B_4^6B_5^3B_6^3B_3^8)\mathbf{Z}^{30} + (651974400B_1^2B_2^5B_4^5B_5^3B_6^5B_3^9 +$$
$$195592320B_1^2B_2^5B_4^6B_5^2B_6^5B_3^9 + 195592320B_1^2B_2^6B_4^5B_5^2B_6^5B_3^9 + 651974400B_1^3B_2^5B_4^5B_5^2B_6^5B_3^9 +$$
$$1644128640B_1^2B_2^5B_4^6B_5^3B_6^4B_3^9 + 1075757760B_1^2B_2^6B_4^5B_5^3B_6^4B_3^9 + 3585859200B_1^3B_2^5B_4^5B_5^3B_6^4B_3^9 +$$
$$292723200B_1^2B_2^6B_4^6B_5^2B_6^4B_3^9 + 1075757760B_1^3B_2^5B_4^6B_5^2B_6^4B_3^9 + 1644128640B_1^3B_2^6B_4^5B_5^2B_6^4B_3^9 +$$
$$413887320B_1^2B_2^5B_4^6B_5^3B_6^5B_3^8 + 356548500B_1^2B_2^6B_4^5B_5^3B_6^5B_3^8 + 1189465200B_1^3B_2^5B_4^5B_5^3B_6^5B_3^8 +$$
$$356548500B_1^3B_2^5B_4^6B_5^2B_6^5B_3^8 + 413887320B_1^3B_2^6B_4^5B_5^2B_6^5B_3^8 + 268939440B_1^2B_2^5B_4^6B_5^4B_6^4B_3^8 +$$

Appendix: the full set of E_6 polynomials VII

$$
\begin{aligned}
&48024900B_1^3B_2^5B_4^5B_5^4B_6^4B_3^8 + 133402500B_1^2B_2^5B_4^7B_5^3B_6^4B_3^8 + 672348600B_1^2B_2^6B_4^6B_5^3B_6^4B_3^8 +\\
&4320547560B_1^3B_2^5B_4^6B_5^3B_6^4B_3^8 + 4320547560B_1^3B_2^6B_4^5B_5^3B_6^4B_3^8 + 48024900B_1^4B_2^5B_4^5B_5^3B_6^4B_3^8 +\\
&672348600B_1^3B_2^6B_4^6B_5^2B_6^4B_3^8 + 133402500B_1^3B_2^7B_4^5B_5^2B_6^4B_3^8 + 268939440B_1^4B_2^6B_4^5B_5^2B_6^4B_3^8 +\\
&35218260B_1^3B_2^5B_4^6B_5^4B_6^3B_3^8 + 180457200B_1^3B_2^6B_4^6B_5^3B_6^3B_3^8 + 35218260B_1^4B_2^6B_4^5B_5^3B_6^3B_3^8 +\\
&119528640B_1^3B_2^5B_4^6B_5^4B_6^4B_3^7 + 398428800B_1^3B_2^6B_4^6B_5^3B_6^4B_3^7 + 119528640B_1^4B_2^6B_4^5B_5^3B_6^4B_3^7)\mathbf{Z}^{29} +\\
&(651974400B_1^2B_2^5B_4^5B_5^2B_6^5B_3^9 + 3585859200B_1^2B_2^5B_4^5B_5^3B_6^4B_3^9 + 1075757760B_1^2B_2^5B_4^6B_5^2B_6^4B_3^9 +\\
&1075757760B_1^2B_2^6B_4^5B_5^2B_6^4B_3^9 + 3585859200B_1^3B_2^5B_4^5B_5^2B_6^4B_3^9 + 54573750B_1^2B_2^4B_4^6B_5^3B_6^5B_3^8 +\\
&1671169500B_1^2B_2^5B_4^5B_5^3B_6^5B_3^8 + 298045440B_1^2B_2^5B_4^6B_5^2B_6^5B_3^8 + 298045440B_1^2B_2^6B_4^5B_5^2B_6^5B_3^8 +\\
&1671169500B_1^3B_2^5B_4^5B_5^2B_6^5B_3^8 + 54573750B_1^3B_2^6B_4^4B_5^2B_6^5B_3^8 + 24502500B_1^2B_2^4B_4^6B_5^4B_6^4B_3^8 +\\
&48024900B_1^2B_2^5B_4^5B_5^4B_6^4B_3^8 + 4609356210B_1^2B_2^5B_4^6B_5^3B_6^4B_3^8 + 300155625B_1^3B_2^4B_4^6B_5^3B_6^4B_3^8 +\\
&3054383640B_1^2B_2^6B_4^5B_5^3B_6^4B_3^8 + 14283282150B_1^3B_2^5B_4^5B_5^3B_6^4B_3^8 + 300155625B_1^3B_2^6B_4^4B_5^3B_6^4B_3^8 +\\
&446054400B_1^2B_2^6B_4^6B_5^2B_6^4B_3^8 + 3054383640B_1^3B_2^5B_4^6B_5^2B_6^4B_3^8 + 4609356210B_1^3B_2^6B_4^5B_5^2B_6^4B_3^8 +\\
&48024900B_1^4B_2^5B_4^5B_5^2B_6^4B_3^8 + 24502500B_1^4B_2^6B_4^4B_5^2B_6^4B_3^8 + 35218260B_1^2B_2^5B_4^6B_5^4B_6^3B_3^8 +\\
&117394200B_1^2B_2^6B_4^6B_5^3B_6^3B_3^8 + 607296690B_1^3B_2^5B_4^6B_5^3B_6^3B_3^8 + 607296690B_1^3B_2^6B_4^5B_5^3B_6^3B_3^8 +\\
&117394200B_1^3B_2^6B_4^6B_5^2B_6^3B_3^8 + 35218260B_1^4B_2^6B_4^5B_5^2B_6^3B_3^8 + 499167900B_1^3B_2^5B_4^5B_5^3B_6^5B_3^7 +\\
&186763500B_1^2B_2^5B_4^6B_5^4B_6^4B_3^7 + 56029050B_1^3B_2^5B_4^5B_5^4B_6^4B_3^7 + 2655776970B_1^3B_2^5B_4^6B_5^3B_6^4B_3^7 +\\
&2655776970B_1^3B_2^6B_4^5B_5^3B_6^4B_3^7 + 56029050B_1^4B_2^5B_4^5B_5^3B_6^4B_3^7 + 186763500B_1^4B_2^6B_4^5B_5^2B_6^4B_3^7 +\\
&41087970B_1^3B_2^5B_4^6B_5^4B_6^3B_3^7 + 136959900B_1^3B_2^6B_4^6B_5^3B_6^3B_3^7 + 41087970B_1^4B_2^6B_4^5B_5^3B_6^3B_3^7)\mathbf{Z}^{28} +\\
&(4079910912B_1^2B_2^5B_4^5B_5^2B_6^4B_3^9 + 323400000B_1^2B_2^4B_4^5B_5^3B_6^5B_3^8 + 2253071744B_1^2B_2^5B_4^5B_5^2B_6^5B_3^8 +\\
&323400000B_1^3B_2^5B_4^4B_5^2B_6^5B_3^8 + 11383680B_1B_2^5B_4^6B_5^3B_6^4B_3^8 + 830060000B_1^2B_2^4B_4^6B_5^3B_6^4B_3^8 +\\
&18476731056B_1^2B_2^5B_4^5B_5^3B_6^4B_3^8 + 1778700000B_1^3B_2^4B_4^5B_5^3B_6^4B_3^8 + 1778700000B_1^3B_2^5B_4^4B_5^3B_6^4B_3^8 +\\
&4063973760B_1^2B_2^5B_4^6B_5^2B_6^4B_3^8 + 4063973760B_1^2B_2^6B_4^5B_5^2B_6^4B_3^8 + 18476731056B_1^3B_2^5B_4^5B_5^2B_6^4B_3^8 +\\
&830060000B_1^3B_2^6B_4^4B_5^2B_6^4B_3^8 + 11383680B_1^3B_2^6B_4^5B_5B_6^4B_3^8 + 871627680B_1^2B_2^5B_4^6B_5^3B_6^3B_3^8 +\\
&766975440B_1^2B_2^6B_4^5B_5^3B_6^3B_3^8 + 2414513024B_1^3B_2^5B_4^5B_5^3B_6^3B_3^8 + 766975440B_1^3B_2^5B_4^6B_5^2B_6^3B_3^8 +\\
&871627680B_1^3B_2^6B_4^5B_5^2B_6^3B_3^8 + 887409600B_1^2B_2^5B_4^5B_5^3B_6^5B_3^7 + 887409600B_1^3B_2^5B_4^5B_5^2B_6^5B_3^7 +
\end{aligned}
$$

Appendix: the full set of E_6 polynomials VIII

$$
\begin{aligned}
&50820000B_1^2B_2^4B_4^6B_5^4B_6^4B_3^7 + 99607200B_1^2B_2^5B_4^5B_5^4B_6^4B_3^7 + 3252073440B_1^2B_2^5B_4^6B_5^3B_6^4B_3^7 + \\
&622545000B_1^3B_2^4B_4^6B_5^3B_6^4B_3^7 + 2075150000B_1^2B_2^6B_4^5B_5^3B_6^4B_3^7 + 19480302576B_1^3B_2^5B_4^5B_5^3B_6^4B_3^7 + \\
&622545000B_1^3B_2^6B_4^4B_5^3B_6^4B_3^7 + 2075150000B_1^3B_2^5B_4^6B_5^2B_6^4B_3^7 + 3252073440B_1^3B_2^6B_4^5B_5^2B_6^4B_3^7 + \\
&99607200B_1^4B_2^5B_4^5B_5^2B_6^4B_3^7 + 50820000B_1^4B_2^6B_4^4B_5^2B_6^4B_3^7 + 73045280B_1^2B_2^5B_4^6B_5^4B_6^3B_3^7 + \\
&21913584B_1^3B_2^5B_4^5B_5^4B_6^3B_3^7 + 1016898960B_1^3B_2^5B_4^6B_5^3B_6^3B_3^7 + 1016898960B_1^3B_2^6B_4^5B_5^3B_6^3B_3^7 + \\
&21913584B_1^4B_2^5B_4^5B_5^3B_6^3B_3^7 + 73045280B_1^4B_2^6B_4^5B_5^2B_6^3B_3^7)\mathbf{Z}^{27} + (356548500B_1^2B_2^4B_4^5B_5^2B_6^5B_3^8 + \\
&356548500B_1^2B_2^5B_4^4B_5^2B_6^5B_3^8 + 48024900B_1B_2^4B_4^6B_5^3B_6^4B_3^8 + 94128804B_1B_2^5B_4^5B_5^3B_6^4B_3^8 + \\
&5042614500B_1^2B_2^4B_4^5B_5^3B_6^4B_3^8 + 1961016750B_1^2B_2^5B_4^4B_5^3B_6^4B_3^8 + 588305025B_1^2B_2^4B_4^6B_5^2B_6^4B_3^8 + \\
&27835512516B_1^2B_2^5B_4^5B_5^2B_6^4B_3^8 + 1961016750B_1^3B_2^4B_4^5B_5^2B_6^4B_3^8 + 588305025B_1^2B_2^6B_4^4B_5^2B_6^4B_3^8 + \\
&5042614500B_1^3B_2^5B_4^4B_5^2B_6^4B_3^8 + 94128804B_1^3B_2^5B_4^5B_5B_6^4B_3^8 + 48024900B_1^3B_2^6B_4^4B_5B_6^4B_3^8 + \\
&264136950B_1^2B_2^4B_4^6B_5^3B_6^3B_3^8 + 4790284884B_1^2B_2^5B_4^5B_5^3B_6^3B_3^8 + 880456500B_1^2B_2^5B_4^6B_5^2B_6^3B_3^8 + \\
&880456500B_1^2B_2^6B_4^5B_5^2B_6^3B_3^8 + 4790284884B_1^3B_2^5B_4^5B_5^2B_6^3B_3^8 + 264136950B_1^3B_2^6B_4^4B_5^2B_6^3B_3^8 + \\
&305613000B_1^2B_2^4B_4^5B_5^3B_6^5B_3^7 + 1669054464B_1^2B_2^5B_4^5B_5^2B_6^5B_3^7 + 305613000B_1^3B_2^5B_4^4B_5^2B_6^5B_3^7 + \\
&34303500B_1^2B_2^4B_4^5B_5^4B_6^4B_3^7 + 1413304200B_1^2B_2^4B_4^6B_5^3B_6^4B_3^7 + 30824064054B_1^2B_2^5B_4^5B_5^3B_6^4B_3^7 + \\
&6565308750B_1^3B_2^4B_4^5B_5^3B_6^4B_3^7 + 6565308750B_1^3B_2^5B_4^4B_5^3B_6^4B_3^7 + 3902976000B_1^2B_2^5B_4^6B_5^2B_6^4B_3^7 + \\
&3902976000B_1^2B_2^6B_4^5B_5^2B_6^4B_3^7 + 30824064054B_1^3B_2^5B_4^5B_5^2B_6^4B_3^7 + 1413304200B_1^3B_2^6B_4^4B_5^2B_6^4B_3^7 + \\
&34303500B_1^4B_2^5B_4^4B_5^2B_6^4B_3^7 + 25155900B_1^2B_2^4B_4^6B_5^4B_6^3B_3^7 + 49305564B_1^2B_2^5B_4^5B_5^4B_6^3B_3^7 + \\
&1445944500B_1^2B_2^5B_4^6B_5^3B_6^3B_3^7 + 308159775B_1^3B_2^4B_4^6B_5^3B_6^3B_3^7 + 1027199250B_1^2B_2^6B_4^5B_5^3B_6^3B_3^7 + \\
&8951787306B_1^3B_2^5B_4^5B_5^3B_6^3B_3^7 + 308159775B_1^3B_2^6B_4^4B_5^3B_6^3B_3^7 + 1027199250B_1^3B_2^5B_4^6B_5^2B_6^3B_3^7 + \\
&1445944500B_1^3B_2^6B_4^5B_5^2B_6^3B_3^7 + 49305564B_1^4B_2^5B_4^5B_5^2B_6^3B_3^7 + 25155900B_1^4B_2^6B_4^4B_5^2B_6^3B_3^7 + \\
&8199664704B_1^3B_2^5B_4^5B_5^3B_6^4B_3^6)\mathbf{Z}^{26} + (409812480B_1B_2^4B_4^5B_5^3B_6^4B_3^8 + 122943744B_1B_2^5B_4^5B_5^2B_6^4B_3^8 + \\
&7991343360B_1^2B_2^4B_4^5B_5^2B_6^4B_3^8 + 7991343360B_1^2B_2^5B_4^4B_5^2B_6^4B_3^8 + 122943744B_1^2B_2^5B_4^5B_5B_6^4B_3^8 + \\
&409812480B_1^3B_2^5B_4^4B_5B_6^4B_3^8 + 2253968640B_1^2B_2^4B_4^5B_5^3B_6^3B_3^8 + 8611029504B_1^2B_2^5B_4^5B_5^2B_6^3B_3^8 + \\
&2253968640B_1^3B_2^5B_4^4B_5^2B_6^3B_3^8 + 599001480B_1^2B_2^4B_4^5B_5^2B_6^5B_3^7 + 599001480B_1^2B_2^5B_4^4B_5^2B_6^5B_3^7 + \\
&114345000B_1B_2^4B_4^6B_5^3B_6^4B_3^7 + 224116200B_1B_2^5B_4^5B_5^3B_6^4B_3^7 + 19609868880B_1^2B_2^4B_4^5B_5^3B_6^4B_3^7 +
\end{aligned}
$$

Appendix: the full set of E_6 polynomials IX

$$
\begin{aligned}
&9158882040B_1^2B_2^5B_4^4B_5^3B_6^4B_3^7 + 2858625000B_1^3B_2^4B_4^4B_5^3B_6^4B_3^7 + 1400726250B_1^2B_2^4B_4^6B_5^2B_6^4B_3^7 + \\
&59798117736B_1^2B_2^5B_4^5B_5^2B_6^4B_3^7 + 9158882040B_1^3B_2^4B_4^5B_5^2B_6^4B_3^7 + 1400726250B_1^2B_2^6B_4^4B_5^2B_6^4B_3^7 + \\
&19609868880B_1^3B_2^5B_4^4B_5^2B_6^4B_3^7 + 224116200B_1^3B_2^5B_4^5B_5B_6^4B_3^7 + 114345000B_1^3B_2^6B_4^4B_5B_6^4B_3^7 + \\
&30187080B_1^2B_2^4B_4^5B_5^4B_6^3B_3^7 + 966735000B_1^2B_2^4B_4^6B_5^3B_6^3B_3^7 + 17946116496B_1^2B_2^5B_4^5B_5^3B_6^3B_3^7 + \\
&3421632060B_1^3B_2^4B_4^5B_5^3B_6^3B_3^7 + 3421632060B_1^3B_2^5B_4^4B_5^3B_6^3B_3^7 + 2096325000B_1^2B_2^5B_4^6B_5^2B_6^3B_3^7 + \\
&2096325000B_1^2B_2^6B_4^5B_5^2B_6^3B_3^7 + 17946116496B_1^3B_2^5B_4^5B_5^2B_6^3B_3^7 + 966735000B_1^3B_2^6B_4^4B_5^2B_6^3B_3^7 + \\
&30187080B_1^4B_2^5B_4^4B_5^2B_6^3B_3^7 + 439267752B_1^3B_2^5B_4^5B_5^3B_6^2B_3^7 + 16734009600B_1^2B_2^5B_4^5B_5^3B_6^4B_3^6 + \\
&5020202880B_1^3B_2^4B_4^5B_5^3B_6^4B_3^6 + 5020202880B_1^3B_2^5B_4^4B_5^3B_6^4B_3^6 + 16734009600B_1^3B_2^5B_4^5B_5^2B_6^4B_3^6 + \\
&6754454784B_1^3B_2^5B_4^5B_5^3B_6^3B_3^6)\mathbf{Z}^{25} + (557800320B_1B_2^4B_4^5B_5^2B_6^4B_3^8 + 1859334400B_1^2B_2^4B_4^4B_5^2B_6^4B_3^8 + \\
&557800320B_1^2B_2^5B_4^4B_5B_6^4B_3^8 + 3067901760B_1^2B_2^4B_4^5B_5^2B_6^3B_3^8 + 3067901760B_1^2B_2^5B_4^4B_5^2B_6^3B_3^8 + \\
&221852400B_1^2B_2^4B_4^4B_5^2B_6^5B_3^7 + 2212838320B_1B_2^4B_4^5B_5^3B_6^4B_3^7 + 1985156250B_1^2B_2^3B_4^5B_5^3B_6^4B_3^7 + \\
&6097637700B_1^2B_2^4B_4^4B_5^3B_6^4B_3^7 + 520396800B_1B_2^5B_4^5B_5^2B_6^4B_3^7 + 40830215370B_1^2B_2^4B_4^5B_5^2B_6^4B_3^7 + \\
&40830215370B_1^2B_2^5B_4^4B_5^2B_6^4B_3^7 + 6097637700B_1^3B_2^4B_4^4B_5^2B_6^4B_3^7 + 1985156250B_1^3B_2^5B_4^3B_5^2B_6^4B_3^7 + \\
&520396800B_1^2B_2^5B_4^5B_5B_6^4B_3^7 + 2212838320B_1^3B_2^5B_4^4B_5B_6^4B_3^7 + 69877500B_1B_2^4B_4^6B_5^3B_6^3B_3^7 + \\
&136959900B_1B_2^5B_4^5B_5^3B_6^3B_3^7 + 16980581310B_1^2B_2^4B_4^5B_5^3B_6^3B_3^7 + 5281747240B_1^2B_2^5B_4^4B_5^3B_6^3B_3^7 + \\
&2862182400B_1^3B_2^4B_4^4B_5^3B_6^3B_3^7 + 855999375B_1^2B_2^4B_4^6B_5^2B_6^3B_3^7 + 41763159900B_1^2B_2^5B_4^5B_5^2B_6^3B_3^7 + \\
&5281747240B_1^3B_2^4B_4^5B_5^2B_6^3B_3^7 + 855999375B_1^2B_2^6B_4^4B_5^2B_6^3B_3^7 + 16980581310B_1^3B_2^5B_4^4B_5^2B_6^3B_3^7 + \\
&136959900B_1^3B_2^5B_4^5B_5B_6^3B_3^7 + 69877500B_1^3B_2^6B_4^4B_5B_6^3B_3^7 + 1220188200B_1^2B_2^5B_4^5B_5^3B_6^2B_3^7 + \\
&1220188200B_1^3B_2^5B_4^5B_5^2B_6^2B_3^7 + 18993314340B_1^2B_2^4B_4^5B_5^3B_6^4B_3^6 + 10676646750B_1^2B_2^5B_4^4B_5^3B_6^4B_3^6 + \\
&3890016900B_1^3B_2^4B_4^4B_5^3B_6^4B_3^6 + 38856294400B_1^2B_2^5B_4^5B_5^2B_6^4B_3^6 + 10676646750B_1^3B_2^4B_4^5B_5^2B_6^4B_3^6 + \\
&18993314340B_1^3B_2^5B_4^4B_5^2B_6^4B_3^6 + 3913140B_1^2B_2^4B_4^5B_5^4B_6^3B_3^6 + 81523750B_1^2B_2^4B_4^6B_5^3B_6^3B_3^6 + \\
&16798204500B_1^2B_2^5B_4^5B_5^3B_6^3B_3^6 + 5039461350B_1^3B_2^4B_4^5B_5^3B_6^3B_3^6 + 5039461350B_1^3B_2^5B_4^4B_5^3B_6^3B_3^6 + \\
&16798204500B_1^3B_2^5B_4^5B_5^2B_6^3B_3^6 + 81523750B_1^3B_2^6B_4^4B_5^2B_6^3B_3^6 + 3913140B_1^4B_2^5B_4^4B_5^2B_6^3B_3^6 + \\
&1423552900B_1^3B_2^5B_4^5B_5^3B_6^2B_3^6)\mathbf{Z}^{24} + (457380000B_1B_2^3B_4^5B_5^3B_6^4B_3^7 + 896464800B_1B_2^4B_4^4B_5^3B_6^4B_3^7 + \\
&4201797600B_1B_2^4B_4^5B_5^2B_6^4B_3^7 + 5602905000B_1^2B_2^3B_4^5B_5^2B_6^4B_3^7 + 268939440B_1B_2^5B_4^4B_5^2B_6^4B_3^7 +
\end{aligned}
$$

$$32647658400 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^7 + 5602905000 B_1^2 B_2^5 B_4^3 B_5^2 B_6^4 B_3^7 + 268939440 B_1^2 B_2^4 B_4^5 B_5 B_6^4 B_3^7 +$$
$$4201797600 B_1^2 B_2^5 B_4^4 B_5 B_6^4 B_3^7 + 896464800 B_1^3 B_2^4 B_4^4 B_5 B_6^4 B_3^7 + 457380000 B_1^3 B_2^5 B_4^3 B_5 B_6^4 B_3^7 +$$
$$1537683840 B_1 B_2^4 B_4^5 B_5^3 B_6^3 B_3^7 + 2515590000 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^7 + 6940533600 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^7 +$$
$$402494400 B_1 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7 + 38328942960 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^7 + 38328942960 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7 +$$
$$6940533600 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^7 + 2515590000 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^7 + 402494400 B_1^2 B_2^5 B_4^5 B_5 B_6^3 B_3^7 +$$
$$1537683840 B_1^3 B_2^5 B_4^4 B_5 B_6^3 B_3^7 + 1075757760 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^7 + 3585859200 B_1^2 B_2^5 B_4^5 B_5^2 B_6^2 B_3^7 +$$
$$1075757760 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^7 + 2561328000 B_1 B_2^4 B_4^5 B_5^3 B_6^4 B_3^6 + 4802490000 B_1^2 B_2^3 B_4^5 B_5^3 B_6^4 B_3^6 +$$
$$14386125600 B_1^2 B_2^4 B_4^4 B_5^3 B_6^4 B_3^6 + 48870138240 B_1^2 B_2^4 B_4^5 B_5^2 B_6^4 B_3^6 + 48870138240 B_1^2 B_2^5 B_4^4 B_5^2 B_6^4 B_3^6 +$$
$$14386125600 B_1^3 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6 + 4802490000 B_1^3 B_2^5 B_4^3 B_5^2 B_6^4 B_3^6 + 2561328000 B_1^3 B_2^5 B_4^4 B_5 B_6^4 B_3^6 +$$
$$28131531120 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6 + 12664602720 B_1^2 B_2^5 B_4^4 B_5^3 B_6^3 B_3^6 + 6411081600 B_1^3 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6 +$$
$$45964195200 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6 + 12664602720 B_1^3 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 28131531120 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6 +$$
$$4183502400 B_1^2 B_2^5 B_4^5 B_5^3 B_6^2 B_3^6 + 1255050720 B_1^3 B_2^4 B_4^5 B_5^3 B_6^2 B_3^6 + 1255050720 B_1^3 B_2^5 B_4^4 B_5^3 B_6^2 B_3^6 +$$
$$4183502400 B_1^3 B_2^5 B_4^5 B_5^2 B_6^2 B_3^6)\mathbf{Z}^{23} + (1400726250 B_1 B_2^3 B_4^5 B_5^2 B_6^4 B_3^7 + 3186932364 B_1 B_2^4 B_4^4 B_5^2 B_6^4 B_3^7 +$$
$$4669087500 B_1^2 B_2^3 B_4^4 B_5^2 B_6^4 B_3^7 + 4669087500 B_1^2 B_2^4 B_4^3 B_5^2 B_6^4 B_3^7 + 3186932364 B_1^2 B_2^4 B_4^4 B_5 B_6^4 B_3^7 +$$
$$1400726250 B_1^2 B_2^5 B_4^3 B_5 B_6^4 B_3^7 + 628897500 B_1 B_2^3 B_4^5 B_5^3 B_6^3 B_3^7 + 1232639100 B_1 B_2^4 B_4^4 B_5^3 B_6^3 B_3^7 +$$
$$3421632060 B_1 B_2^4 B_4^5 B_5^2 B_6^3 B_3^7 + 7703994375 B_1^2 B_2^3 B_4^5 B_5^2 B_6^3 B_3^7 + 369791730 B_1 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7 +$$
$$38630489436 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^7 + 7703994375 B_1^2 B_2^5 B_4^3 B_5^2 B_6^3 B_3^7 + 369791730 B_1^2 B_2^4 B_4^5 B_5 B_6^3 B_3^7 +$$
$$3421632060 B_1^2 B_2^5 B_4^4 B_5 B_6^3 B_3^7 + 1232639100 B_1^3 B_2^4 B_4^4 B_5 B_6^3 B_3^7 + 628897500 B_1^3 B_2^5 B_4^3 B_5 B_6^3 B_3^7 +$$
$$3294508140 B_1^2 B_2^4 B_4^5 B_5^2 B_6^2 B_3^7 + 3294508140 B_1^2 B_2^5 B_4^4 B_5^2 B_6^2 B_3^7 + 1200622500 B_1 B_2^3 B_4^5 B_5^3 B_6^4 B_3^6 +$$
$$2353220100 B_1 B_2^4 B_4^4 B_5^3 B_6^4 B_3^6 + 4002075000 B_1^2 B_2^3 B_4^4 B_5^3 B_6^4 B_3^6 + 6556999680 B_1 B_2^4 B_4^5 B_5^2 B_6^4 B_3^6 +$$
$$14707625625 B_1^2 B_2^3 B_4^5 B_5^2 B_6^4 B_3^6 + 76881768516 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6 + 14707625625 B_1^2 B_2^5 B_4^3 B_5^2 B_6^4 B_3^6 +$$
$$4002075000 B_1^3 B_2^4 B_4^3 B_5^2 B_6^4 B_3^6 + 6556999680 B_1^2 B_2^5 B_4^4 B_5 B_6^4 B_3^6 + 2353220100 B_1^3 B_2^4 B_4^4 B_5 B_6^4 B_3^6 +$$
$$1200622500 B_1^3 B_2^5 B_4^3 B_5 B_6^4 B_3^6 + 3447314640 B_1 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6 + 10017315000 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^6 +$$
$$27874845306 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6 + 80030488230 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 80030488230 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6 +$$
$$27874845306 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 + 10017315000 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^6 + 3447314640 B_1^3 B_2^5 B_4^4 B_5 B_6^3 B_3^6 +$$

$$
\begin{aligned}
&8519617260 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^6 + 3843592830 B_1^2 B_2^5 B_4^4 B_5^3 B_6^2 B_3^6 + 1967099904 B_1^3 B_2^4 B_4^4 B_5^3 B_6^2 B_3^6 + \\
&13340250000 B_1^2 B_2^5 B_4^5 B_5^2 B_6^2 B_3^6 + 3843592830 B_1^3 B_2^4 B_4^5 B_5^2 B_6^2 B_3^6 + 8519617260 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^6 + \\
&2629630080 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^5 + 788889024 B_1^3 B_2^4 B_4^4 B_5^3 B_6^3 B_3^5 + 2629630080 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^5) \mathbf{Z}^{22} + \\
&(1328096000 B_1 B_2^3 B_4^4 B_5^2 B_6^4 B_3^7 + 398428800 B_1 B_2^4 B_4^4 B_5 B_6^4 B_3^7 + 1328096000 B_1^2 B_2^4 B_4^3 B_5 B_6^4 B_3^7 + \\
&2191358400 B_1 B_2^3 B_4^5 B_5^2 B_6^3 B_3^7 + 4881115008 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^7 + 7304528000 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^7 + \\
&7304528000 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^7 + 4881115008 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^7 + 2191358400 B_1^2 B_2^5 B_4^3 B_5 B_6^3 B_3^7 + \\
&3123681792 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^7 + 1138368000 B_1 B_2^3 B_4^4 B_5^3 B_6^4 B_3^6 + 3890016900 B_1 B_2^3 B_4^5 B_5^2 B_6^4 B_3^6 + \\
&10875161528 B_1 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6 + 28921662000 B_1^2 B_2^3 B_4^4 B_5^2 B_6^4 B_3^6 + 28921662000 B_1^2 B_2^4 B_4^3 B_5^2 B_6^4 B_3^6 + \\
&10875161528 B_1^2 B_2^4 B_4^4 B_5 B_6^4 B_3^6 + 3890016900 B_1^2 B_2^5 B_4^3 B_5 B_6^4 B_3^6 + 1138368000 B_1^3 B_2^4 B_4^3 B_5 B_6^4 B_3^6 + \\
&2752521200 B_1 B_2^3 B_4^5 B_5^3 B_6^3 B_3^6 + 5394941552 B_1 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6 + 10284615000 B_1^2 B_2^3 B_4^4 B_5^3 B_6^3 B_3^6 + \\
&97828500 B_1^2 B_2^4 B_4^3 B_5^3 B_6^3 B_3^6 + 10284615000 B_1 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 33718384700 B_1^2 B_2^3 B_4^5 B_5^2 B_6^3 B_3^6 + \\
&97828500 B_1 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6 + 163830961008 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 + 97828500 B_1^3 B_2^3 B_4^4 B_5^2 B_6^3 B_3^6 + \\
&33718384700 B_1^2 B_2^5 B_4^3 B_5^2 B_6^3 B_3^6 + 10284615000 B_1^3 B_2^4 B_4^3 B_5^2 B_6^3 B_3^6 + 97828500 B_1^2 B_2^4 B_4^5 B_5 B_6^3 B_3^6 + \\
&10284615000 B_1^2 B_2^5 B_4^4 B_5 B_6^3 B_3^6 + 5394941552 B_1^3 B_2^4 B_4^4 B_5 B_6^3 B_3^6 + 2752521200 B_1^3 B_2^5 B_4^3 B_5 B_6^3 B_3^6 + \\
&1138368000 B_1 B_2^4 B_4^5 B_5^3 B_6^2 B_3^6 + 3890016900 B_1^2 B_2^3 B_4^5 B_5^3 B_6^2 B_3^6 + 10875161528 B_1^2 B_2^4 B_4^4 B_5^3 B_6^2 B_3^6 + \\
&28921662000 B_1^2 B_2^4 B_4^5 B_5^2 B_6^2 B_3^6 + 28921662000 B_1^2 B_2^5 B_4^4 B_5^2 B_6^2 B_3^6 + 10875161528 B_1^3 B_2^4 B_4^4 B_5^2 B_6^2 B_3^6 + \\
&3890016900 B_1^3 B_2^5 B_4^3 B_5^2 B_6^2 B_3^6 + 1138368000 B_1^3 B_2^5 B_4^4 B_5 B_6^2 B_3^6 + 3123681792 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^5 + \\
&2191358400 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^5 + 4881115008 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^5 + 7304528000 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^5 + \\
&7304528000 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^5 + 4881115008 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5 + 2191358400 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^5 + \\
&1328096000 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^5 + 398428800 B_1^3 B_2^4 B_4^4 B_5^3 B_6^2 B_3^5 + 1328096000 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^5) \mathbf{Z}^{21} + \\
&(2629630080 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^7 + 788889024 B_1 B_2^4 B_4^4 B_5 B_6^3 B_3^7 + 2629630080 B_1^2 B_2^4 B_4^3 B_5 B_6^3 B_3^7 + \\
&8519617260 B_1 B_2^3 B_4^4 B_5^2 B_6^4 B_3^6 + 3843592830 B_1 B_2^4 B_4^3 B_5^2 B_6^4 B_3^6 + 13340250000 B_1^2 B_2^3 B_4^3 B_5^2 B_6^4 B_3^6 + \\
&1967099904 B_1 B_2^4 B_4^4 B_5 B_6^4 B_3^6 + 3843592830 B_1^2 B_2^3 B_4^4 B_5 B_6^4 B_3^6 + 8519617260 B_1^2 B_2^4 B_4^3 B_5 B_6^4 B_3^6 + \\
&3447314640 B_1 B_2^3 B_4^4 B_5^3 B_6^3 B_3^6 + 10017315000 B_1 B_2^3 B_4^5 B_5^2 B_6^3 B_3^6 + 27874845306 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 + \\
&80030488230 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^6 + 80030488230 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^6 + 27874845306 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^6 +
\end{aligned}
$$

$$10017315000B_1^2B_2^5B_4^3B_5B_6^3B_3^6 + 3447314640B_1^3B_2^4B_4^3B_5B_6^3B_3^6 + 1200622500B_1B_2^3B_4^5B_5^3B_6^2B_3^6 +$$
$$2353220100B_1B_2^4B_4^4B_5^3B_6^2B_3^6 + 6556999680B_1^2B_2^3B_4^4B_5^3B_6^2B_3^6 + 4002075000B_1B_2^4B_4^5B_5^2B_6^2B_3^6 +$$
$$14707625625B_1^2B_2^3B_4^5B_5^2B_6^2B_3^6 + 76881768516B_1^2B_2^4B_4^4B_5^2B_6^2B_3^6 + 14707625625B_1^2B_2^5B_4^3B_5^2B_6^2B_3^6 +$$
$$6556999680B_1^3B_2^4B_4^3B_5^2B_6^2B_3^6 + 4002075000B_1^2B_2^5B_4^4B_5B_6^2B_3^6 + 2353220100B_1^3B_2^4B_4^4B_5B_6^2B_3^6 +$$
$$1200622500B_1^3B_2^5B_4^3B_5B_6^2B_3^6 + 3294508140B_1^2B_2^3B_4^4B_5^2B_6^4B_3^5 + 3294508140B_1^2B_2^4B_4^3B_5^2B_6^4B_3^5 +$$
$$628897500B_1B_2^3B_4^5B_5^3B_6^3B_3^5 + 1232639100B_1B_2^4B_4^4B_5^3B_6^3B_3^5 + 3421632060B_1^2B_2^3B_4^4B_5^3B_6^3B_3^5 +$$
$$369791730B_1^2B_2^4B_4^3B_5^3B_6^3B_3^5 + 7703994375B_1^2B_2^3B_4^5B_5^2B_6^3B_3^5 + 38630489436B_1^2B_2^4B_4^4B_5^2B_6^3B_3^5 +$$
$$369791730B_1^3B_2^3B_4^4B_5^2B_6^3B_3^5 + 7703994375B_1^2B_2^5B_4^3B_5^2B_6^3B_3^5 + 3421632060B_1^3B_2^4B_4^3B_5^2B_6^3B_3^5 +$$
$$1232639100B_1^3B_2^4B_4^4B_5B_6^3B_3^5 + 628897500B_1^3B_2^5B_4^3B_5B_6^3B_3^5 + 1400726250B_1^2B_2^3B_4^5B_5^3B_6^2B_3^5 +$$
$$3186932364B_1^2B_2^4B_4^4B_5^3B_6^2B_3^5 + 4669087500B_1^2B_2^4B_4^5B_5^2B_6^2B_3^5 + 4669087500B_1^2B_2^5B_4^4B_5^2B_6^2B_3^5 +$$
$$3186932364B_1^3B_2^4B_4^4B_5^2B_6^2B_3^5 + 1400726250B_1^3B_2^5B_4^3B_5^2B_6^2B_3^5)\mathbf{Z}^{20} + (4183502400B_1B_2^3B_4^3B_5^2B_6^4B_3^6 +$$
$$1255050720B_1B_2^3B_4^4B_5B_6^4B_3^6 + 1255050720B_1B_2^4B_4^3B_5B_6^4B_3^6 + 4183502400B_1^2B_2^3B_4^3B_5B_6^4B_3^6 +$$
$$28131531120B_1B_2^3B_4^4B_5^2B_6^3B_3^6 + 12664602720B_1B_2^4B_4^3B_5^2B_6^3B_3^6 + 45964195200B_1^2B_2^3B_4^3B_5^2B_6^3B_3^6 +$$
$$6411081600B_1B_2^4B_4^4B_5B_6^3B_3^6 + 12664602720B_1^2B_2^3B_4^4B_5B_6^3B_3^6 + 28131531120B_1^2B_2^4B_4^3B_5B_6^3B_3^6 +$$
$$2561328000B_1B_2^3B_4^4B_5^3B_6^2B_3^6 + 4802490000B_1B_2^3B_4^5B_5^2B_6^2B_3^6 + 14386125600B_1B_2^4B_4^4B_5^2B_6^2B_3^6 +$$
$$48870138240B_1^2B_2^3B_4^4B_5^2B_6^2B_3^6 + 48870138240B_1^2B_2^4B_4^3B_5^2B_6^2B_3^6 + 14386125600B_1^2B_2^4B_4^4B_5B_6^2B_3^6 +$$
$$4802490000B_1^2B_2^5B_4^3B_5B_6^2B_3^6 + 2561328000B_1^3B_2^4B_4^3B_5B_6^2B_3^6 + 1075757760B_1B_2^3B_4^4B_5^2B_6^4B_3^5 +$$
$$3585859200B_1^2B_2^3B_4^3B_5^2B_6^4B_3^5 + 1075757760B_1^2B_2^4B_4^3B_5B_6^4B_3^5 + 1537683840B_1B_2^3B_4^4B_5^3B_6^3B_3^5 +$$
$$402494400B_1^2B_2^3B_4^3B_5^3B_6^3B_3^5 + 2515590000B_1B_2^3B_4^5B_5^2B_6^3B_3^5 + 6940533600B_1B_2^4B_4^4B_5^2B_6^3B_3^5 +$$
$$38328942960B_1^2B_2^3B_4^4B_5^2B_6^3B_3^5 + 38328942960B_1^2B_2^4B_4^3B_5^2B_6^3B_3^5 + 402494400B_1^3B_2^3B_4^3B_5^2B_6^3B_3^5 +$$
$$6940533600B_1^2B_2^4B_4^4B_5B_6^3B_3^5 + 2515590000B_1^2B_2^5B_4^3B_5B_6^3B_3^5 + 1537683840B_1^3B_2^4B_4^3B_5B_6^3B_3^5 +$$
$$457380000B_1B_2^3B_4^5B_5^3B_6^2B_3^5 + 896464800B_1B_2^4B_4^4B_5^3B_6^2B_3^5 + 4201797600B_1^2B_2^3B_4^4B_5^3B_6^2B_3^5 +$$
$$268939440B_1^2B_2^4B_4^3B_5^3B_6^2B_3^5 + 5602905000B_1^2B_2^3B_4^5B_5^2B_6^2B_3^5 + 32647658400B_1^2B_2^4B_4^4B_5^2B_6^2B_3^5 +$$
$$268939440B_1^3B_2^3B_4^4B_5^2B_6^2B_3^5 + 5602905000B_1^2B_2^5B_4^3B_5^2B_6^2B_3^5 + 4201797600B_1^3B_2^4B_4^3B_5^2B_6^2B_3^5 +$$
$$896464800B_1^3B_2^4B_4^4B_5B_6^2B_3^5 + 457380000B_1^3B_2^5B_4^3B_5B_6^2B_3^5)\mathbf{Z}^{19} + (1423552900B_1B_2^3B_4^3B_5B_6^4B_3^6 +$$

Appendix: the full set of E_6 polynomials XIII

$3913140 B_1^2 B_2^4 B_4^3 B_6^3 B_3^6 + 3913140 B_2^3 B_4^4 B_5^2 B_6^3 B_3^6 + 81523750 B_1 B_2^2 B_4^4 B_5^2 B_6^3 B_3^6 +$
$16798204500 B_1 B_2^3 B_4^3 B_5^2 B_6^3 B_3^6 + 5039461350 B_1 B_2^3 B_4^4 B_5 B_6^3 B_3^6 + 5039461350 B_1 B_2^4 B_4^3 B_5 B_6^3 B_3^6 +$
$16798204500 B_1^2 B_2^3 B_4^3 B_5 B_6^3 B_3^6 + 81523750 B_1^2 B_2^4 B_4^2 B_5 B_6^3 B_3^6 + 18993314340 B_1 B_2^3 B_4^4 B_5^2 B_6^2 B_3^6 +$
$10676646750 B_1 B_2^4 B_4^3 B_5^2 B_6^2 B_3^6 + 38856294400 B_1^2 B_2^3 B_4^3 B_5^2 B_6^2 B_3^6 + 3890016900 B_1 B_2^4 B_4^4 B_5 B_6^2 B_3^6 +$
$10676646750 B_1^2 B_2^3 B_4^4 B_5 B_6^2 B_3^6 + 18993314340 B_1^2 B_2^4 B_4^3 B_5 B_6^2 B_3^6 + 1220188200 B_1 B_2^3 B_4^3 B_5^2 B_6^4 B_3^5 +$
$1220188200 B_1^2 B_2^3 B_4^3 B_5 B_6^4 B_3^5 + 69877500 B_1 B_2^2 B_4^4 B_5^3 B_6^3 B_3^5 + 136959900 B_1 B_2^3 B_4^3 B_5^3 B_6^3 B_3^5 +$
$16980581310 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5 + 855999375 B_1^2 B_2^2 B_4^4 B_5^2 B_6^3 B_3^5 + 5281747240 B_1 B_2^4 B_4^3 B_5^2 B_6^3 B_3^5 +$
$41763159900 B_1^2 B_2^3 B_4^3 B_5^2 B_6^3 B_3^5 + 855999375 B_1^2 B_2^4 B_4^2 B_5^2 B_6^3 B_3^5 + 2862182400 B_1 B_2^4 B_4^4 B_5 B_6^3 B_3^5 +$
$5281747240 B_1^2 B_2^3 B_4^4 B_5 B_6^3 B_3^5 + 16980581310 B_1^2 B_2^4 B_4^3 B_5 B_6^3 B_3^5 + 136959900 B_1^3 B_2^3 B_4^3 B_5 B_6^3 B_3^5 +$
$69877500 B_1^3 B_2^4 B_4^2 B_5 B_6^3 B_3^5 + 2212838320 B_1 B_2^3 B_4^4 B_5^3 B_6^2 B_3^5 + 520396800 B_1^2 B_2^3 B_4^3 B_5^3 B_6^2 B_3^5 +$
$1985156250 B_1 B_2^3 B_4^5 B_5^2 B_6^2 B_3^5 + 6097637700 B_1 B_2^4 B_4^4 B_5^2 B_6^2 B_3^5 + 40830215370 B_1^2 B_2^3 B_4^4 B_5^2 B_6^2 B_3^5 +$
$40830215370 B_1^2 B_2^4 B_4^3 B_5^2 B_6^2 B_3^5 + 520396800 B_1^3 B_2^3 B_4^3 B_5^2 B_6^2 B_3^5 + 6097637700 B_1^2 B_2^4 B_4^4 B_5 B_6^2 B_3^5 +$
$1985156250 B_1^2 B_2^5 B_4^3 B_5 B_6^2 B_3^5 + 2212838320 B_1^3 B_2^4 B_4^3 B_5 B_6^2 B_3^5 + 221852400 B_1^2 B_2^4 B_4^4 B_5^2 B_6 B_3^5 +$
$3067901760 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^4 + 3067901760 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^4 + 557800320 B_1^2 B_2^3 B_4^4 B_5^3 B_6^2 B_3^4 +$
$1859334400 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^4 + 557800320 B_1^3 B_2^4 B_4^3 B_5^2 B_6^2 B_3^4) \mathbf{Z}^{18} + (6754454784 B_1 B_2^3 B_4^3 B_5 B_6^3 B_3^6 +$
$16734009600 B_1 B_2^3 B_4^3 B_5^2 B_6^2 B_3^6 + 5020202880 B_1 B_2^3 B_4^4 B_5 B_6^2 B_3^6 + 5020202880 B_1 B_2^4 B_4^3 B_5 B_6^2 B_3^6 +$
$16734009600 B_1^2 B_2^3 B_4^3 B_5 B_6^2 B_3^6 + 439267752 B_1 B_2^3 B_4^3 B_5 B_6^4 B_3^5 + 30187080 B_1^2 B_2^4 B_4^3 B_6^3 B_3^5 +$
$30187080 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5 + 966735000 B_1 B_2^2 B_4^4 B_5^2 B_6^3 B_3^5 + 17946116496 B_1 B_2^3 B_4^3 B_5^2 B_6^3 B_3^5 +$
$2096325000 B_1^2 B_2^2 B_4^3 B_5^2 B_6^3 B_3^5 + 2096325000 B_1^2 B_2^3 B_4^2 B_5^2 B_6^3 B_3^5 + 3421632060 B_1 B_2^3 B_4^4 B_5 B_6^3 B_3^5 +$
$3421632060 B_1 B_2^4 B_4^3 B_5 B_6^3 B_3^5 + 17946116496 B_1^2 B_2^3 B_4^3 B_5 B_6^3 B_3^5 + 966735000 B_1^2 B_2^4 B_4^2 B_5 B_6^3 B_3^5 +$
$114345000 B_1 B_2^2 B_4^4 B_5^3 B_6^2 B_3^5 + 224116200 B_1 B_2^3 B_4^3 B_5^3 B_6^2 B_3^5 + 19609868880 B_1 B_2^3 B_4^4 B_5^2 B_6^2 B_3^5 +$
$1400726250 B_1^2 B_2^2 B_4^4 B_5^2 B_6^2 B_3^5 + 9158882040 B_1 B_2^4 B_4^3 B_5^2 B_6^2 B_3^5 + 59798117736 B_1^2 B_2^3 B_4^3 B_5^2 B_6^2 B_3^5 +$
$1400726250 B_1^2 B_2^4 B_4^2 B_5^2 B_6^2 B_3^5 + 2858625000 B_1 B_2^4 B_4^4 B_5 B_6^2 B_3^5 + 9158882040 B_1^2 B_2^3 B_4^4 B_5 B_6^2 B_3^5 +$
$19609868880 B_1^2 B_2^4 B_4^3 B_5 B_6^2 B_3^5 + 224116200 B_1^3 B_2^3 B_4^3 B_5 B_6^2 B_3^5 + 114345000 B_1^3 B_2^4 B_4^2 B_5 B_6^2 B_3^5 +$
$599001480 B_1^2 B_2^3 B_4^4 B_5^2 B_6 B_3^5 + 599001480 B_1^2 B_2^4 B_4^3 B_5^2 B_6 B_3^5 + 2253968640 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^4 +$

Appendix: the full set of E_6 polynomials XIV

$8611029504 B_1^2 B_2^3 B_4^3 B_5^2 B_6^3 B_3^4 + 2253968640 B_1^2 B_2^4 B_4^3 B_5 B_6^3 B_3^4 + 409812480 B_1 B_2^3 B_4^4 B_5^3 B_6^2 B_3^4 +$
$122943744 B_1^2 B_2^3 B_4^3 B_5^3 B_6^2 B_3^4 + 7991343360 B_1^2 B_2^3 B_4^4 B_5^2 B_6^2 B_3^4 + 7991343360 B_1^2 B_2^4 B_4^3 B_5^2 B_6^2 B_3^4 +$
$122943744 B_1^3 B_2^3 B_4^3 B_5^2 B_6^2 B_3^4 + 409812480 B_1^3 B_2^4 B_4^3 B_5 B_6^2 B_3^4)\mathbf{Z}^{17} + (8199664704 B_1 B_2^3 B_4^3 B_5 B_6^2 B_3^6 +$
$49305564 B_1^2 B_2^3 B_4^3 B_6^3 B_3^5 + 25155900 B_1^2 B_2^4 B_4^2 B_6^3 B_3^5 + 25155900 B_2^2 B_4^4 B_5^2 B_6^3 B_3^5 + 49305564 B_2^3 B_4^3 B_5^2 B_6^3 B_3^5 +$
$1445944500 B_1 B_2^2 B_4^3 B_5^2 B_6^3 B_3^5 + 1027199250 B_1 B_2^3 B_4^2 B_5^2 B_6^3 B_3^5 + 308159775 B_1 B_2^2 B_4^4 B_5 B_6^3 B_3^5 +$
$8951787306 B_1 B_2^3 B_4^3 B_5 B_6^3 B_3^5 + 1027199250 B_1^2 B_2^2 B_4^3 B_5 B_6^3 B_3^5 + 308159775 B_1 B_2^4 B_4^2 B_5 B_6^3 B_3^5 +$
$1445944500 B_1^2 B_2^3 B_4^2 B_5 B_6^3 B_3^5 + 34303500 B_1^2 B_2^4 B_4^3 B_6^2 B_3^5 + 34303500 B_2^3 B_4^4 B_5^2 B_6^2 B_3^5 +$
$1413304200 B_1 B_2^2 B_4^4 B_5^2 B_6^2 B_3^5 + 30824064054 B_1 B_2^3 B_4^3 B_5^2 B_6^2 B_3^5 + 3902976000 B_1^2 B_2^2 B_4^3 B_5^2 B_6^2 B_3^5 +$
$3902976000 B_1^2 B_2^3 B_4^2 B_5^2 B_6^2 B_3^5 + 6565308750 B_1 B_2^3 B_4^4 B_5 B_6^2 B_3^5 + 6565308750 B_1 B_2^4 B_4^3 B_5 B_6^2 B_3^5 +$
$30824064054 B_1^2 B_2^3 B_4^3 B_5 B_6^2 B_3^5 + 1413304200 B_1^2 B_2^4 B_4^2 B_5 B_6^2 B_3^5 + 305613000 B_1 B_2^3 B_4^4 B_5^2 B_6 B_3^5 +$
$1669054464 B_1^2 B_2^3 B_4^3 B_5^2 B_6 B_3^5 + 305613000 B_1^2 B_2^4 B_4^3 B_5 B_6 B_3^5 + 264136950 B_1 B_2^2 B_4^4 B_5^2 B_6^3 B_3^4 +$
$4790284884 B_1 B_2^3 B_4^3 B_5^2 B_6^3 B_3^4 + 880456500 B_1^2 B_2^2 B_4^3 B_5^2 B_6^3 B_3^4 + 880456500 B_1^2 B_2^3 B_4^2 B_5^2 B_6^3 B_3^4 +$
$4790284884 B_1^2 B_2^3 B_4^3 B_5 B_6^3 B_3^4 + 264136950 B_1^2 B_2^4 B_4^2 B_5 B_6^3 B_3^4 + 48024900 B_1 B_2^2 B_4^4 B_5^3 B_6^2 B_3^4 +$
$94128804 B_1 B_2^3 B_4^3 B_5^3 B_6^2 B_3^4 + 5042614500 B_1 B_2^3 B_4^4 B_5^2 B_6^2 B_3^4 + 588305025 B_1^2 B_2^2 B_4^4 B_5^2 B_6^2 B_3^4 +$
$1961016750 B_1 B_2^4 B_4^3 B_5^2 B_6^2 B_3^4 + 27835512516 B_1^2 B_2^3 B_4^3 B_5^2 B_6^2 B_3^4 + 588305025 B_1^2 B_2^4 B_4^2 B_5^2 B_6^2 B_3^4 +$
$1961016750 B_1^2 B_2^3 B_4^4 B_5 B_6^2 B_3^4 + 5042614500 B_1^2 B_2^4 B_4^3 B_5 B_6^2 B_3^4 + 94128804 B_1^3 B_2^3 B_4^3 B_5 B_6^2 B_3^4 +$
$48024900 B_1^3 B_2^4 B_4^2 B_5 B_6^2 B_3^4 + 356548500 B_1^2 B_2^3 B_4^4 B_5^2 B_6 B_3^4 + 356548500 B_1^2 B_2^4 B_4^3 B_5^2 B_6 B_3^4)\mathbf{Z}^{16} +$
$(21913584 B_1 B_2^3 B_4^3 B_6^3 B_3^5 + 73045280 B_1^2 B_2^3 B_4^2 B_6^3 B_3^5 + 73045280 B_2^2 B_4^3 B_5^2 B_6^3 B_3^5 +$
$21913584 B_2^3 B_4^3 B_5 B_6^3 B_3^5 + 1016898960 B_1 B_2^2 B_4^3 B_5 B_6^3 B_3^5 + 1016898960 B_1 B_2^3 B_4^2 B_5 B_6^3 B_3^5 +$
$99607200 B_1^2 B_2^3 B_4^3 B_6^2 B_3^5 + 50820000 B_1^2 B_2^4 B_4^2 B_6^2 B_3^5 + 50820000 B_2^2 B_4^4 B_5^2 B_6^2 B_3^5 +$
$99607200 B_2^3 B_4^3 B_5^2 B_6^2 B_3^5 + 3252073440 B_1 B_2^2 B_4^3 B_5^2 B_6^2 B_3^5 + 2075150000 B_1 B_2^3 B_4^2 B_5^2 B_6^2 B_3^5 +$
$622545000 B_1 B_2^2 B_4^4 B_5 B_6^2 B_3^5 + 19480302576 B_1 B_2^3 B_4^3 B_5 B_6^2 B_3^5 + 2075150000 B_1^2 B_2^2 B_4^3 B_5 B_6^2 B_3^5 +$
$622545000 B_1 B_2^4 B_4^2 B_5 B_6^2 B_3^5 + 3252073440 B_1^2 B_2^3 B_4^2 B_5 B_6^2 B_3^5 + 887409600 B_1 B_2^3 B_4^3 B_5^2 B_6 B_3^5 +$
$887409600 B_1^2 B_2^3 B_4^3 B_5 B_6 B_3^5 + 871627680 B_1 B_2^2 B_4^3 B_5^2 B_6^3 B_3^4 + 766975440 B_1 B_2^3 B_4^2 B_5^2 B_6^3 B_3^4 +$
$2414513024 B_1 B_2^3 B_4^3 B_5 B_6^3 B_3^4 + 766975440 B_1^2 B_2^2 B_4^3 B_5 B_6^3 B_3^4 + 871627680 B_1^2 B_2^3 B_4^2 B_5 B_6^3 B_3^4 +$

Appendix: the full set of E_6 polynomials XV

$$
\begin{aligned}
&11383680 B_1 B_2^2 B_4^3 B_5^3 B_6^2 B_3^4 + 830060000 B_1 B_2^2 B_4^4 B_5^2 B_6^2 B_3^4 + 18476731056 B_1 B_2^3 B_4^3 B_5^2 B_6^2 B_3^4 + \\
&4063973760 B_1^2 B_2^2 B_4^3 B_5^2 B_6^2 B_3^4 + 4063973760 B_1^2 B_2^3 B_4^2 B_5^2 B_6^2 B_3^4 + 1778700000 B_1 B_2^3 B_4^4 B_5 B_6^2 B_3^4 + \\
&1778700000 B_1 B_2^4 B_4^3 B_5 B_6^2 B_3^4 + 18476731056 B_1^2 B_2^3 B_4^3 B_5 B_6^2 B_3^4 + 830060000 B_1^2 B_2^4 B_4^2 B_5 B_6^2 B_3^4 + \\
&11383680 B_1^3 B_2^3 B_4^2 B_5 B_6^2 B_3^4 + 323400000 B_1 B_2^3 B_4^4 B_5^2 B_6 B_3^4 + 2253071744 B_1^2 B_2^3 B_4^3 B_5^2 B_6 B_3^4 + \\
&323400000 B_1^2 B_2^4 B_4^3 B_5 B_6 B_3^4 + 4079910912 B_1^2 B_2^3 B_4^3 B_5^2 B_6^2 B_3^3)\mathbf{Z}^{15} + (41087970 B_1 B_2^3 B_4^2 B_6^3 B_3^5 + \\
&41087970 B_2^2 B_4^3 B_5 B_6^3 B_3^5 + 136959900 B_1 B_2^2 B_4^2 B_5 B_6^3 B_3^5 + 56029050 B_1 B_2^3 B_4^3 B_6^2 B_3^5 + \\
&186763500 B_1^2 B_2^3 B_4^2 B_6^2 B_3^5 + 186763500 B_2^2 B_4^3 B_5^2 B_6^2 B_3^5 + 56029050 B_2^3 B_4^3 B_5 B_6^2 B_3^5 + \\
&2655776970 B_1 B_2^2 B_4^3 B_5 B_6^2 B_3^5 + 2655776970 B_1 B_2^3 B_4^2 B_5 B_6^2 B_3^5 + 499167900 B_1 B_2^3 B_4^3 B_5 B_6 B_3^5 + \\
&35218260 B_1^2 B_2^3 B_4^2 B_6^3 B_3^4 + 35218260 B_2^2 B_4^3 B_5^2 B_6^3 B_3^4 + 117394200 B_1 B_2^2 B_4^2 B_5^2 B_6^3 B_3^4 + \\
&607296690 B_1 B_2^2 B_4^3 B_5 B_6^3 B_3^4 + 607296690 B_1 B_2^3 B_4^2 B_5 B_6^3 B_3^4 + 117394200 B_1^2 B_2^2 B_4^2 B_5 B_6^3 B_3^4 + \\
&48024900 B_1^2 B_2^3 B_4^3 B_6^2 B_3^4 + 24502500 B_1^2 B_2^4 B_4^2 B_6^2 B_3^4 + 24502500 B_2^2 B_4^4 B_5^2 B_6^2 B_3^4 + \\
&48024900 B_2^3 B_4^3 B_5^2 B_6^2 B_3^4 + 4609356210 B_1 B_2^2 B_4^3 B_5^2 B_6^2 B_3^4 + 3054383640 B_1 B_2^3 B_4^2 B_5^2 B_6^2 B_3^4 + \\
&446054400 B_1^2 B_2^2 B_4^2 B_5^2 B_6^2 B_3^4 + 300155625 B_1 B_2^2 B_4^4 B_5 B_6^2 B_3^4 + 14283282150 B_1 B_2^3 B_4^3 B_5 B_6^2 B_3^4 + \\
&3054383640 B_1^2 B_2^2 B_4^3 B_5 B_6^2 B_3^4 + 300155625 B_1 B_2^4 B_4^2 B_5 B_6^2 B_3^4 + 4609356210 B_1^2 B_2^3 B_4^2 B_5 B_6^2 B_3^4 + \\
&54573750 B_1 B_2^2 B_4^4 B_5^2 B_6 B_3^4 + 1671169500 B_1 B_2^3 B_4^3 B_5^2 B_6 B_3^4 + 298045440 B_1^2 B_2^2 B_4^3 B_5^2 B_6 B_3^4 + \\
&298045440 B_1^2 B_2^3 B_4^2 B_5^2 B_6 B_3^4 + 1671169500 B_1^2 B_2^3 B_4^3 B_5 B_6 B_3^4 + 54573750 B_1^2 B_2^4 B_4^2 B_5 B_6 B_3^4 + \\
&3585859200 B_1 B_2^3 B_4^3 B_5^2 B_6^2 B_3^3 + 1075757760 B_1^2 B_2^2 B_4^3 B_5^2 B_6^2 B_3^3 + 1075757760 B_1^2 B_2^3 B_4^2 B_5^2 B_6^2 B_3^3 + \\
&3585859200 B_1^2 B_2^3 B_4^3 B_5 B_6^2 B_3^3 + 651974400 B_1^2 B_2^3 B_4^3 B_5^2 B_6 B_3^3)\mathbf{Z}^{14} + (119528640 B_1 B_2^3 B_4^2 B_6^2 B_3^5 + \\
&119528640 B_2^2 B_4^3 B_5 B_6^2 B_3^5 + 398428800 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^5 + 35218260 B_1 B_2^3 B_4^2 B_6^3 B_3^4 + \\
&35218260 B_2^2 B_4^3 B_5 B_6^3 B_3^4 + 180457200 B_1 B_2^2 B_4^2 B_5 B_6^3 B_3^4 + 48024900 B_1 B_2^3 B_4^3 B_6^2 B_3^4 + \\
&268939440 B_1^2 B_2^3 B_4^2 B_6^2 B_3^4 + 268939440 B_2^2 B_4^3 B_5^2 B_6^2 B_3^4 + 133402500 B_1 B_2 B_4^3 B_5^2 B_6^2 B_3^4 + \\
&672348600 B_1 B_2^2 B_4^2 B_5^2 B_6^2 B_3^4 + 48024900 B_2^3 B_4^3 B_5 B_6^2 B_3^4 + 4320547560 B_1 B_2^2 B_4^3 B_5 B_6^2 B_3^4 + \\
&4320547560 B_1 B_2^3 B_4^2 B_5 B_6^2 B_3^4 + 672348600 B_1^2 B_2^2 B_4^2 B_5 B_6^2 B_3^4 + 133402500 B_1^2 B_2^3 B_4 B_5 B_6^2 B_3^4 + \\
&413887320 B_1 B_2^2 B_4^3 B_5^2 B_6 B_3^4 + 356548500 B_1 B_2^3 B_4^2 B_5^2 B_6 B_3^4 + 1189465200 B_1 B_2^3 B_4^3 B_5 B_6 B_3^4 + \\
&356548500 B_1^2 B_2^2 B_4^3 B_5 B_6 B_3^4 + 413887320 B_1^2 B_2^3 B_4^2 B_5 B_6 B_3^4 + 1644128640 B_1 B_2^2 B_4^3 B_5^2 B_6^2 B_3^3 +
\end{aligned}
$$

Appendix: the full set of E_6 polynomials XVI

$$1075757760 B_1 B_2^3 B_4^2 B_5^2 B_6^2 B_3^3 + 292723200 B_1^2 B_2^2 B_4^2 B_5^2 B_6^2 B_3^3 + 3585859200 B_1 B_2^3 B_4^3 B_5 B_6^2 B_3^3 +$$
$$1075757760 B_1^2 B_2^2 B_4^3 B_5 B_6^2 B_3^3 + 1644128640 B_1^2 B_2^3 B_4^2 B_5 B_6^2 B_3^3 + 651974400 B_1 B_2^3 B_4^3 B_5^2 B_6 B_3^3 +$$
$$195592320 B_1^2 B_2^2 B_4^3 B_5^2 B_6 B_3^3 + 195592320 B_1^2 B_2^3 B_4^2 B_5^2 B_6 B_3^3 + 651974400 B_1^2 B_2^3 B_4^3 B_5 B_6 B_3^3)\mathbf{Z}^{13} +$$
$$(6391462 B_1 B_2^2 B_4^2 B_6^3 B_3^4 + 6391462 B_2^2 B_4^2 B_5 B_6^3 B_3^4 + 8715630 B_1 B_2^2 B_4^3 B_6^2 B_3^4 + 253998360 B_1 B_2^3 B_4^2 B_6^2 B_3^4 +$$
$$29052100 B_1^2 B_2^2 B_4^2 B_6^2 B_3^4 + 14822500 B_2 B_4^3 B_5^2 B_6^2 B_3^4 + 29052100 B_2^2 B_4^2 B_5^2 B_6^2 B_3^4 +$$
$$14822500 B_1^2 B_2^3 B_4 B_6^2 B_3^4 + 253998360 B_2^2 B_4^3 B_5 B_6^2 B_3^4 + 181575625 B_1 B_2 B_4^3 B_5 B_6^2 B_3^4 +$$
$$8715630 B_2^3 B_4^2 B_5 B_6^2 B_3^4 + 1554121926 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^4 + 181575625 B_1 B_2^3 B_4 B_5 B_6^2 B_3^4 +$$
$$20212500 B_1^2 B_2^3 B_4^2 B_6 B_3^4 + 20212500 B_2^2 B_4^3 B_5^2 B_6 B_3^4 + 110387200 B_1 B_2^2 B_4^2 B_5^2 B_6 B_3^4 +$$
$$388031490 B_1 B_2^2 B_4^3 B_5 B_6 B_3^4 + 388031490 B_1 B_2^3 B_4^2 B_5 B_6 B_3^4 + 110387200 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^4 +$$
$$5478396 B_1 B_2^2 B_4^2 B_5 B_6^3 B_3^3 + 111143340 B_1^2 B_2^3 B_4^2 B_6^2 B_3^3 + 111143340 B_2^2 B_4^3 B_5^2 B_6^2 B_3^3 +$$
$$155636250 B_1 B_2 B_4^3 B_5^2 B_6^2 B_3^3 + 542666124 B_1 B_2^2 B_4^2 B_5^2 B_6^2 B_3^3 + 1941993130 B_1 B_2^2 B_4^3 B_5 B_6^2 B_3^3 +$$
$$1941993130 B_1 B_2^3 B_4^2 B_5 B_6^2 B_3^3 + 542666124 B_1^2 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 155636250 B_1^2 B_2^3 B_4 B_5 B_6^2 B_3^3 +$$
$$353089660 B_1 B_2^2 B_4^3 B_5^2 B_6 B_3^3 + 247546530 B_1 B_2^3 B_4^2 B_5^2 B_6 B_3^3 + 60555264 B_1^2 B_2^2 B_4^2 B_5^2 B_6 B_3^3 +$$
$$707437500 B_1 B_2^3 B_4^3 B_5 B_6 B_3^3 + 247546530 B_1^2 B_2^2 B_4^3 B_5 B_6 B_3^3 + 353089660 B_1^2 B_2^3 B_4^2 B_5 B_6 B_3^3)\mathbf{Z}^{12} +$$
$$(66594528 B_1 B_2^2 B_4^2 B_6^2 B_3^4 + 21344400 B_1 B_2^3 B_4 B_6^2 B_3^4 + 21344400 B_2 B_4^3 B_5 B_6^2 B_3^4 +$$
$$66594528 B_2^2 B_4^2 B_5 B_6^2 B_3^4 + 71148000 B_1 B_2 B_4^2 B_5 B_6^2 B_3^4 + 71148000 B_1 B_2^2 B_4 B_5 B_6^2 B_3^4 +$$
$$29106000 B_1 B_2^3 B_4^2 B_6 B_3^4 + 29106000 B_2^2 B_4^3 B_5 B_6 B_3^4 + 191866752 B_1 B_2^2 B_4^2 B_5 B_6 B_3^4 +$$
$$137214000 B_1 B_2^3 B_4^2 B_6^2 B_3^3 + 35858592 B_1^2 B_2^2 B_4^2 B_6^2 B_3^3 + 18295200 B_2 B_4^3 B_5^2 B_6^2 B_3^3 +$$
$$35858592 B_2^2 B_4^2 B_5^2 B_6^2 B_3^3 + 60984000 B_1 B_2 B_4^2 B_5^2 B_6^2 B_3^3 + 18295200 B_1^2 B_2^3 B_4 B_6^2 B_3^3 +$$
$$137214000 B_2^2 B_4^3 B_5 B_6^2 B_3^3 + 224116200 B_1 B_2 B_4^3 B_5 B_6^2 B_3^3 + 1339753968 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^3 +$$
$$224116200 B_1 B_2^3 B_4 B_5 B_6^2 B_3^3 + 60984000 B_1^2 B_2^2 B_4 B_5 B_6^2 B_3^3 + 24948000 B_1^2 B_2^3 B_4^2 B_6 B_3^3 +$$
$$24948000 B_2^2 B_4^3 B_5^2 B_6 B_3^3 + 40748400 B_1 B_2 B_4^3 B_5^2 B_6 B_3^3 + 177031008 B_1 B_2^2 B_4^2 B_5^2 B_6 B_3^3 +$$
$$467082000 B_1 B_2^2 B_4^3 B_5 B_6 B_3^3 + 467082000 B_1 B_2^3 B_4^2 B_5 B_6 B_3^3 + 177031008 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^3 +$$
$$40748400 B_1^2 B_2^3 B_4 B_5 B_6 B_3^3)\mathbf{Z}^{11} + (2614689 B_2^2 B_4^2 B_6^2 B_3^4 + 8715630 B_1 B_2^2 B_4 B_6^2 B_3^4 +$$
$$8715630 B_2 B_4^2 B_5 B_6^2 B_3^4 + 11884950 B_1 B_2^2 B_4^2 B_6 B_3^4 + 11884950 B_2^2 B_4^2 B_5 B_6 B_3^4 + 87268104 B_1 B_2^2 B_4^2 B_6^2 B_3^3 +$$

$$7470540 B_2 B_4^2 B_5^2 B_6^2 B_3^3 + 28586250 B_1 B_2^3 B_4 B_6^2 B_3^3 + 7470540 B_1^2 B_2^2 B_4 B_6^2 B_3^3 + 28586250 B_2 B_4^3 B_5 B_6^2 B_3^3 +$$
$$87268104 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 202848030 B_1 B_2 B_4^2 B_5 B_6^2 B_3^3 + 202848030 B_1 B_2^2 B_4 B_5 B_6^2 B_3^3 +$$
$$38981250 B_1 B_2^3 B_4^2 B_6 B_3^3 + 10187100 B_1^2 B_2^2 B_4^2 B_6 B_3^3 + 5197500 B_2 B_4^3 B_5^2 B_6 B_3^3 + 10187100 B_2^2 B_4^2 B_5^2 B_6 B_3^3 +$$
$$28385280 B_1 B_2 B_4^2 B_5^2 B_6 B_3^3 + 5197500 B_1^2 B_2^3 B_4 B_6 B_3^3 + 38981250 B_2^2 B_4^3 B_5 B_6 B_3^3 +$$
$$63669375 B_1 B_2 B_4^3 B_5 B_6 B_3^3 + 440527626 B_1 B_2^2 B_4^2 B_5 B_6 B_3^3 + 63669375 B_1 B_2^3 B_4 B_5 B_6 B_3^3 +$$
$$28385280 B_1^2 B_2^2 B_4 B_5 B_6 B_3^3 + 30735936 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^2 + 5588352 B_1 B_2^2 B_4^2 B_5^2 B_6 B_3^2 +$$
$$5588352 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^2) \mathbf{Z}^{10} + (6225450 B_2^2 B_4^2 B_6^2 B_3^3 + 13280960 B_1 B_2 B_4^2 B_6^2 B_3^3 +$$
$$27544440 B_1 B_2^2 B_4 B_6^2 B_3^3 + 27544440 B_2 B_4^2 B_5 B_6^2 B_3^3 + 13280960 B_2^2 B_4 B_5 B_6^2 B_3^3 +$$
$$41164200 B_1 B_2 B_4 B_5 B_6^2 B_3^3 + 205800 B_1 B_2^2 B_4^2 B_5 B_3^3 + 37560600 B_1 B_2^2 B_4^2 B_6 B_3^3 + 3773000 B_2 B_4^2 B_5^2 B_6 B_3^3 +$$
$$9240000 B_1 B_2^3 B_4 B_6 B_3^3 + 3773000 B_1^2 B_2^2 B_4 B_6 B_3^3 + 9240000 B_2 B_4^3 B_5 B_6 B_3^3 + 37560600 B_2^2 B_4^2 B_5 B_6 B_3^3 +$$
$$82222140 B_1 B_2 B_4^2 B_5 B_6 B_3^3 + 82222140 B_1 B_2^2 B_4 B_5 B_6 B_3^3 + 11383680 B_1 B_2 B_4^2 B_5 B_6^2 B_3^2 +$$
$$11383680 B_1 B_2^2 B_4 B_5 B_6^2 B_3^2 + 2069760 B_1 B_2 B_4^2 B_5^2 B_6 B_3^2 + 24147200 B_1 B_2^2 B_4^2 B_5 B_6 B_3^2 +$$
$$2069760 B_1^2 B_2^2 B_4 B_5 B_6 B_3^2) \mathbf{Z}^9 + (1867635 B_2 B_4^2 B_6^2 B_3^3 + 1867635 B_2^2 B_4 B_6^2 B_3^3 + 6225450 B_1 B_2 B_4 B_6^2 B_3^3 +$$
$$6225450 B_2 B_4 B_5 B_6^2 B_3^3 + 2546775 B_2^2 B_4^2 B_6 B_3^3 + 8489250 B_1 B_2 B_4^2 B_6 B_3^3 + 13222440 B_1 B_2^2 B_4 B_6 B_3^3 +$$
$$13222440 B_2 B_4^2 B_5 B_6 B_3^3 + 8489250 B_2^2 B_4 B_5 B_6 B_3^3 + 23654400 B_1 B_2 B_4 B_5 B_6 B_3^3 +$$
$$1600830 B_1 B_2^2 B_4 B_6^2 B_3^2 + 1600830 B_2 B_4^2 B_5 B_6^2 B_3^2 + 5336100 B_1 B_2 B_4 B_5 B_6^2 B_3^2 + 396900 B_1 B_2^2 B_4^2 B_5 B_3^2 +$$
$$2182950 B_1 B_2^2 B_4^2 B_6 B_3^2 + 291060 B_2 B_4^2 B_5^2 B_6 B_3^2 + 291060 B_1^2 B_2^2 B_4 B_6 B_3^2 + 2182950 B_2^2 B_4^2 B_5 B_6 B_3^2 +$$
$$9168390 B_1 B_2 B_4^2 B_5 B_6 B_3^2 + 9168390 B_1 B_2^2 B_4 B_5 B_6 B_3^2) \mathbf{Z}^8 + (996072 B_2 B_4 B_6^2 B_3^3 +$$
$$1358280 B_2 B_4^2 B_6 B_3^3 + 1358280 B_2^2 B_4 B_6 B_3^3 + 4527600 B_1 B_2 B_4 B_6 B_3^3 + 4527600 B_2 B_4 B_5 B_6 B_3^3 +$$
$$853776 B_1 B_2 B_4 B_6^2 B_3^2 + 853776 B_2 B_4 B_5 B_6^2 B_3^2 + 211680 B_1 B_2 B_4^2 B_5 B_3^2 + 211680 B_1 B_2^2 B_4 B_5 B_3^2 +$$
$$1164240 B_1 B_2 B_4^2 B_6 B_3^2 + 1853280 B_1 B_2^2 B_4 B_6 B_3^2 + 1853280 B_2 B_4^2 B_5 B_6 B_3^2 + 1164240 B_2^2 B_4 B_5 B_6 B_3^2 +$$
$$6044544 B_1 B_2 B_4 B_5 B_6 B_3^2) \mathbf{Z}^7 + (916839 B_2 B_4 B_6 B_3^3 + 148225 B_2 B_4 B_6^2 B_3^2 + 36750 B_1 B_2^2 B_4 B_3^2 +$$
$$36750 B_2 B_4^2 B_5 B_3^2 + 200704 B_1 B_2 B_4 B_5 B_3^2 + 26950 B_1 B_2^2 B_6 B_3^2 + 202125 B_2 B_4^2 B_6 B_3^2 + 202125 B_2^2 B_4 B_6 B_3^2 +$$
$$1539384 B_1 B_2 B_4 B_6 B_3^2 + 26950 B_4^2 B_5 B_6 B_3^2 + 1539384 B_2 B_4 B_5 B_6 B_3^2 + 369600 B_1 B_2 B_4 B_5 B_6 B_3) \mathbf{Z}^6 +$$
$$(44100 B_1 B_2 B_4 B_3^2 + 44100 B_2 B_4 B_5 B_3^2 + 32340 B_1 B_2 B_6 B_3^2 + 407484 B_2 B_4 B_6 B_3^2 + 32340 B_4 B_5 B_6 B_3^2 +$$

$24192 B_1 B_2 B_4 B_5 B_3 + 133056 B_1 B_2 B_4 B_6 B_3 + 133056 B_2 B_4 B_5 B_6 B_3)\mathbf{Z}^5 + (11025 B_2 B_4 B_3^2 +$
$8085 B_2 B_6 B_3^2 + 8085 B_4 B_6 B_3^2 + 9450 B_1 B_2 B_4 B_3 + 9450 B_2 B_4 B_5 B_3 + 6930 B_1 B_2 B_6 B_3 +$
$51975 B_2 B_4 B_6 B_3 + 6930 B_4 B_5 B_6 B_3)\mathbf{Z}^4 + (560 B_1 B_2 B_3 + 4200 B_2 B_4 B_3 + 560 B_4 B_5 B_3 +$
$3080 B_2 B_6 B_3 + 3080 B_4 B_6 B_3)\mathbf{Z}^3 + (315 B_2 B_3 + 315 B_4 B_3 + 231 B_6 B_3)\mathbf{Z}^2 + 42 B_3 \mathbf{Z} + 1$

$\boxed{H_4} =$

$B_1^3 B_2^6 B_3^8 B_4^6 B_5^3 B_6^4 \mathbf{Z}^{30} + 30 B_1^3 B_2^5 B_3^8 B_4^6 B_5^3 B_6^4 \mathbf{Z}^{29} + (120 B_1^2 B_2^5 B_4^6 B_5^3 B_6^4 B_3^8 + 315 B_1^3 B_2^5 B_4^6 B_5^3 B_6^4 B_3^7)\mathbf{Z}^{28} +$
$(2240 B_1^2 B_2^5 B_4^6 B_5^3 B_6^4 B_3^7 + 1050 B_1^3 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7 + 770 B_1^3 B_2^5 B_4^6 B_5^3 B_6^3 B_3^7)\mathbf{Z}^{27} + (4200 B_1^2 B_2^4 B_4^6 B_5^3 B_6^4 B_3^7 +$
$9450 B_1^2 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7 + 1050 B_1^3 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7 + 6930 B_1^2 B_2^5 B_4^6 B_5^3 B_6^3 B_3^7 + 5775 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7)\mathbf{Z}^{26} +$
$(31500 B_1^2 B_2^4 B_4^5 B_5^3 B_6^4 B_3^7 + 10752 B_1^2 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7 + 23100 B_1^2 B_2^4 B_4^6 B_5^3 B_6^3 B_3^7 + 59136 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7 +$
$8316 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7 + 9702 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6)\mathbf{Z}^{25} + (45360 B_1^2 B_2^4 B_4^5 B_5^2 B_6^4 B_3^7 +$
$249480 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^7 + 92400 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7 + 36750 B_1^2 B_2^4 B_4^5 B_5^3 B_6^4 B_3^6 + 26950 B_1^2 B_2^4 B_4^6 B_5^3 B_6^3 B_3^6 +$
$107800 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6 + 8085 B_1^3 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6 + 26950 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6)\mathbf{Z}^{24} +$
$(443520 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^7 + 94080 B_1^2 B_2^4 B_4^5 B_5^2 B_6^4 B_3^6 + 1132560 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6 +$
$316800 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6 + 32340 B_1^3 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 16500 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6)\mathbf{Z}^{23} +$
$(44100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6 + 32340 B_1 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6 + 495000 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^6 +$
$242550 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6 + 3256110 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 202125 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6 +$
$44550 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 + 177870 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^6 + 1358280 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^5)\mathbf{Z}^{22} +$
$(178200 B_1 B_2^3 B_4^5 B_5^3 B_6^3 B_3^6 + 168960 B_1 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 2182950 B_1^2 B_2^3 B_4^5 B_5^2 B_6^3 B_3^6 +$
$2674100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 + 711480 B_1^2 B_2^4 B_4^5 B_5^2 B_6^2 B_3^6 + 1478400 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^5 +$
$1131900 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^5 + 4928000 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^5 + 23100 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5 +$
$830060 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^5)\mathbf{Z}^{21} + (970200 B_1 B_2^3 B_4^5 B_5^2 B_6^3 B_3^6 + 349272 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 +$
$3234000 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^6 + 155232 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^6 + 853776 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^6 +$
$577500 B_1 B_2^3 B_4^5 B_5^3 B_6^3 B_3^5 + 1559250 B_1^2 B_2^3 B_4^4 B_5^3 B_6^3 B_3^5 + 7074375 B_1^2 B_2^3 B_4^5 B_5^2 B_6^3 B_3^5 +$
$9315306 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5 + 1143450 B_1^2 B_2^3 B_4^5 B_5^3 B_6^2 B_3^5 + 996072 B_1^2 B_2^4 B_4^4 B_5^3 B_6^2 B_3^5 +$

Appendix: the full set of E_6 polynomials XIX

$$3811500 B_1^2 B_2^4 B_4^5 B_5^2 B_6^2 B_3^5 + 5082 B_1^3 B_2^4 B_4^4 B_5^2 B_6^2 B_3^5)\mathbf{Z}^{20} + (2069760 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^6 +$$
$$693000 B_1 B_2^3 B_4^4 B_5^3 B_6^3 B_3^5 + 3326400 B_1 B_2^3 B_4^5 B_5^2 B_6^3 B_3^5 + 369600 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5 +$$
$$21801780 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5 + 4331250 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^5 + 1478400 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^5 +$$
$$508200 B_1 B_2^3 B_4^5 B_5^3 B_6^2 B_3^5 + 2439360 B_1^2 B_2^3 B_4^4 B_5^3 B_6^2 B_3^5 + 6225450 B_1^2 B_2^3 B_4^5 B_5^2 B_6^2 B_3^5 +$$
$$11384100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^5)\mathbf{Z}^{19} + (14314300 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5 + 14437500 B_1^2 B_2^3 B_4^3 B_5^2 B_6^3 B_3^5 +$$
$$3056130 B_1^2 B_2^3 B_4^4 B_5 B_6^3 B_3^5 + 1559250 B_1^2 B_2^4 B_4^3 B_5 B_6^3 B_3^5 + 1372140 B_1 B_2^3 B_4^4 B_5^3 B_6^2 B_3^5 +$$
$$3176250 B_1 B_2^3 B_4^5 B_5^2 B_6^2 B_3^5 + 127050 B_1 B_2^4 B_4^4 B_5^2 B_6^2 B_3^5 + 28420210 B_1^2 B_2^3 B_4^4 B_5^2 B_6^2 B_3^5 +$$
$$8575875 B_1^2 B_2^4 B_4^3 B_5^2 B_6^2 B_3^5 + 2032800 B_1^2 B_2^4 B_4^4 B_5 B_6^2 B_3^5 + 6338640 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^4 +$$
$$711480 B_1^2 B_2^3 B_4^4 B_5^3 B_6^2 B_3^4 + 2371600 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^4)\mathbf{Z}^{18} + (577500 B_1 B_2^2 B_4^4 B_5^2 B_6^3 B_3^5 +$$
$$10187100 B_1 B_2^3 B_4^3 B_5^2 B_6^3 B_3^5 + 1774080 B_1 B_2^3 B_4^4 B_5 B_6^3 B_3^5 + 5913600 B_1^2 B_2^3 B_4^3 B_5 B_6^3 B_3^5 +$$
$$18705960 B_1 B_2^3 B_4^4 B_5^2 B_6^2 B_3^5 + 32524800 B_1^2 B_2^3 B_4^3 B_5^2 B_6^2 B_3^5 + 7470540 B_1^2 B_2^3 B_4^4 B_5 B_6^2 B_3^5 +$$
$$3811500 B_1^2 B_2^4 B_4^3 B_5 B_6^2 B_3^5 + 8279040 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^4 + 8731800 B_1^2 B_2^3 B_4^3 B_5^2 B_6^3 B_3^4 +$$
$$711480 B_1 B_2^3 B_4^4 B_5^3 B_6^2 B_3^4 + 16625700 B_1^2 B_2^3 B_4^4 B_5^2 B_6^2 B_3^4 + 4446750 B_1^2 B_2^4 B_4^3 B_5^2 B_6^2 B_3^4)\mathbf{Z}^{17} +$$
$$(4527600 B_1 B_2^3 B_4^3 B_5 B_6^3 B_3^5 + 508200 B_1 B_2^2 B_4^4 B_5^2 B_6^2 B_3^5 + 24901800 B_1 B_2^3 B_4^3 B_5^2 B_6^2 B_3^5 +$$
$$5488560 B_1 B_2^3 B_4^4 B_5 B_6^2 B_3^5 + 18295200 B_1^2 B_2^3 B_4^3 B_5 B_6^2 B_3^5 + 2182950 B_1 B_2^2 B_4^4 B_5^2 B_6^3 B_3^4 +$$
$$11884950 B_1 B_2^3 B_4^3 B_5^2 B_6^3 B_3^4 + 3880800 B_1^2 B_2^3 B_4^3 B_5 B_6^3 B_3^4 + 108900 B_1 B_2^2 B_4^4 B_5^3 B_6^2 B_3^4 +$$
$$18478980 B_1 B_2^3 B_4^4 B_5^2 B_6^2 B_3^4 + 1334025 B_1^2 B_2^2 B_4^4 B_5^2 B_6^2 B_3^4 + 45530550 B_1^2 B_2^3 B_4^3 B_5^2 B_6^2 B_3^4 +$$
$$4446750 B_1^2 B_2^3 B_4^4 B_5 B_6^2 B_3^4 + 2268750 B_1^2 B_2^4 B_4^3 B_5 B_6^2 B_3^4 + 1584660 B_1^2 B_2^3 B_4^4 B_5^2 B_6 B_3^4)\mathbf{Z}^{16} +$$
$$(15937152 B_1 B_2^3 B_4^3 B_5 B_6^2 B_3^5 + 3234000 B_1 B_2^2 B_4^3 B_5^2 B_6^3 B_3^4 + 5588352 B_1 B_2^3 B_4^3 B_5 B_6^3 B_3^4 +$$
$$6203600 B_1 B_2^2 B_4^4 B_5^2 B_6^2 B_3^4 + 53742416 B_1 B_2^3 B_4^3 B_5^2 B_6^2 B_3^4 + 5808000 B_1^2 B_2^2 B_4^3 B_5^2 B_6^2 B_3^4 +$$
$$5808000 B_1 B_2^3 B_4^4 B_5 B_6^2 B_3^4 + 34036496 B_1^2 B_2^3 B_4^3 B_5 B_6^2 B_3^4 + 3234000 B_1 B_2^3 B_4^4 B_5^2 B_6 B_3^4 +$$
$$5588352 B_1^2 B_2^3 B_4^3 B_5^2 B_6 B_3^4 + 15937152 B_1^2 B_2^3 B_4^3 B_5^2 B_6^2 B_3^3)\mathbf{Z}^{15} + (1584660 B_1 B_2^2 B_4^3 B_5 B_6^3 B_3^4 +$$
$$108900 B_2^2 B_4^4 B_5^2 B_6^2 B_3^4 + 18478980 B_1 B_2^2 B_4^3 B_5^2 B_6^2 B_3^4 + 1334025 B_1 B_2^2 B_4^4 B_5 B_6^2 B_3^4 +$$
$$45530550 B_1 B_2^3 B_4^3 B_5 B_6^2 B_3^4 + 4446750 B_1^2 B_2^2 B_4^3 B_5 B_6^2 B_3^4 + 2268750 B_1^2 B_2^3 B_4^2 B_5 B_6^2 B_3^4 +$$
$$2182950 B_1 B_2^2 B_4^4 B_5^2 B_6 B_3^4 + 11884950 B_1 B_2^3 B_4^3 B_5^2 B_6 B_3^4 + 3880800 B_1^2 B_2^3 B_4^3 B_5 B_6 B_3^4 +$$

Appendix: the full set of E_6 polynomials XX

$$508200 B_1 B_2^2 B_4^4 B_5^2 B_6^2 B_3^3 + 24901800 B_1 B_2^3 B_4^3 B_5^2 B_6^2 B_3^3 + 5488560 B_1^2 B_2^2 B_4^3 B_5^2 B_6^2 B_3^3 +$$
$$18295200 B_1^2 B_2^3 B_4^3 B_5 B_6^2 B_3^3 + 4527600 B_1^2 B_2^3 B_4^3 B_5^2 B_6 B_3^3)\mathbf{Z}^{14} + (711480 B_2^2 B_4^3 B_5^2 B_6^2 B_3^4 +$$
$$16625700 B_1 B_2^2 B_4^3 B_5 B_6^2 B_3^4 + 4446750 B_1 B_2^3 B_4^2 B_5 B_6^2 B_3^4 + 8279040 B_1 B_2^2 B_4^3 B_5^2 B_6 B_3^4 +$$
$$8731800 B_1 B_2^3 B_4^3 B_5 B_6 B_3^4 + 18705960 B_1 B_2^2 B_4^3 B_5^2 B_6^2 B_3^3 + 32524800 B_1 B_2^3 B_4^3 B_5 B_6^2 B_3^3 +$$
$$7470540 B_1^2 B_2^2 B_4^3 B_5 B_6^2 B_3^3 + 3811500 B_1^2 B_2^3 B_4^2 B_5 B_6^2 B_3^3 + 577500 B_1 B_2^2 B_4^4 B_5^2 B_6 B_3^3 +$$
$$10187100 B_1 B_2^3 B_4^3 B_5^2 B_6 B_3^3 + 1774080 B_1^2 B_2^2 B_4^3 B_5^2 B_6 B_3^3 + 5913600 B_1^2 B_2^3 B_4^3 B_5 B_6 B_3^3)\mathbf{Z}^{13} +$$
$$(711480 B_2^2 B_4^3 B_5 B_6^2 B_3^4 + 2371600 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^4 + 6338640 B_1 B_2^2 B_4^3 B_5 B_6 B_3^4 +$$
$$1372140 B_2^2 B_4^3 B_5^2 B_6^2 B_3^3 + 3176250 B_1 B_2 B_4^3 B_5^2 B_6^2 B_3^3 + 127050 B_1 B_2^2 B_4^2 B_5^2 B_6^2 B_3^3 +$$
$$28420210 B_1 B_2^2 B_4^3 B_5 B_6^2 B_3^3 + 8575875 B_1 B_2^3 B_4^2 B_5 B_6^2 B_3^3 + 2032800 B_1^2 B_2^2 B_4^2 B_5 B_6^2 B_3^3 +$$
$$14314300 B_1 B_2^2 B_4^3 B_5^2 B_6 B_3^3 + 14437500 B_1 B_2^3 B_4^3 B_5 B_6 B_3^3 + 3056130 B_1^2 B_2^2 B_4^3 B_5 B_6 B_3^3 +$$
$$1559250 B_1^2 B_2^3 B_4^2 B_5 B_6 B_3^3)\mathbf{Z}^{12} + (508200 B_2 B_4^3 B_5^2 B_6^2 B_3^3 + 2439360 B_2^2 B_4^3 B_5 B_6^2 B_3^3 +$$
$$6225450 B_1 B_2 B_4^3 B_5 B_6^2 B_3^3 + 11384100 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 693000 B_2^2 B_4^3 B_5^2 B_6 B_3^3 +$$
$$3326400 B_1 B_2 B_4^3 B_5^2 B_6 B_3^3 + 369600 B_1 B_2^2 B_4^2 B_5^2 B_6 B_3^3 + 21801780 B_1 B_2^2 B_4^3 B_5 B_6 B_3^3 +$$
$$4331250 B_1 B_2^3 B_4^2 B_5 B_6 B_3^3 + 1478400 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^3 + 2069760 B_1 B_2^2 B_4^3 B_5^2 B_6 B_3^2)\mathbf{Z}^{11} +$$
$$(5082 B_1 B_2^2 B_4^2 B_6^2 B_3^3 + 1143450 B_2 B_4^3 B_5 B_6^2 B_3^3 + 996072 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 3811500 B_1 B_2 B_4^2 B_5 B_6^2 B_3^3 +$$
$$577500 B_2 B_4^3 B_5^2 B_6 B_3^3 + 1559250 B_2^2 B_4^3 B_5 B_6 B_3^3 + 7074375 B_1 B_2 B_4^3 B_5 B_6 B_3^3 + 9315306 B_1 B_2^2 B_4^2 B_5 B_6 B_3^3 +$$
$$853776 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^2 + 970200 B_1 B_2 B_4^3 B_5^2 B_6 B_3^2 + 349272 B_1 B_2^2 B_4^2 B_5^2 B_6 B_3^2 +$$
$$3234000 B_1 B_2^2 B_4^3 B_5 B_6 B_3^2 + 155232 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^2)\mathbf{Z}^{10} + (830060 B_2 B_4^2 B_5 B_6^2 B_3^3 +$$
$$23100 B_1 B_2^2 B_4^2 B_6 B_3^3 + 1478400 B_2 B_4^3 B_5 B_6 B_3^3 + 1131900 B_2^2 B_4^2 B_5 B_6 B_3^3 + 4928000 B_1 B_2 B_4^2 B_5 B_6 B_3^3 +$$
$$711480 B_1 B_2 B_4^2 B_5 B_6^2 B_3^2 + 178200 B_2 B_4^3 B_5^2 B_6 B_3^2 + 168960 B_1 B_2 B_4^2 B_5^2 B_6 B_3^2 + 2182950 B_1 B_2 B_4^3 B_5 B_6 B_3^2 +$$
$$2674100 B_1 B_2^2 B_4^2 B_5 B_6 B_3^2)\mathbf{Z}^9 + (1358280 B_2 B_4^2 B_5 B_6 B_3^3 + 177870 B_2 B_4^2 B_5 B_6^2 B_3^2 +$$
$$44100 B_1 B_2^2 B_4^2 B_5 B_3^2 + 44550 B_1 B_2^2 B_4^2 B_6 B_3^2 + 32340 B_2 B_4^2 B_5^2 B_6 B_3^2 + 495000 B_2 B_4^3 B_5 B_6 B_3^2 +$$
$$242550 B_2^2 B_4^2 B_5 B_6 B_3^2 + 3256110 B_1 B_2 B_4^2 B_5 B_6 B_3^2 + 202125 B_1 B_2^2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^8 +$$
$$(94080 B_1 B_2 B_4^2 B_5 B_3^2 + 32340 B_1 B_2 B_4^2 B_6 B_3^2 + 16500 B_1 B_2^2 B_4 B_6 B_3^2 + 1132560 B_2 B_4^2 B_5 B_6 B_3^2 +$$
$$316800 B_1 B_2 B_4 B_5 B_6 B_3^2 + 443520 B_1 B_2 B_4^2 B_5 B_6 B_3)\mathbf{Z}^7 + (36750 B_2 B_4^2 B_5 B_3^2 + 8085 B_2 B_4^2 B_6 B_3^2 +$$

Appendix: the full set of E_6 polynomials XXI

$26950 B_1 B_2 B_4 B_6 B_3^2 + 26950 B_4^2 B_5 B_6 B_3^2 + 107800 B_2 B_4 B_5 B_6 B_3^2 + 45360 B_1 B_2 B_4^2 B_5 B_3 + 249480 B_2 B_4^2 B_5 B_6 B_3 + 92400 B_1 B_2 B_4 B_5 B_6 B_3)\mathbf{Z}^6 + (9702 B_2 B_4 B_6 B_3^2 + 31500 B_2 B_4^2 B_5 B_3 + 10752 B_1 B_2 B_4 B_5 B_3 + 8316 B_1 B_2 B_4 B_6 B_3 + 23100 B_4^2 B_5 B_6 B_3 + 59136 B_2 B_4 B_5 B_6 B_3)\mathbf{Z}^5 + (4200 B_3 B_5 B_4^2 + 1050 B_1 B_2 B_3 B_4 + 9450 B_2 B_3 B_5 B_4 + 5775 B_2 B_3 B_6 B_4 + 6930 B_3 B_5 B_6 B_4)\mathbf{Z}^4 + (1050 B_2 B_3 B_4 + 2240 B_3 B_5 B_4 + 770 B_3 B_6 B_4)\mathbf{Z}^3 + (315 B_3 B_4 + 120 B_5 B_4)\mathbf{Z}^2 + 30 B_4 \mathbf{Z} + 1$

$\boxed{H_5} = B_1^2 B_2^3 B_3^4 B_4^3 B_5^2 B_6^2 \mathbf{Z}^{16} + 16 B_1 B_2^3 B_3^4 B_4^3 B_5^2 B_6^2 \mathbf{Z}^{15} + 120 B_1 B_2^2 B_3^4 B_4^3 B_5^2 B_6^2 \mathbf{Z}^{14} + 560 B_1 B_2^2 B_3^3 B_4^3 B_5^2 B_6^2 \mathbf{Z}^{13} + (1050 B_1 B_2^2 B_4^2 B_5^2 B_6^2 B_3^3 + 770 B_1 B_2^2 B_4^3 B_5^2 B_6 B_3^3)\mathbf{Z}^{12} + (672 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 3696 B_1 B_2^2 B_4^2 B_5^2 B_6 B_3^3)\mathbf{Z}^{11} + (3696 B_1 B_2^2 B_4^2 B_5 B_6 B_3^3 + 4312 B_1 B_2^2 B_4^2 B_5^2 B_6 B_3^2)\mathbf{Z}^{10} + (2640 B_1 B_2 B_3^2 B_5^2 B_6 B_4^2 + 8800 B_1 B_2^2 B_3^2 B_5 B_6 B_4^2)\mathbf{Z}^9 + (660 B_2 B_4^2 B_5^2 B_6 B_3^2 + 8085 B_1 B_2 B_4^2 B_5 B_6 B_3^2 + 4125 B_1 B_2^2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^8 + (2640 B_2 B_4^2 B_5 B_6 B_3^2 + 8800 B_1 B_2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^7 + (4312 B_2 B_4 B_5 B_6 B_3^2 + 3696 B_1 B_2 B_4 B_5 B_6 B_3)\mathbf{Z}^6 + (672 B_1 B_2 B_3 B_4 B_5 + 3696 B_2 B_3 B_4 B_6 B_5)\mathbf{Z}^5 + (1050 B_2 B_3 B_4 B_5 + 770 B_3 B_4 B_6 B_5)\mathbf{Z}^4 + 560 B_3 B_4 B_5 \mathbf{Z}^3 + 120 B_4 B_5 \mathbf{Z}^2 + 16 B_5 \mathbf{Z} + 1$

$\boxed{H_6} = B_1^2 B_2^4 B_3^6 B_4^4 B_5^2 B_6^4 \mathbf{Z}^{22} + 22 B_1^2 B_2^4 B_3^6 B_4^4 B_5^2 B_6^3 \mathbf{Z}^{21} + 231 B_1^2 B_2^4 B_3^5 B_4^4 B_5^2 B_6^3 \mathbf{Z}^{20} + (770 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5 + 770 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^5)\mathbf{Z}^{19} + (770 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5 + 5775 B_1^2 B_2^3 B_4^3 B_5^2 B_6^3 B_3^5 + 770 B_1^2 B_2^4 B_4^3 B_5 B_6^3 B_3^5)\mathbf{Z}^{18} + (8316 B_1 B_2^3 B_4^3 B_5^2 B_6^3 B_3^5 + 8316 B_1^2 B_2^3 B_4^3 B_5 B_6^3 B_3^5 + 9702 B_1^2 B_2^3 B_4^3 B_5^2 B_6^3 B_3^4)\mathbf{Z}^{17} + (14784 B_1 B_2^3 B_4^3 B_5 B_6^3 B_3^5 + 26950 B_1 B_2^3 B_4^3 B_5^2 B_6^3 B_3^4 + 26950 B_1^2 B_2^3 B_4^3 B_5 B_6^3 B_3^4 + 5929 B_1^2 B_2^3 B_4^3 B_5^2 B_6^2 B_3^4)\mathbf{Z}^{16} + (16500 B_1 B_2^2 B_4^3 B_5^2 B_6^3 B_3^4 + 90112 B_1 B_2^3 B_4^3 B_5 B_6^3 B_3^4 + 16500 B_1^2 B_2^3 B_4^2 B_5 B_6^3 B_3^4 + 23716 B_1 B_2^3 B_4^3 B_5^2 B_6^2 B_3^4 + 23716 B_1^2 B_2^3 B_4^3 B_5 B_6^2 B_3^4)\mathbf{Z}^{15} + (72765 B_1 B_2^2 B_4^3 B_5 B_6^3 B_3^4 + 72765 B_1 B_2^3 B_4^2 B_5 B_6^3 B_3^4 +$

Appendix: the full set of E_6 polynomials XXII

$$32670 B_1 B_2^2 B_4^3 B_5^2 B_6^2 B_3^4 + 108900 B_1 B_2^3 B_4^3 B_5 B_6^2 B_3^4 + 32670 B_1^2 B_2^3 B_4^2 B_5 B_6^2 B_3^4)\mathbf{Z}^{14} +$$
$$(107800 B_1 B_2^2 B_4^2 B_5 B_6^3 B_3^4 + 177870 B_1 B_2^2 B_4^3 B_5 B_6^2 B_3^4 + 177870 B_1 B_2^3 B_4^2 B_5 B_6^2 B_3^4 +$$
$$16940 B_1 B_2^2 B_4^3 B_5^2 B_6^2 B_3^3 + 16940 B_1^2 B_2^3 B_4^2 B_5 B_6^2 B_3^3)\mathbf{Z}^{13} + (379456 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^4 +$$
$$45276 B_1 B_2^2 B_4^2 B_5 B_6^3 B_3^3 + 5082 B_1 B_2^2 B_4^2 B_5^2 B_6^2 B_3^3 + 105875 B_1 B_2^2 B_4^3 B_5 B_6^2 B_3^3 + 105875 B_1 B_2^3 B_4^2 B_5 B_6^2 B_3^3 +$$
$$5082 B_1^2 B_2^2 B_4^2 B_5 B_6^2 B_3^3)\mathbf{Z}^{12} + 705432 B_1 B_2^2 B_3^3 B_4^2 B_5 B_6^2 \mathbf{Z}^{11} + (5082 B_1 B_2^2 B_4^2 B_6^2 B_3^3 +$$
$$5082 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 105875 B_1 B_2 B_4^2 B_5 B_6^2 B_3^3 + 105875 B_1 B_2^2 B_4 B_5 B_6^2 B_3^3 + 45276 B_1 B_2^2 B_4^2 B_5 B_6 B_3^3 +$$
$$379456 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^2)\mathbf{Z}^{10} + (16940 B_1 B_2^2 B_4 B_6^2 B_3^3 + 16940 B_2 B_4^2 B_5 B_6^2 B_3^3 + 177870 B_1 B_2 B_4^2 B_5 B_6^2 B_3^2 +$$
$$177870 B_1 B_2^2 B_4 B_5 B_6^2 B_3^2 + 107800 B_1 B_2^2 B_4^2 B_5 B_6 B_3^2)\mathbf{Z}^9 + (32670 B_1 B_2^2 B_4 B_6^2 B_3^2 + 32670 B_2 B_4^2 B_5 B_6^2 B_3^2 +$$
$$108900 B_1 B_2 B_4 B_5 B_6^2 B_3^2 + 72765 B_1 B_2 B_4^2 B_5 B_6 B_3^2 + 72765 B_1 B_2^2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^8 + (23716 B_1 B_2 B_4 B_6^2 B_3^2 +$$
$$23716 B_2 B_4 B_5 B_6^2 B_3^2 + 16500 B_1 B_2^2 B_4 B_6 B_3^2 + 16500 B_2 B_4^2 B_5 B_6 B_3^2 + 90112 B_1 B_2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^7 +$$
$$(5929 B_2 B_4 B_6^2 B_3^2 + 26950 B_1 B_2 B_4 B_6 B_3^2 + 26950 B_2 B_4 B_5 B_6 B_3^2 + 14784 B_1 B_2 B_4 B_5 B_6 B_3)\mathbf{Z}^6 +$$
$$(9702 B_2 B_4 B_6 B_3^2 + 8316 B_1 B_2 B_4 B_6 B_3 + 8316 B_2 B_4 B_5 B_6 B_3)\mathbf{Z}^5 + (770 B_1 B_2 B_3 B_6 +$$
$$5775 B_2 B_3 B_4 B_6 + 770 B_3 B_4 B_5 B_6)\mathbf{Z}^4 + (770 B_2 B_3 B_6 + 770 B_3 B_4 B_6)\mathbf{Z}^3 + 231 B_3 B_6 \mathbf{Z}^2 + 22 B_6 \mathbf{Z} + 1$$

Thank you for your attention!