CPT breaking and electric charge non-conservation

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Sources:

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- V. Novikov, Nuovo Cim. **C035N1** (2012) 243.

Summary

- We demonstrate that an inequality of masses for particle and antiparticle implies a non-conservation of the usually conserved charges.
- We demonstrate that CPT-violation due to the e^+e^- mass difference generates a photon mass.
- We demonstrate that cosmological bound on the photon mass leads to the bound on e^+e^- mass difference that more than 10 orders of magnitude stronger than the direct experimental bound (CERN ?!!).

Introduction into CPT

- Prehistory
- C-symmetry. Kramers(1937).
- CPT-symmetry. J.Schwinger (1951)
- First Proof of *CPT*
  Lüders (1954) and Pauli (1955) (Bell?) within the Hamiltonian formulation of quantum field theory with local and Lorentz invariant interaction.
- General Proof of *CPT*
  Jost (1957) within the axiomatic formulation of quantum field theory. The "local commutativity" condition was relaxed to "weak local commutativity".
- Lorentz symmetry has been an essential ingredient of the proof, both in the Hamiltonian QFT and in the axiomatic QFT.

Experiment

- The weak interactions break both P (1956) and C (1957) symmetries.
- Individual CP (1964) and T symmetries have been observed to be violated in hadrons.
- Combined product, CPT, remarkably remains as an exact symmetry (still).

Violation of Lorentz symmetry and CPT.

- The interplay of Lorentz symmetry and CPT symmetry was considered in the literature for decades
- A long list of references includes Coleman,Glashow, Okun,Colladay,Kostelecky, Cohen,Lehner ...
- Relation between the CPT and Lorentz invariance. Does the violation of any of symmetry automatically imply the violation of the other one?

CPT-violating Quantum Mechanics

- There is widely spread habit to parametrize CPT violation by attributing different masses to particle and antiparticle.
- This tradition is traced to a good old theory of $K - \bar{K}$-mesons oscillation.
- For a given momenta $\mathbf{q}$ the theory of oscillation is equivalent to a non-hermitian Quantum Mechanics (QM) with two degrees of freedom.
- Diagonal elements of 2×2 Hamiltonian matrix represent masses for particle and antiparticle. Their unequality breaks CPT-symmetry.
- Such strategy has no explicit loop-holes and is still used for parametrization of CPT-symmetry violation in D and B meson oscillations.

CPT-violating Quantum Field Theory.

(QFT) deals with an infinite sum over all momenta.
The set of plane waves with all possible momenta for particle and antiparticle is a complete set of orthogonal modes and an arbitrary field operator can be decomposed over this set.
Naive generalization of CPT-conserving QFT to CPT-violating QFT was to attribute different masses for particle and antiparticle.

- Phenomenology: different masses for neutrino and antineutrino (Murayama and Yanagida(2001)) .
- *CPT*-violating quantum field theory.
  By Barenboim et al (2001) and later by Greenberg (2002) .
- Greenberg conclusion: *CPT* violation implies violation of Lorentz invariance.
- This result was given as a "theorem"!!

The Simplest Example

For a complex scalar field one gets the infinite sum

$$\phi(x) = \sum_{\mathbf{q}} \left\{ a(\mathbf{q}) \frac{1}{\sqrt{2E}} e^{-i(Et-\mathbf{qx})} + b^+(\mathbf{q}) \frac{1}{\sqrt{2\tilde{E}}} e^{i(\tilde{E}t-\mathbf{qx})} \right\} \quad , \tag{1}$$

where $(a(\mathbf{q}), a^+(\mathbf{q}))$, $(b(\mathbf{q}),\ b^+(\mathbf{q}))$ are annihilation and creation operators, and (m, E) and $(\tilde{m}, \tilde{E})$ are masses and energies of particle and antiparticle respectively.

- Bose commutation relations for particle $a(p), a^+(p^{'})$ with mass m;
- Bose commutation relations for antiparticle $b(p), b^+(p^{'})$ with masse $\tilde{m}$
- Hamiltonian is a sum over free oscillators

Borenboim Calculations

- In this formalism one can calculate the Wightman functions:

$$< \phi(x), \phi(y)^+ >= D^+(x-y; m)\,, \tag{2}$$

$$< \phi(x)^+, \phi(y) >= D^-(x-y; \tilde{m})\,. \tag{3}$$

They are given by the standard Lorentz-invariant Pauli-Jordan functions but with different masses.

Greenberg arguments

- The commutator of two fields is equal to the difference $D^+(x-y;m) - D^-(x-y;\tilde{m})$ and does not vanish for space-like separation The theory can not be a Lorentz-invariant one.
- Propagator is not a Lorentz covariant one.

$$D_F(q) = \frac{1}{(2E(\mathbf{q}))}\frac{1}{(q_0 - E(\mathbf{q}))} - \frac{1}{(2\tilde{E}(\mathbf{q}))}\frac{1}{(q_0 + \tilde{E}(\mathbf{q}))} \quad (4)$$

Charge non-conservation

- Greenberg: Dynamic of fields breaks a Lorentz symmetry.
- ADD+VAN: Dynamivs of fields breaks the electric charge conservation!.
- For the standard QED the operator of electric charge

$$\hat{Q}(t) = \sum_{\mathbf{q}} \left\{ a^+(\mathbf{q})a(\mathbf{q}) - b^+(\mathbf{q})b(\mathbf{q}) \right\} \quad . \tag{5}$$

is a diagonal one. The modes with different momenta are orthogonal to each other and disappear after integration over space.

- For electron the modes with different momenta are still orthogonal to each other.
- The same is true for the modes of positron, they are also orthogonal among
- There is no reason for wave function of electron with mass m be orthogonal to wave functions for positron with mass $\tilde{m}$. Thus

$$Q(t) = Q_{loc} + \ C \sum_{\mathbf{q}} \frac{(E - \tilde{E})}{\sqrt{4E\tilde{E}}} \left[b(\mathbf{q}) a(-\mathbf{q}) e^{-i(E+\tilde{E})t} + \mathrm{h.c.} \right] ,$$

Constant C depends on the sorts of particles and on the definition of the charge.

- Non-conservation of charge exhibits itself in annihilation processes but not in the scattering processes
- Next step: Non-zero mass difference for charged particle and antiparticle Δm generates a non-zero mass for photon.
- In the theory where photon interacts with non-conserving electric current nothing protect photon from being massive.

At the first loop one gets non-zero m_γ:

$$m_\gamma^2 = C\,\frac{\alpha}{\pi}\,\Delta m^2\,. \tag{6}$$

The coefficient C can be calculated for any given convention about QFT with different masses for particle and antiparticle.
There are no reliable theoretical frameworks for calculations of C!!!

Numbers

According to the PDG:

$$|m_{e^+} - m_{e^-}|/m_{e^-} < 8 \cdot 10^{-9} \tag{7}$$

The mass difference Δm generates the photon mass of the order:

$$m_\gamma^2 \sim C \left(\frac{\alpha}{\pi}\right) \Delta m^2 \leq 10^{-8} C \ \mathrm{eV}^2 \,. \tag{8}$$

For any reasonable coefficient C and Δm which is not too far from the experimental upper bound this value of m_γ is huge,i.e. it is much larger than the existing limits.

The opposite way of reading:

$$\Delta m_e < 20\, m_\gamma/\sqrt{C}\,, \tag{9}$$

We have to substitute for m_γ the upper limit on the photon mass.

- The Earth based experiments: $m_\gamma < 3 \cdot 10^{-13}$ eV, $\Delta m < 6 \cdot 10^{-12}$ eV, i.e. 9 orders of magnitude stronger
- Magnetic field of the Jupiter: $m_\gamma < 4 \cdot 10^{-16}$ eV, $\Delta m < 8 \cdot 10^{-15}$ eV, i.e. 12 orders of magnitude stronger
- Solar wind at Pluto orbit: $m_\gamma < 10^{-18}$-eV (PDG) $\Delta m < 2 \cdot 10^{-17}$ eV, i.e. almost 14 orders of magnitude stronger .
- Large scale magnetic fields in galaxies: $m_\gamma < 2 \cdot 10^{-27}$ eV. $\Delta m < 4 \cdot 10^{-26}$ eV, which is 23 orders of magnitude stronger than the direct limit on the mass difference.
- Similar bounds can be derived on the CPT-odd mass differences of any other electrically charged particles and

Actual "Calculations"

- No one calculation can be done in a formal self-consistent way.
- 'a la Barenboim-Greenberg decomposition for an electron-positron spinor field operator $\Psi(x)$.

$$\Psi(x) = \sum_{\mathbf{p}} \left\{ a(\mathbf{p}) \frac{u(p)e^{-ipx}}{\sqrt{2\omega(p)}} + b^+(\mathbf{p}) \frac{u(-\mathbf{p})e^{i\tilde{p}x}}{\sqrt{2\tilde{\omega}(p)}} \right\} \qquad (10)$$

$$\left\{ a(\mathbf{p}), a^+(\mathbf{p}') \right\} = \delta_{\mathbf{p},\mathbf{p}'} \ , \quad \text{etc.} \qquad (11)$$

- The first term annihilates electron with mass m,
- The second term creates positron with mass $\tilde{m}$.

Actual "Calculations"

- Local product of field operators for the electric current

$$j_\mu(x) = \bar{\Psi}(x)\gamma_\mu\Psi(x) \quad .$$

Because of the electron-positron mass difference this current is not conserved, $\partial_\mu j(x) \neq 0$.

Actual "Calculations"

Consider the electron-positron pair contribution into the photon propagator, i.e. the polarization operator. Actually one can argue that polarization operator is still given by the text-book formula with covariant propagator with different masses m_1 and m_2 (we take $m_1 = m$, $m_2 = \tilde{m}$):

$$(ie^2) \int \frac{d^D p}{(2\pi)^D} \mathrm{Tr} \frac{1}{\hat{p} - m_1} \gamma_\nu \frac{1}{\hat{p} - \hat{q} - m_2} \gamma_\mu = \tilde{g}_{\mu\nu} \Pi_T(q^2) + g_{\mu\nu} \Pi_L(q^2), \tag{12}$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2$. The divergent integral has to be regularized and we choose covariant dimensional regularization.

Actual "Calculations"

For $m_1 = m_2$ the current is conserved and only transversal part of polarization operator $\Pi_T(q^2)$ is non-zero. Thus Π_T gives no contribution into the photon mass.
For non-conserving currents a longitudinal function $\Pi_L(q^2)$ would be generated. Nonzero $\Pi_L(0) \neq 0$ corresponds to non-zero photon mass.
Consider the divergence of $\Pi_{\mu\nu}$, i.e. $q_\mu \Pi_{\mu\nu} = q_\nu \Pi_L(q^2)$:

$$q_\nu \Pi_L(q^2) = ie^2 \int \frac{d^D p}{(2\pi)^D} \mathrm{Tr} \left[\frac{1}{\hat{p} - m_1} \right] \gamma_\nu \left[\frac{1}{\hat{p} - \hat{q} - m_L} \right] \hat{q} \ . \quad (13)$$

In the standard case of equal masses the $\hat{q}$ is a difference of two inverse propagators and we reproduce the standard Ward identity. In our case $\hat{q}$ is a difference of inverse propagators plus mass difference:

$$\hat{q} = (\hat{p} - m_1) - (\hat{p} - \hat{q} - m_2) + (m_1 - m_2) \ . \tag{14}$$

Substituting this formula into (13) we get a sum of 3 integrals.

The first one is zero due to Lorentz covariance.

$$\int \frac{d^D p}{(2\pi)^D} \mathrm{Tr}\gamma_\nu \frac{1}{\hat{p} - m_1} \equiv 0 \ . \tag{15}$$

The second one

$$\int \frac{d^D p}{(2\pi)^D} \mathrm{Tr}\gamma_\nu \frac{1}{\hat{p} - \hat{q} - m_2} = 0 \tag{16}$$

is zero if regularization allows to make a shift of variables.

The third integral

$$q_\nu \Pi_L(q^2) = ie^2(m_1 - m_2) \int \frac{d^D p}{(2\pi)^D} \frac{\mathrm{Tr}\,[(\hat{p} + m_1)\gamma_\nu(\hat{p} - \hat{q} + m_2]}{(p^2 - m_1^2)[(p-q)^2 - m_2^2]} =$$

$$= 4e^2(m_1 - m_2) q_\nu \int_0^1 dx \int \frac{d^D p}{(2\pi)^D} \frac{m_2 x - m_1(1-x)}{[p^2 + \Delta^2]^2} \quad (17)$$

where $\Delta^2 = m_1^2(1-x) + m_2^2 x - q^2 x(1-x)$.

This integral is logarithmically divergent. At this moment we can forget about dimensional regularization. For $q^2 = 0$ this integral is trivial and one gets that

$$m_\gamma^2 = \Pi_L(0) = \frac{\alpha}{2\pi}\left[m_2 - m_1\right]^2 \left[\ln\frac{\Lambda^2}{m^2} - \frac{5}{3}\right] \quad , \qquad (18)$$

where Λ is a cut-off, i.e. the photon mass is divergent and has to be renormalized. There is no principle that protect m_γ from being non-zero. Formally it can be an arbitrary number. But if loops have any physical sense for such theories this number has to be proportional to fine coupling constant and disappear for equal mass, i.e. we arrive to eq. (5)

Conclusion

We demonstrate that CPT-violation due to the e^+e^- mass difference generates a photon mass.