A practical parametrisation of line shapes of near-threshold resonances

A.V. Nefediev

ITEP, MEPhI, MIPT

The International Conference on Particle Physics and Astrophysics
October 5-10, 2015, Moscow, Russia

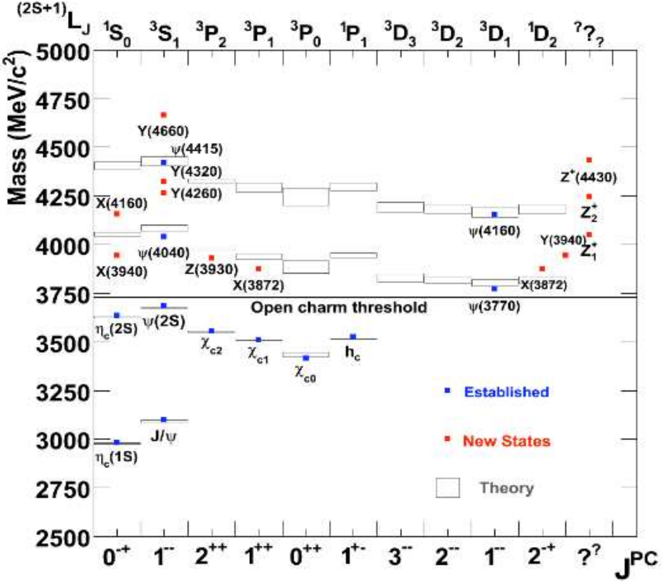
Acknowlegments

I would like to gratefully acknowledge:

- Kind invitation of the Organising Committee to participate in this conference and to present this talk
- Collaboration with colleagues from ITEP (Russia) and Forschungszentrum Juelich (Germany):

  C.Hanhart, Yu.Kalashnikova, P.Matuschek, R.Mizuk, Q.Wang
- Financial support from the Russian Science Foundation (Grant No. 15-12-30014)

Challenge: hadronic states above open-flavour thresholds



Ambitious programme

To build a parametrisation for threshold phenomena which is

- motivated by phenomenology and simple enough to be used in data analysis (no blind replication of parameters)

- powerful enough to describe all relevant data sets simultaneously

Interaction potential

The setup: a coupled-channel problem for

(i) bare pole (elementary state)

(ii) N_e elastic (open-flavour) channels

(iii) N_i inelastic (hidden-flavour) channels

$$\hat{V} = \begin{array}{cccc} \text{Pole} & \beta=\overline{1,N_e} & i=\overline{1,N_{in}} & \\ 0 & f_\beta(\boldsymbol{p}') & f_i(\boldsymbol{k}) & \text{Pole} \\ f_\alpha(\boldsymbol{p}) & v_{\alpha\beta}(\boldsymbol{p},\boldsymbol{p}') & v_{\alpha i}(\boldsymbol{p},\boldsymbol{k}) & \alpha=\overline{1,N_e} \\ f_j(\boldsymbol{k}') & v_{j\beta}(\boldsymbol{k}',\boldsymbol{p}') & 0 & j=\overline{1,N_{in}} \end{array}$$

Example 1: $B \to K X(3872)$ with $X(3872) \to D\bar{D}^*$, $\rho J/\psi$, $\omega J/\psi$

Example 2: $\Upsilon(5S) \to \pi Z_b^{(\prime)}$ with $Z_b^{(\prime)} \to B\bar{B}^*$, $B^*\bar{B}^*$, $\pi\Upsilon(1S)$, $\pi\Upsilon(2S)$, $\pi\Upsilon(3S)$, $\pi h_b(1P)$, $\pi h_b(2P)$

Effective elastic scattering potential

Assume separable form of the elastic-to-inelastic vertex

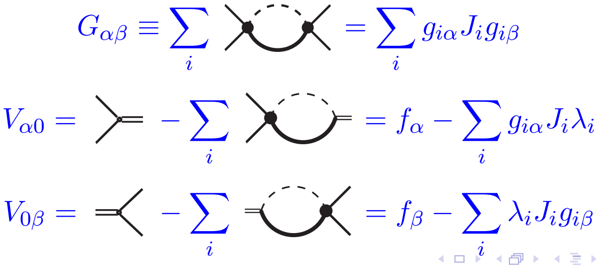
$$v_{\alpha i}(\boldsymbol{p},\boldsymbol{k}) = \chi_\alpha(\boldsymbol{p})\varphi_{i\alpha}(\boldsymbol{k}) \qquad \chi_\alpha(\boldsymbol{p}=0)=1$$

and use parametrisation of the vertices as

$$f_\alpha(\boldsymbol{p}) = f_\alpha \quad \chi_\alpha(\boldsymbol{p}) = 1 \quad \varphi_{i\alpha}(\boldsymbol{k}) = g_{i\alpha}|\boldsymbol{k}|^{l_i} \quad f_i(\boldsymbol{k}) = \lambda_i|\boldsymbol{k}|^{l_i}$$

Inelastic channels enter additively, for example,

$$V^{\text{eff}}_{\alpha\beta} = v_{\alpha\beta} - G_{\alpha\beta} - V_{\alpha 0}G_0V_{0\beta}$$



$$G_{\alpha\beta} \equiv \sum_i \qquad = \sum_i g_{i\alpha}J_i g_{i\beta}$$

$$V_{\alpha 0} = \qquad - \sum_i \qquad = f_\alpha - \sum_i g_{i\alpha}J_i\lambda_i$$

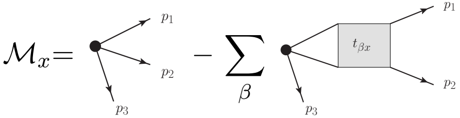
$$V_{0\beta} = \qquad - \sum_i \qquad = f_\beta - \sum_i \lambda_i J_i g_{i\beta}$$

Line shapes of near-threshold resonance(s)

- Solve Lippmann–Schwinger equation for the full t-matrix

$$t = \hat{V} - \hat{V} S t$$

- Consider production from point-like sources



- Construct differential BF's $d\mathrm{Br}_x/dM \propto |\mathcal{M}_x|^2 p_3 k_x$
- Fit parameters of the model:
  (i) elastic direct interaction potential ($v_{\alpha\beta}$)
  (ii) couplings (couplings g's, λ's and f's)
  (iii) ratios of production sources (ξ's)
  (iv) range of force in elastic channels (κ)
  (v) bare elementary state mass (M_0)
  (vi) overall norm (Λ)

Line shapes and parameters

Simultaneous fit to the data in elastic and inelastic production channels can be done with the help of the formulae

$$\frac{dBr^e_\alpha}{dM} = \Lambda\Big|\sum_\beta \xi_\beta t_{\beta\alpha}\Big|^2 p_3 k_\alpha \quad \alpha = \overline{1, N_e}$$

$$\frac{dBr^{in}_i}{dM} = \Lambda\Big|\sum_\alpha \xi_\alpha t_{\alpha i}\Big|^2 p_3 k^{in}_i \quad i = \overline{1, N_{in}}$$

with the fitting parameters

$$\{\Lambda,\ \xi_\alpha,\ f_\alpha,\ \lambda_i,\ g_{i\alpha},\ M_0,\ \kappa,\ t^v\}$$

which might obey various symmetries constrains

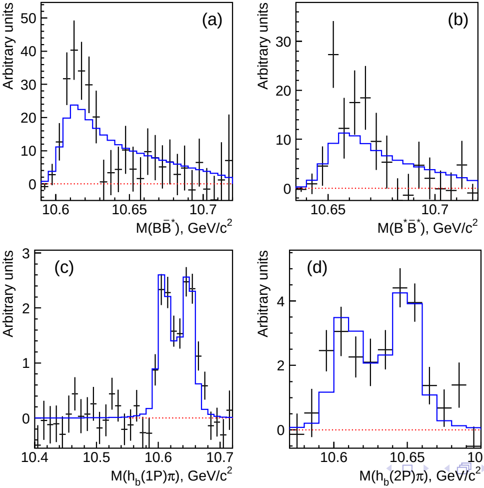
A typical data set to be described:

$$\begin{aligned}
\Upsilon(5S) &\to \pi Z_b^{(\prime)} \to \pi B^{(*)}\bar{B}^* \\
\Upsilon(5S) &\to \pi Z_b^{(\prime)} \to \pi\pi\Upsilon(nS) \qquad n = 1,2,3 \\
\Upsilon(5S) &\to \pi Z_b^{(\prime)} \to \pi\pi h_b(mP) \qquad m = 1,2
\end{aligned}$$

Paradigmatic example

Line shapes of the $Z_b(10610)$ and $Z_b(10650)$ in the $B^{(*)}\bar{B}^*$ and $\pi h_b(mP)$ $(m = 1, 2)$ channels with the strict HQSS constrains imposed: CL 50%

New Belle data announced in September 2015



Conclusions

- Description of the experimental data for near-threshold states requires adequate parametrisations which respect requirements from unitarity and analiticity
- To employ the full information contained in the data all relevant channels should be analysed simultaneously
- Parameters extracted from the fit should be used to find renormalisation group invariant quantities describing the near-threshold resonance(s)