Chiral extrapolation of the $X(3872)$ binding energy

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$X(3872)$ as a typical near-threshold state

$$M_X = 3871.68 \pm 0.17 \text{ MeV} \approx m(D^0) + m(D^{*0})$$

$$M_{D^0} + M_{D^{*0}} - M_X = (0.12 \pm 0.26) \text{ MeV}$$

$$\Gamma_X < 1.2 \text{ MeV}$$

\[ \begin{array}{ccc}
D^0\bar{D}^0\pi^0 & X & D^+D^{*-} + \text{c.c.} \\
\sim 7 \text{ MeV} & & \sim 8 \text{ MeV}
\end{array} \]
Observation of $X(3872)$

- Observation of the $X(3872)$ in the mode
  \[ B^+ \to K^+ X \to K^+ \left( \pi^+ \pi^- J/\psi \right) \]
  (Belle 2003, CDF, DØ, BABAR 2004-2006, LHCb 2013)

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  \[ B^+ \to K^+ X \to K^+ \left( D^0 \bar{D}^0 \pi^0 \right) \]
  (Belle 2006, BABAR 2007)

- Studies of the $X$ radiative decays (large branchings!)
  \[ X \to \gamma J/\psi \quad X \to \gamma \psi'(3686) \]
  (BABAR 2008, Belle 2010/2011)
Conclusions from the theoretical analysis

- $X(3872)$ is a bound state generated dynamically by a strong coupling of the genuine charmonium $\chi'_{c1}$ to the $D\bar{D}^*$ hadronic channel.

- Admixture of the $c\bar{c}$ charmonium in the w.f. is $\sim 50\%$.

- $X(3872)$ is bound by short-range forces, one pion exchange (OPE) and related three-body dynamics playing an important role for the $X$ properties.
One pion exchange in $X(3872)$

\[ \left( V_{\text{OPE}}(p, p') \right)_{ij} = -g^2 \frac{q_i q'_j}{D_3(p, p')} \]

\[ q \approx q' \approx p_\pi \]

\[ D_3(p, p') = \left( E_D(p) + E_D(p') + E_\pi(p_\pi) \right) - (m_* + m + E) \]

\[ V_{\text{OPE}} \propto \frac{p^2_{\pi}}{p^2_{\pi} + \mu^2} \] \[ p_\pi \to \infty \] \[ = O(1) \]

Therefore OPE in $X(3872)$ contains short-range physics which results in divergent loops
EFT approach to the $X$

- Short-range interaction (including the short-range part of OPE) is described by a constant contact term $C_0$

$$V_{\text{full}} = C_0(\Lambda) + V_{\text{OPE}}^{\text{reg}}(\Lambda)$$

- The contact term is fixed to generate $X$ as a bound state with a given binding energy $E_B$

$$\frac{\partial E_B}{\partial \Lambda} = 0 \implies C_0(\Lambda) \text{ is fixed}$$

- Relativised approach is renormalisable in LO
- The full dynamical relativised problem is solved for $V_{\text{full}}$:
  - Three-body dynamics included
  - Unitarity preserved
  - Renormalisation group equation satisfied
  - Renormalisable approach
**$X(3872)$ on the lattice**

Lattice predictions for $M_X - m_{D^0} - m_{D^*0}$:

<table>
<thead>
<tr>
<th>Authors</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prelovsek, Leskovec, PRL 111, 192001 (2013)</td>
<td>$-(11 \pm 7)$ MeV</td>
</tr>
<tr>
<td>Fermilab Lattice and MILC Collabs, arXiv:1411.1389 [hep-lat]</td>
<td>$-(13 \pm 6)$ MeV</td>
</tr>
<tr>
<td>Padmanath, Lang, Prelovsek, arXiv:1503.03257 [hep-lat]</td>
<td>$-(8 \pm 15)$ MeV, $-(9 \pm 8)$ MeV</td>
</tr>
</tbody>
</table>

Experimental value:

$$M_X - m_{D^0} - m_{D^*0} = -(0.12 \pm 0.26) \text{ MeV}$$

Thus lattice simulations systematically predict $X(3872)$ bound stronger than observed experimentally and it is a challenge for the theory to provide an extrapolation formula for the $E_B$ to proceed from $m_\pi \simeq 300$ MeV to $m_\pi = 140$ MeV.
Leading order prediction for the $X$ binding energy

At LO dependence on $m_\pi$ comes only from the long-range OPE and from renormalised selfenergy loops $\implies$ **Prediction!**

Black solid curve — full dynamical theory with 3-body effects

Blue dotted curve — simplified formulation with static OPE

Conclusions:

- 3-body effects are important
- In LO binding energy decreases fast with the increase of $m_\pi$
- To arrive at stronger bound state one has to proceed to NLO
Contact interaction “running” law

\[ C_0(\Lambda, m_\pi) = C_0^{\text{ph}}(\Lambda, m_{\pi}^{\text{ph}}) + \delta C_0(\Lambda, m_\pi) \]

What do we do with \( \delta C_0(\Lambda, m_\pi) \)?

- **Approach #1:** \( \delta C_0(\Lambda, m_\pi) \) is prescribed using a suitable model; \( E_B \) and \( \partial E_B / \partial m_\pi \) at the physical point are treated as input
  Outcome — prediction for \( E_B(m_{\pi}^{\text{unph}}) \) to compare with lattice simulations

- **Approach #2:** Lattice data are used to fix \( E_B(m_{\pi}^{\text{unph}}) \) for two values of \( m_{\pi}^{\text{unph}} \)
  Outcome — extrapolation formula to \( m_{\pi}^{\text{ph}} \)
Approach #1

- Dashed-dotted line: pionless theory
- Dashed line: natural assumption \( \left( \frac{\partial E_B}{\partial m^2_\pi} \right) \bigg|_{m_\pi = m^{ph}_\pi} = \frac{E_B^{ph}}{m^{ph}_\pi} \)
- Red band: relativised approach \( (\Lambda \to \infty) \) with resonance saturation
- Black band: nonrelativistic approach \( (\Lambda \in [500,700] \text{ MeV}) \) in heavy-meson formulation
- Blue dot with error bar: lattice result by Prelovsek & Leskovec’2013
Approach #2

Red band — the full calculation with dynamical pions in NLO
Blue band — static OPE

physical pion mass

gedanken lattice data
Conclusions

- Three-body dynamics is important in the $X(3872)$

- Lattice predictions of stronger bound $X$ for unphysical pion masses is compatible with the $X$ formed by short-range forces

- Extrapolation formula from unphysically large pion masses to the physical point is nontrivial and it is strongly influenced by the three-body dynamics

- Simulations of the $X$ on the lattice may provide valuable information on binding mechanisms in it

- Suggested approach supports generalisations to other near-threshold states