Chiral extrapolation of the $X(3872)$ binding energy

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The International Conference on Particle Physics and Astrophysics
October 5-10, 2015, Moscow, Russia

Acknowlegments

I would like to gratefully acknowledge:

- Kind invitation of the Organising Committee to participate in this conference and to present this talk

- Collaboration with colleagues from Forschungszentrum Juelich, Bonn Univ., and Bochum Univ. (Germany):

  V.Baru, E.Epelbaum, A.Filin, C.Hanhart, U.-G.Meißner

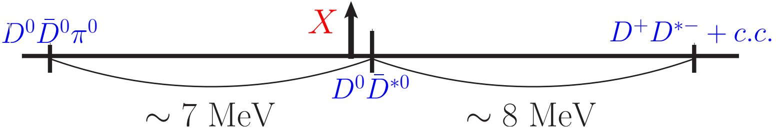
- Financial support from the Russian Science Foundation (Grant No. 15-12-30014)

$X(3872)$ as a typical near-threshold state

$$M_X = 3871.68 \pm 0.17 \text{ MeV} \approx m(D^0) + m(\bar{D}^{*0})$$

$$M_{D^0} + M_{\bar{D}^{*0}} - M_X = (0.12 \pm 0.26) \text{ MeV}$$

$$\Gamma_X < 1.2 \text{ MeV}$$



Observation of $X(3872)$

- Observation of the $X(3872)$ in the mode

$$B^+ \to K^+ X \to K^+ \left(\pi^+\pi^- J/\psi\right)$$

(Belle 2003, CDF, D∅, BABAR 2004-2006, LHCb 2013)

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(Belle 2005, BABAR 2010)

- Observation of the $X(3872)$ in the mode

$$B^+ \to K^+ X \to K^+ \left(D^0 \bar{D}^0 \pi^0\right)$$

(Belle 2006, BABAR 2007)

- Studies of the X radiative decays (large branchings!)

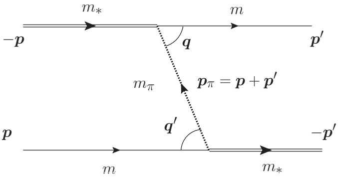
$$X \to \gamma J/\psi \qquad X \to \gamma \psi'(3686)$$

(BABAR 2008, Belle 2010/2011)

Conclusions from the theoretical analysis

- $X(3872)$ is a bound state generated dynamically by a strong coupling of the genuine charmonium χ'_{c1} to the $D\bar{D}^*$ hadronic channel
- Admixture of the $c\bar{c}$ charmonium in the w.f. is $\sim 50\%$
- $X(3872)$ is bound by short-range forces, one pion exchange (OPE) and related three-body dynamics playing an important role for the X properties

One pion exchange in $X(3872)$



$$\left(V_{\mathrm{OPE}}(\boldsymbol{p},\boldsymbol{p}')\right)_{ij} = -g^2 \frac{q_i q_j'}{D_3(\boldsymbol{p},\boldsymbol{p}')}$$

$$\boldsymbol{q} \approx \boldsymbol{q}' \approx \boldsymbol{p}_\pi$$

$$D_3(\boldsymbol{p},\boldsymbol{p}') = \Big(E_D(\boldsymbol{p}) + E_D(\boldsymbol{p}') + E_\pi(\boldsymbol{p}_\pi)\Big) - (m_* + m + E)$$

$$V_{\mathrm{OPE}} \propto \frac{p_\pi^2}{p_\pi^2 + \mu^2} \underset{p_\pi \to \infty}{=} O(1)$$

Therefore OPE in $X(3872)$ contains short-range physics which results in divergent loops

EFT approach to the X

- Short-range interaction (including the short-range part of OPE) is described by a constant contact term C_0

$$V_{\rm full} = C_0(\Lambda) + V_{\rm OPE}^{\rm reg}(\Lambda)$$

- The contact term is fixed to generate X as a bound state with a given binding energy E_B

$$\frac{\partial E_B}{\partial \Lambda} = 0 \quad \Longrightarrow \quad C_0(\Lambda) \text{ is fixed}$$

- Relativised approach is renormalisable in LO
- The full dynamical relativised problem is solved for $V_{\rm full}$:
  - Three-body dynamics included
  - Unitarity preserved
  - Renormalisation group equation satisfied
  - Renormalisable approach

$X(3872)$ on the lattice

Lattice predictions for $M_X - m_{D^0} - m_{D^{*0}}$:

Prelovsek, Leskovec, PRL **111**, 192001 (2013)	$-(11 \pm 7)$ MeV
Fermilab Lattice and MILC Collabs, arXiv:1411.1389 [hep-lat]	$-(13 \pm 6)$ MeV
Padmanath, Lang, Prelovsek arXiv:1503.03257 [hep-lat]	$-(8 \pm 15)$ MeV $-(9 \pm 8)$ MeV

Experimental value:

$$M_X - m_{D^0} - m_{D^{*0}} = -(0.12 \pm 0.26)\ \text{MeV}$$

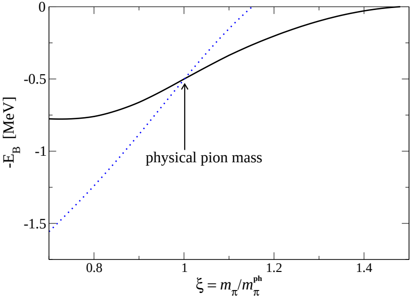
Thus lattice simulations systematically predict $X(3872)$ bound stronger than observed experimentally and it is a challenge for the theory to provide an extrapolation formula for the $\boldsymbol{E}_B$ to proceed from $m_\pi \simeq 300$ MeV to $m_\pi = 140$ MeV

Leading order prediction for the X binding energy

At LO dependence on m_π comes only from the long-range OPE and from renormalised selfenergy loops $\Longrightarrow$ Prediction!

Black solid curve — full dynamical theory with 3-body effects

Blue dotted curve — simplified formulation with static OPE



Conclusions:

- 3-body effects are important
- In LO binding energy decreases fast with the increase of m_π
- To arrive at stronger bound state one has to proceed to NLO

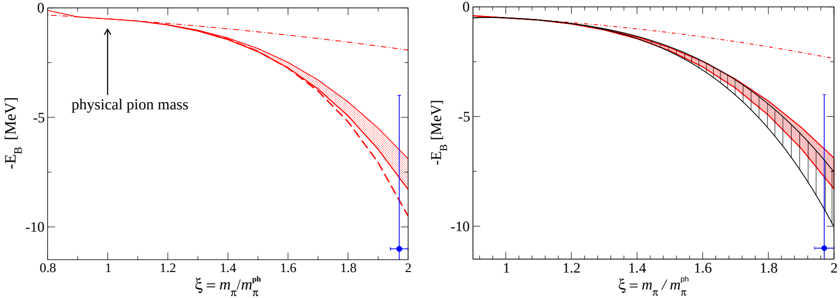
Contact interaction “running” law

$$C_0(\Lambda, m_\pi) = C_0^{\rm ph}(\Lambda, m_\pi^{\rm ph}) + \delta C_0(\Lambda, m_\pi)$$

What do we do with $\delta C_0(\Lambda, m_\pi)$?

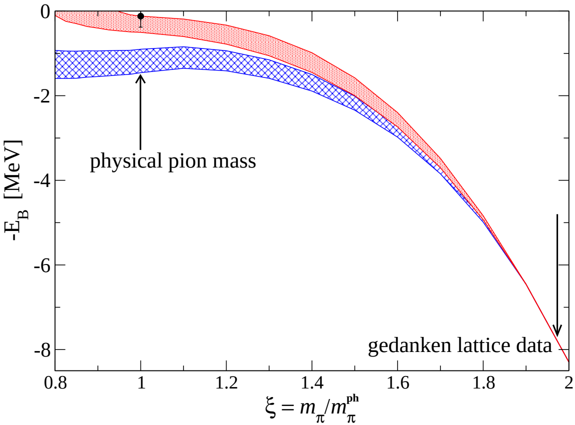
- Approach #1: $\delta C_0(\Lambda, m_\pi)$ is prescribed using a suitable model; E_B and $\partial E_B/\partial m_\pi$ at the physical point are treated as input
  Outcome — prediction for $E_B(m_\pi^{\rm unph})$ to compare with lattice simulations
- Approach #2: Lattice data are used to fix $E_B(m_\pi^{\rm unph})$ for two values of $m_\pi^{\rm unph}$
  Outcome — extrapolaton formula to $m_\pi^{\rm ph}$

Approach #1



- Dashed-dotted line: pionless theory
- Dashed line: natural assumption $(\partial E_B/\partial m_\pi^2)\big|_{m_\pi=m_\pi^{\rm ph}} = E_B^{\rm ph}/{m_\pi^{\rm ph}}^2$
- Red band: relativised approach ($\Lambda \to \infty$) with resonance saturation
- Black band: nonrelativistic approach ($\Lambda \in$ [500,700] MeV) in heavy-meson formulation
- Blue dot with error bar: lattice result by Prelovsek & Leskovec'2013

Approach #2



Red band — the full calculation with dynamical pions in NLO
Blue band — static OPE

Conclusions

- Three-body dynamics is important in the $X(3872)$
- Lattice predictions of stronger bound X for unphysical pion masses is compatible with the X formed by short-range forces
- Extrapolation formula from uphysically large pion masses to the physical point is nontrivial and it is strongly influenced by the three-body dynamics
- Simulations of the X on the lattice may provide valuable information on binding mechanisms in it
- Suggested approach supports generalisations to other near-threshold states