Fermion scattering on deformed extra space

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Intoduction

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The point-like defect on a space with topology of sphere leads to fermion interaction with such defect and calculate cross section of a fermion scattering on such defect.

Deformed extra space

Consider the manifold $M^4 \times S^2$ with a metric:

$$ds^2 = G_{ab} dX^a dX^b = \eta_{\mu\nu} dx^\mu dx^\nu - r^2(x,\theta)(d\theta^2 + \sin^2(\theta) d\phi^2), \qquad (1)$$

where $a, b = 0..5$, $\mu, \nu = 0..3$.
The function $r(\theta, x)$ determined by numerical solution of multidimensional equations for gravity.

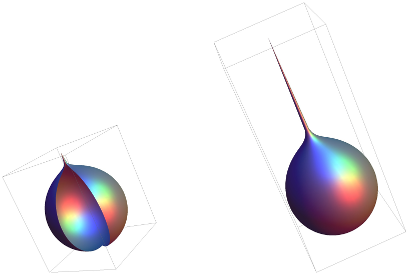

Figure: The extra space configurations

Action and Dirac equation

Lets start from 6-dimensional action for fermion on curved space-time:

$$S = \int d^6x \sqrt{|G|} i\overline{\Psi} e_a^A \Gamma_a \nabla_A \Psi, \quad (2)$$

where Ψ - is a six-dimensional Dirac spinor, e_a^A - is a "vielbein".

The covariant derivative has the form:

$$\nabla_A = \partial_A + \frac{1}{4}\omega_A^{ab}\Gamma_{ab} \quad (3)$$

The variational principle leads to the Dirac equation:

$$i\left(\Gamma^\mu\left(\partial_\mu - \frac{\partial_\mu r}{r}\right) + \frac{\Gamma_4}{r}\left(\partial_\theta + \frac{1}{2}\left[\cot(\theta) + \frac{r'}{r}\right]\right) + \frac{\Gamma_5}{r\sin(\theta)}\partial_\varphi\right)\Psi = 0. \quad (4)$$

4-dimensional effective action

After standard calculations the expression for the 4-dimensional effective action acquires a form:

$$S = \int d^4x(i\overline{\psi}\gamma_\mu\partial_\mu\psi I(x) - \overline{\psi}\gamma_\mu\psi A_\mu(x)), \tag{5}$$

where

$$I(x) = 2\pi \int r^2 \sin(\theta) d\theta,\ A_\mu(x) = \frac{1}{2}\partial_\mu I(x). \tag{6}$$

Thus Lagrangian gets a coupling between fermion and vector and scalar fields originated from point-like defect. The size of extra space is supposed to vary depending on 4-dimensional coordinates.

$$r(\theta, x) = a(\theta) + b(\theta)e^{-cr}. \tag{7}$$

a and b functions are defined by numerical solution of multidimensional equations for gravity, c parameter is found from principle of least action.

Amplitude and cross-section

To obtain an expression for the cross-section one should write down a Feynman rules, considering vector and scalar fields classically. Invariante amplitude in the first order of perturbation theory is:

$$M = i\overline{u_2}\gamma_\mu u_1 p_{1\mu}\chi(q) + \overline{u_2}\gamma_\mu u_1 A_\mu(q), \tag{8}$$

where

$$\chi(q) = \frac{32\pi^2 c\alpha}{(q^2+c^2)^2} + \frac{32\pi^2 c\beta}{(q^2+4c^2)^2}, \; A_\mu(q) = \frac{i}{2}\chi(q)q_\mu. \tag{9}$$

Now one can calculate the square of invariante amplitude:

$$\overline{|M|}^2 = 4m^2\chi^2(m^2 + (E^2 - m^2)\sin^2\left(\frac{\theta_4}{2}\right)). \tag{10}$$

Cross-section of the fermion scattering on classical potential is given by:

$$\frac{d\sigma}{d\Omega_4} = \frac{1}{16\pi^2}\overline{|M|}^2. \tag{11}$$

Electron scattering cross-section

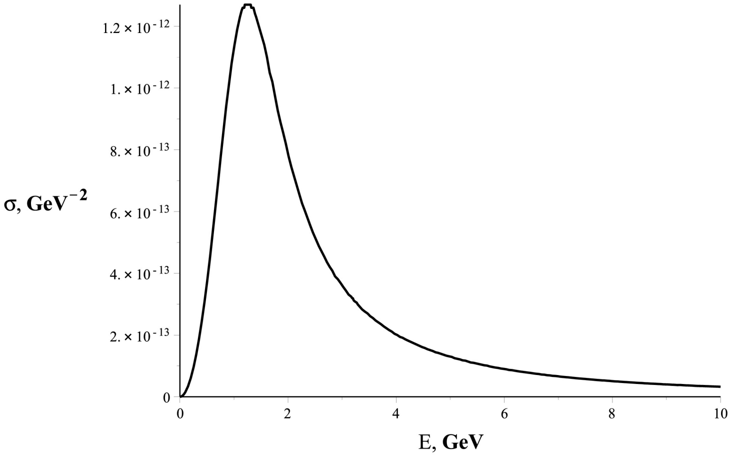

Figure: Electron scattering cross-section vs full energy of the electron, $m_D = 10\ TeV$, $\alpha \sim 10^{-2}$,$\beta \sim 10^{-2}$ and $c = 10^{-3} m_D$

Conclusion

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Thank you!