Visible, invisible and trapped ghosts as sources of wormholes and black universes.

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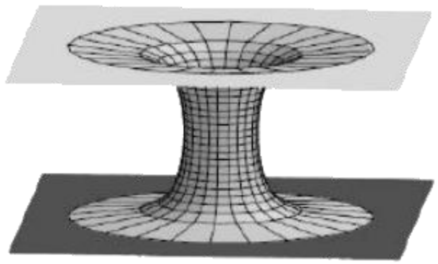
Motivation

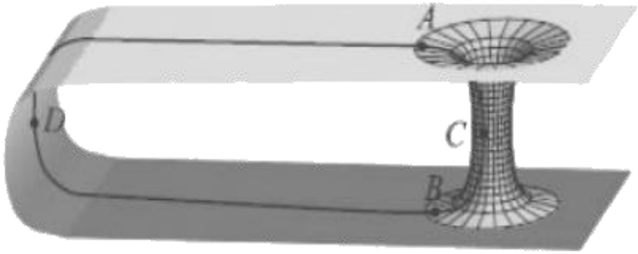
- Search for non-singular BH-like solutions in classical gravity
- Exact solutions combining properties of BH, WH, and non-singular cosmological models
- Phantom scalar fields in the context of the accelerated cosmological expansion (estimates give w . -1, e.g., Planck-2015)
- Possible existence of a global primordial magnetic field ($\sim$ 10−15 G) causing correlated orientations of quasars distant from each other (Poltis and Stojkovic, 2010). Possible global anisotropy
- Different geometric and causal structures and their connection with some cosmological scenarios
- Phantom fields are not observed $\Rightarrow$ the "trapped ghost" concept
- Problems with "trapped ghosts" (instability) $\Rightarrow$ possible "invisible ghosts"

What is a wormhole?

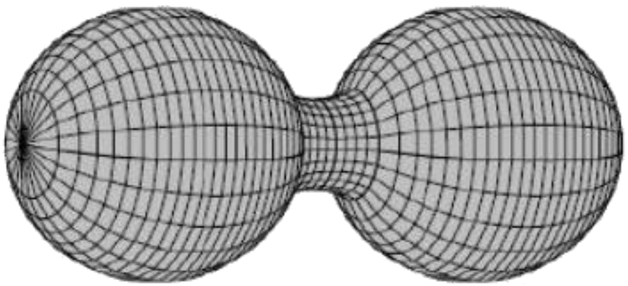
Wormholes (Spherical vs. Cylindrical)



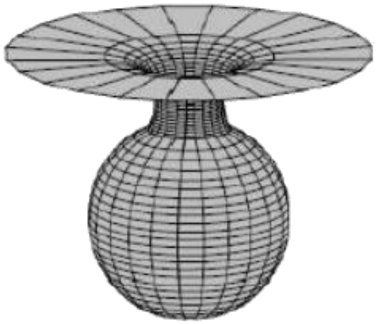
SPH: twice asymptotically flat WH topology (**S^2** x **R**)
CYL: topology (**S^1** x **R** x **R**)



A wormhole is a "handle"? A shortcut between remote parts of the universe (or a time machine if times at A and B are essentially different)



A "hanging drop" wormhole.
Topology: **R^3** for **SPH** and **CYL**



A "dumbbell" wormhole.
SPH: topology S^3
CYL: topology **S^2** x **R**

Black Universes

A black universe (BU) is a regular black hole where, beyond the horizon, instead of a singularity there is an expanding, asymptotically isotropic space-time. It combines the properties of the following objects:

- A black hole (BH) — a Killing horizon separating static and non-static space-time regions;
- A wormhole (WH) — no center and a regular minimum of the area of coordinate spheres;
- A nonsingular cosmological model — at large times the nonstatic region reaches a de Sitter (dS) mode of isotropic expansion;

The Model

Action[1]: $S = \frac{1}{2}\int \sqrt{-g}\, d^4x \Big[R + 2\epsilon g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - F^{\mu\nu}F_{\mu\nu} - 2V(\phi)\Big]$

- $F_{\mu\nu}$ — electromagnetic field tensor;
- ϕ — phantom scalar field ($\epsilon = -1$) with a potential $V(\phi)$.

General (formally) static sph. symm. metric in quasiglobal gauge ($g_{00}g_{11} = -1$):

$$ds^2 = A(\rho)dt^2 - \frac{d\rho^2}{A(\rho)} - r^2(\rho)d\Omega^2$$

$\rho \in \mathbb{R}$ — quasiglobal coordinate, $\partial\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$,
$r(\rho)$ — "area function",
$A(\rho)$ — "redshift function",
$A(\rho) > 0 \quad \Rightarrow \quad$ R-region,
$A(\rho) < 0 \quad \Rightarrow \quad$ T-region.

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[1] $\hbar = c = 8\pi G = 1$, signature $(+\ -\ -\ -)$

Properties of function A(ρ), r(ρ) for WH's and BU's

Both $A(\rho)$ and $r(\rho)$ should be regular, $r(\rho) > 0$ everywhere, and $r(\pm\infty) \to \infty$.

Regions with $A > 0$ (R-regions) $\Rightarrow$ static,
with $A < 0$ (T-regions) $\Rightarrow$ cosmological (Kantowski-Sachs).

Flat, de Sitter or AdS asymptotic behavior as $\rho \to \pm\infty$:

- **WH**: no horizons ($A(\rho) > 0$ everywhere), and flat or AdS asymptotics at both ends.
- **BU**: flat or AdS asymptotic at one end; de Sitter asymptotic at the other end.

Fields

Scalar field $\phi(x)$:

$$T_\mu^\nu[\phi] = \epsilon A(\rho)\phi'(\rho)^2 \,\mathrm{diag}(1,\ -1,\ 1,\ 1) + \delta_\mu^\nu V(\rho)$$

$\epsilon = -1$ — phantom field

Electromagnetic field $F_{\mu\nu}(x)$ (conforms to Wheeler's idea of a "charge without charge"):

$$F_{01} = -F_{10} \ \text{(electric)}, \quad F_{01}F^{01} = -q_e^2/r^4(\rho)$$

$$F_{23} = -F_{32} \ \text{(magnetic)}, \quad F_{23}F^{23} = q_m^2/r^4(\rho)$$

$$T_\mu^\nu[F] = \frac{q^2}{r^4(\rho)} \,\mathrm{diag}(1,\ 1,\ -1,\ -1), \qquad q^2 = q_e^2 + q_m^2.$$

(Maxwell's equations have been solved)

Einstein and Scalar Field Equations

There are three independent equations [Eqs. (4), (5) follow from (1) – (3)] for 4 functions $r(\rho)$, $A(\rho)$, $V(\phi)$, $\phi(\rho)$:

$$
\begin{aligned}
r''/r &= -\epsilon\phi'^2, && (1)\\
(A'r^2)' &= -2r^2V + 2q^2/r^2, && (2)\\
A(r^2)'' - r^2A'' &= 2 - 4q^2/r^2, && (3)\\
2(Ar^2\phi')' &= \epsilon r^2 dV/d\phi, && (4)\\
-1 + A'rr' + Ar'^2 &= r^2(\epsilon A\phi'^2 - V) - q^2/r^2. && (5)
\end{aligned}
$$

Possible approaches:

Specify the potential $V(\phi)$, find the functions r, A, ϕ (very hard technically).

Specify $r(\rho)$, find the functions V, A, ϕ (*inverse problem method*)

To find examples of solutions possessing particular properties, the inverse problem method is quite suitable.

Solution

Choose a function $r(\rho)$ that can provide WH and BU solutions:

$$r = (\rho^2 + b^2)^{1/2} \equiv b\sqrt{1+x^2}, \quad x \equiv \rho/b, \quad b = \text{const} = 1$$

Integrate Eq. (3) twice, denoting $B(x) \equiv A/r^2$:

$$B(x) = B_0 + \frac{1+q^2+px}{1+x^2} + \left(p + \frac{2q^2 x}{1+x^2}\right)\arctan x + q^2 \arctan^2 x$$

Fix the integration constants B_0, p using the asymptotical flatness condition $\lim\limits_{x\to+\infty} B = 0$ and comparing asymptotic of $A(x)$ with the Schwarzschild-like one, $A(x) \simeq 1 - 2m/x$:

$$B_0 = -\frac{\pi p}{2} - \frac{\pi^2 q^2}{4}, \qquad p = 3m - \pi q^2$$

Solution 2

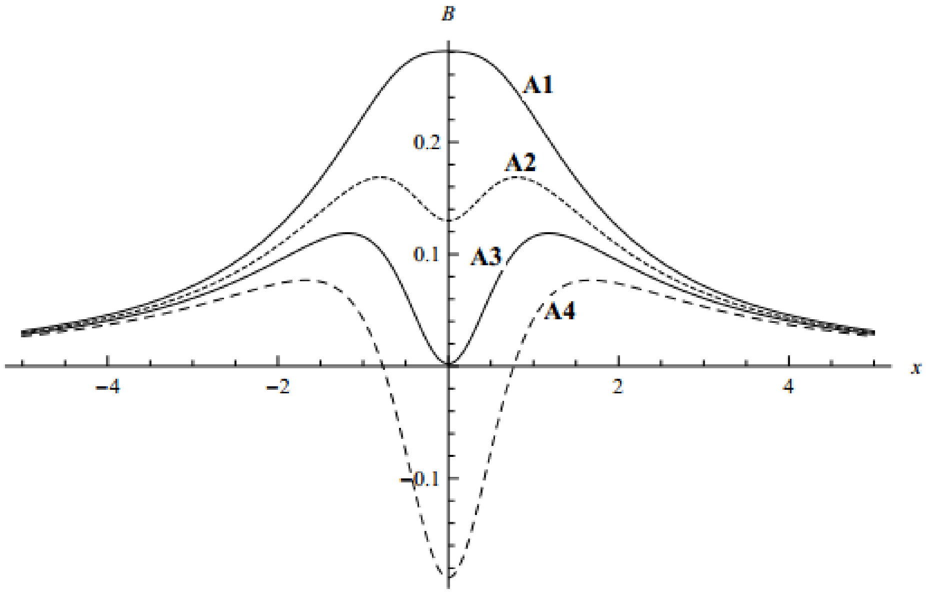
There is a two-parametric family of curves $B(x, q, m)$ with the parameters q, m.

For the scalar field and potential from Eqs. (1), (2) we have:

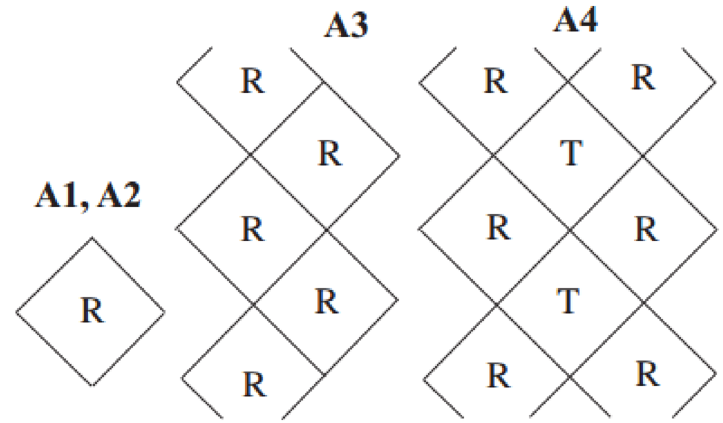
$$\phi = \pm\sqrt{2}\arctan x + \phi_0;$$

$$V = \frac{q^2}{(1+x^2)^2} - \frac{1}{2(1+x^2)}\Big[2q^2\arctan^2 x\,(3x^2+1) + \arctan x\left(18x^2 m - 6\pi x^2 q^2 + 12xq^2 - 2\pi q^2 + 6m\right) + q^2\left(\frac{3}{2}\pi^2 x^2 - 6\pi x + \frac{1}{2}\pi^2 + 6\right) - m(9\pi x^2 - 18x + 3\pi)\Big].$$

Symmetric Asymptotically Flat Configurations (A)

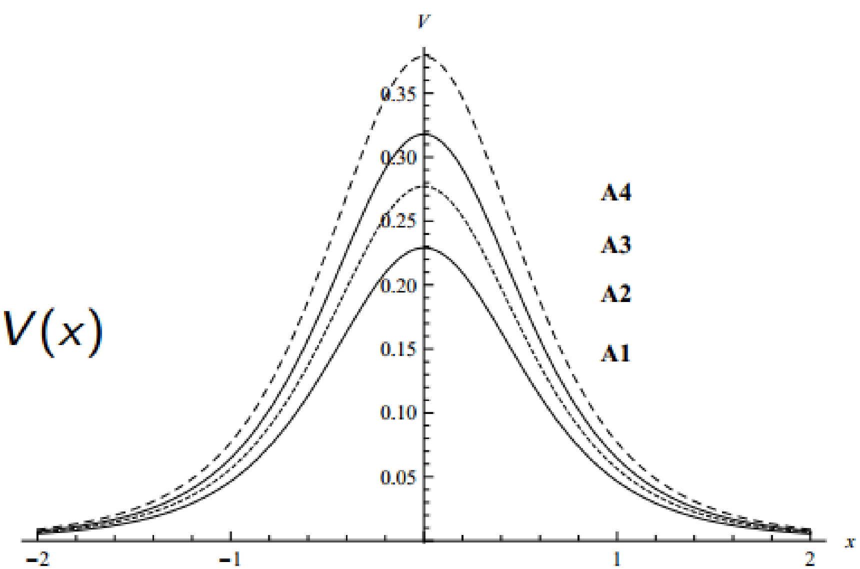


B(x) curves

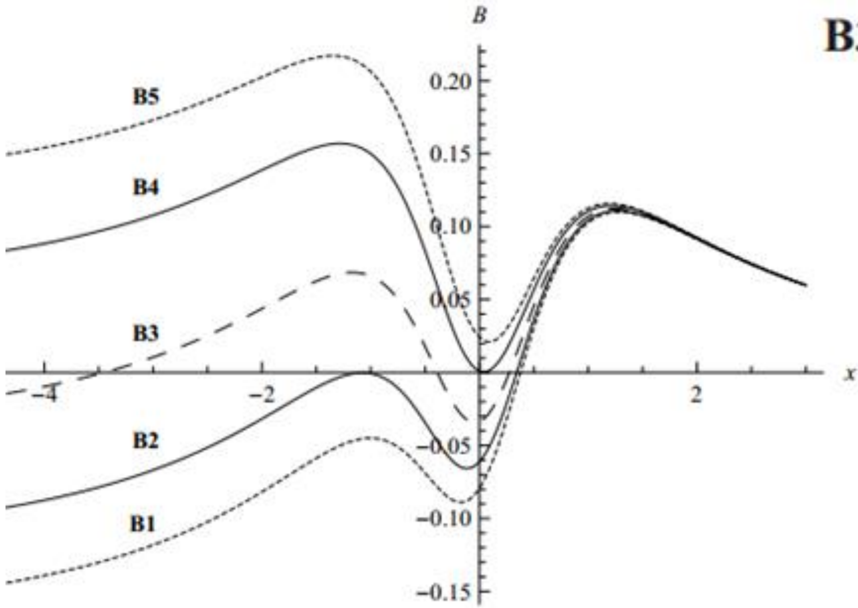


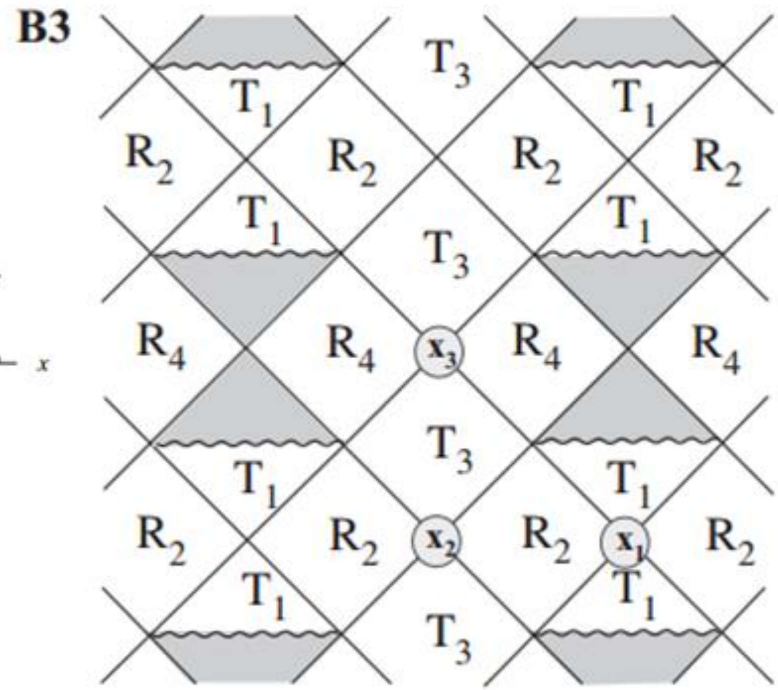
Carter-Penrose diagrams

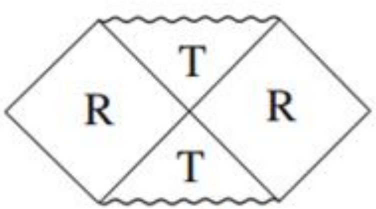
Potentials $V(x)$



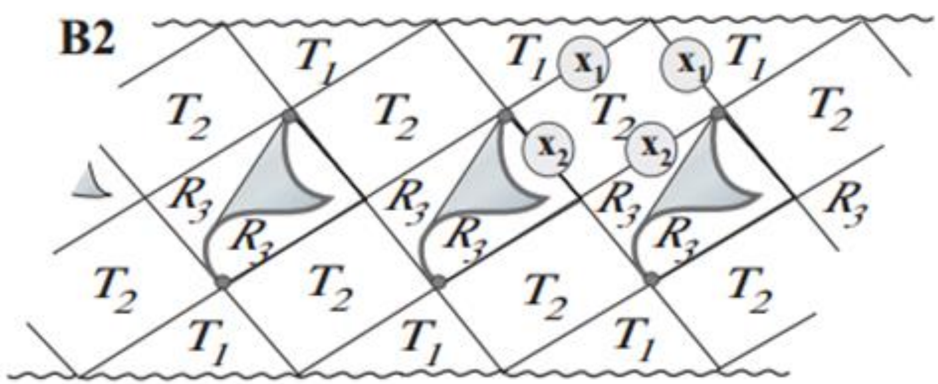
Asymmetric Asymptotically Flat Configurations (B)

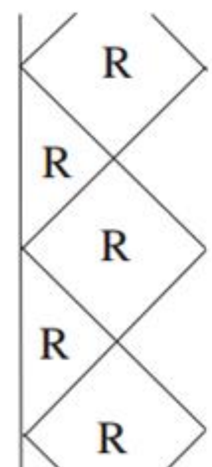






B1





B4

Asymptotically Flat Configurations With m > 0

Solution type (curve number)	Configuration $(x \to -\infty)$—$(x \to +\infty)$	Horizons: order n, R-T-region disposition
A1,A2	M - M WH	none [R]
A3	M - M extr. BH	1 hor., $n = 2$ [RR]
A4	M - M BH	2 hor., $n = 1$ [RTR]
B1, C1, C4, D1, D2	dS - M BU	1 hor., $n = 1$ [TR]
B2, C2	dS - M BU	2 hor., $n = 2; 1$ [TTR]
C4	dS - M BU	2 hor., $n = 1; 2$ [TRR]
B3, C3	dS - M BU	3 hor., $n = 1$, [TRTR]
D3	AdS - M BH	2 hor., $n = 1$ [RTR]
B4	AdS - M extr. BH	1 hor., $n = 2$ [RR]
B5	AdS - M WH	none [R]

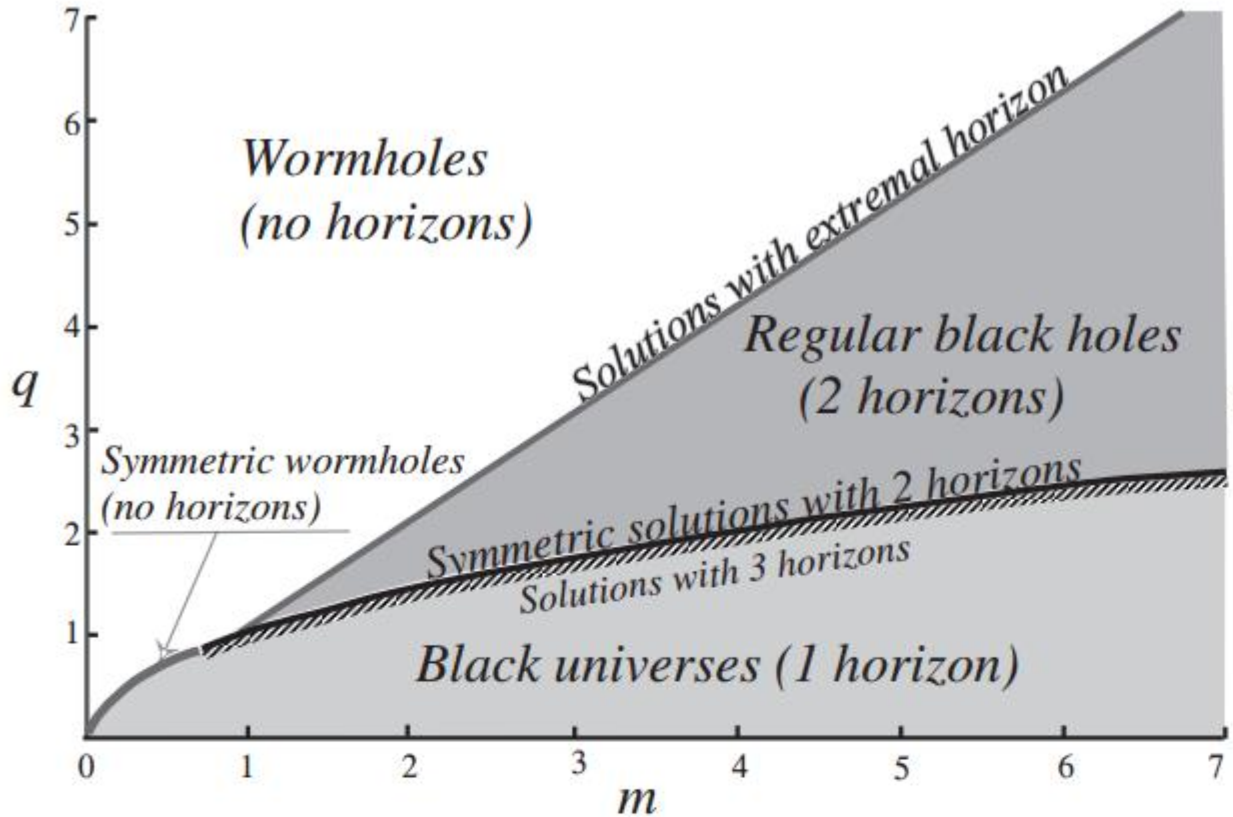
Asymptotic behavior of $B(x, q, m)$ at $x \to -\infty$:

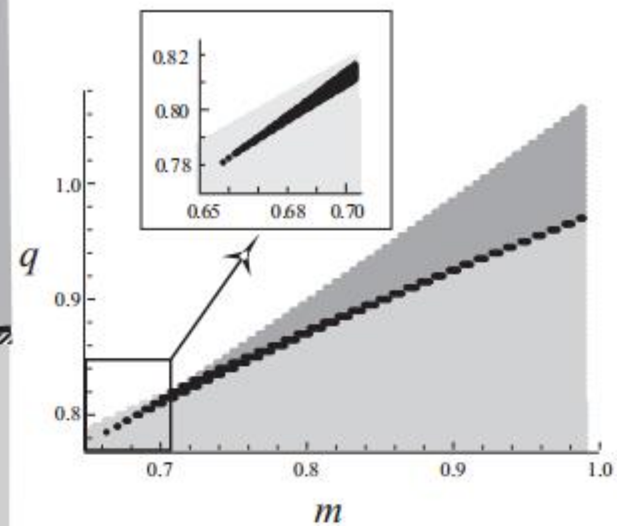
$$B(-\infty) = 0 \;\Rightarrow\; \mathrm{M}$$

$$B(-\infty) < 0 \;\Rightarrow\; \mathrm{dS}$$

$$B(-\infty) > 0 \;\Rightarrow\; \mathrm{AdS}$$

Parametric Map of Asymptotically Flat Solutions on The (q, m) Plane





Right: zoomed-in part of the map showing that configurations with 3 horizons are generic but occupy a very narrow band.

Trapped Ghosts

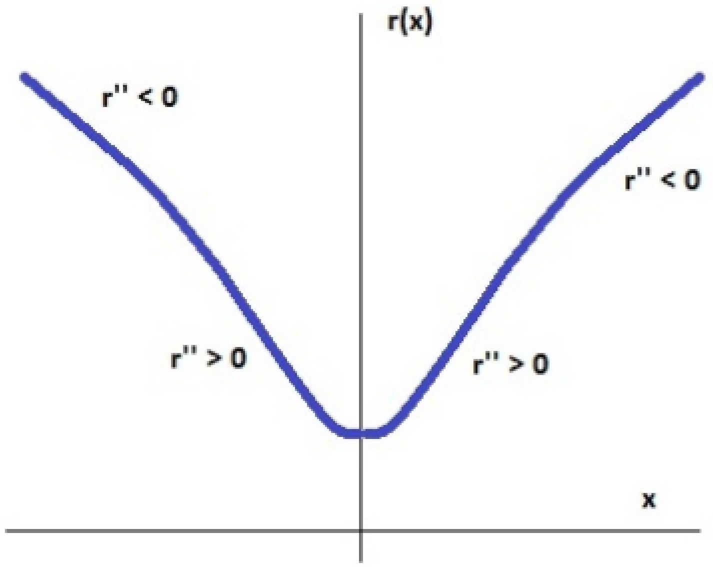
Problem: phantom fields are not observed.

Suggested solution: a field ϕ in the action

$$S = \frac{1}{2}\int \sqrt{-g}\, d^4x \left[R + 2h(\phi) g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - F^{\mu\nu}F_{\mu\nu} - 2V(\phi) \right],$$

($h(\phi)$ is a smooth function) which is phantom ($h(\phi) < 0$) in a strong-field region (say, $|\phi| > \phi_{\rm crit}$) and normal otherwise.

How to realize that in our problem setting? Recall that in our metric
$r'' > 0 \;\; \Rightarrow \;\;$ phantom,
$r'' > 0 \;\; \Rightarrow \;\;$ normal field



Trapped Ghosts 2

Field equations:

$$\begin{aligned} 2(Ar^2h\phi')' - Ar^2h'\phi' &= r^2 dV/d\phi, & (6)\\ (A'r^2)' &= -2r^2V + 2q^2/r^2; & (7)\\ r''/r &= -h(\phi)\phi'^2; & (8)\\ A(r^2)'' - r^2A'' &= 2 - 4q^2/r^2, & (9)\\ -1 + A'rr' + Ar'^2 &= r^2(hA\phi'^2 - V) - q^2/r^2. & (10) \end{aligned}$$

Ansatz for $r(u)$ realizing the trapped ghost idea $[x := u/a]$:

$$r(u) = a\frac{x^2+1}{\sqrt{x^2+n}}, \quad n = \mathrm{const} > 2, \quad a = \mathrm{const} > 0. \qquad (11)$$

Since $r''(x) = \dfrac{1}{a}\dfrac{x^2(2-n)+n(2n-1)}{(x^2+n)^{5/2}}$, we have

$r'' > 0$ at $x^2 < n(2n-1)/(n-2)$ and $r'' < 0$ at larger $|x|$,

as required; also, $r \approx a|x|$ at large $|x|$.

Trapped Ghosts - Solutions

Solutions ($n = 3$ for definiteness):

$$B(x) = B_0 + \frac{26 + 24x^2 + 6x^4 + 3px(69 + 100x^2 + 39x^4)}{6(1+x^2)^3} + \frac{39p}{2}\arctan x$$
$$+ \frac{q^2[107 + 383x^2 + 375x^4 + 117x^6 + 6x(69 + 169x^2 + 139x^4 + 39x^6)\arctan x]}{9(1+x^2)^4}$$
$$+ 13q^2 \arctan^2 x, \tag{12}$$

where p and B_0 are integration constants. Asymptotic flatness $\Rightarrow$

$$B_0 = -\frac{13}{4}\pi(3p + \pi q^2). \qquad p = m - \frac{2}{3}\pi q^2. \tag{13}$$

Thus B is a function of x and two parameters, m = mass and q = charge.
Other unknowns are, as before, found from the field equations.
Using the arbitrariness in ϕ definition, it is convenient to assume

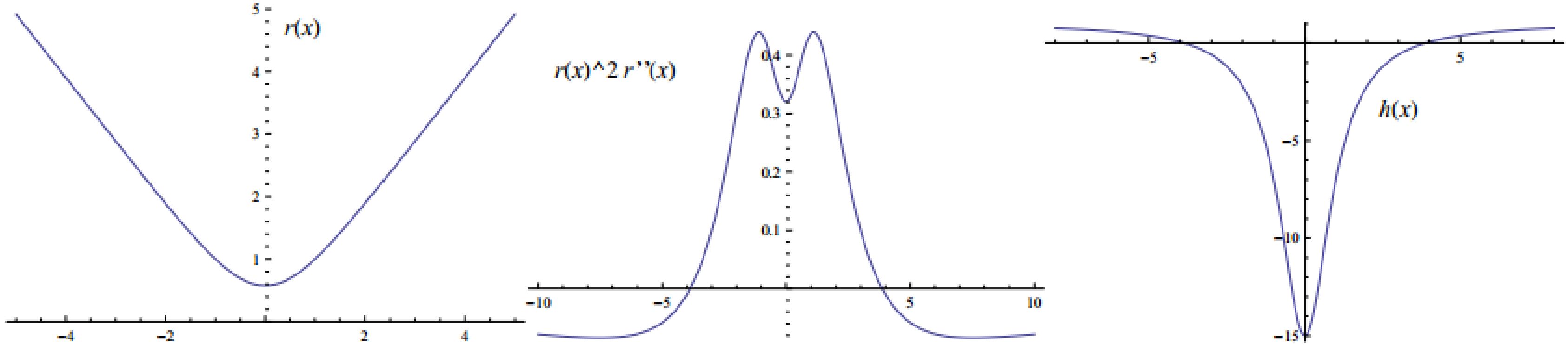
$$\phi(x) = \frac{1}{\sqrt{3}}\arctan\frac{x}{\sqrt{3}}, \tag{14}$$

so that ϕ has a finite range: $\phi \in (-\phi_0, \phi_0)$. Then,

$$h(\phi) = \frac{x^2 - 15}{x^2 + 1} = \frac{3\tan^2(\sqrt{3}\phi) - 15}{3\tan^2(\sqrt{3}\phi) + 1}. \tag{15}$$

The Null Energy Condition is violated only where $h(\phi) < 0$.

Trapped Ghosts – Solutions 2



Plots of $r(x)$ (left), $r^2\, r''(x)$ (middle) and $h(x)$ (right) for $n = 3$.

The diversity of configurations described by the solutions, depending on the parameters m and q, is similar to that with a "visible" ghost.

As before, shifts in B_0 allow for obtaining non-asymptotically flat configurations.

Invisible Ghosts

Another possible explanation of the unobservability of ghosts is their short range, i.e., a sufficiently rapid decay at large distances.
Example with two fields, normal (ϕ) and phantom (ψ):

$$\mathcal{S} = \frac{1}{16\pi} \int \sqrt{-g} d^4x \Big[R + 2h_{ab}(\partial\phi^a, \partial\phi^b) - 2V(\phi^a) - F_{\mu\nu}F^{\mu\nu} \Big],$$

in the same static, spherically symmetric space-time, where $\{\phi^a\}$ is a set of scalar fields, $h_{ab} = h_{ab}(\phi^a)$ is a nondegenerate target space metric, $(\partial\phi^a, \partial\phi^b) \equiv g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b$, and $V(\phi^a)$ is an interaction potential.
The same static, spherically symmetric problem as before but now with two fields: $\phi^1 = \phi$ and $\phi_2 = \psi$, with $h_{ab} = \mathrm{diag}(1, -1)$.
One of the equations reads $r''/r = -\phi'^2 + \psi'^2$.
We choose the same $r(x)$ as for a trapped ghost$(x = u/a)$:

$$r(u) = a\frac{x^2+1}{\sqrt{x^2+n}}, \quad n = \mathrm{const} > 2, \quad a = \mathrm{const} > 0$$

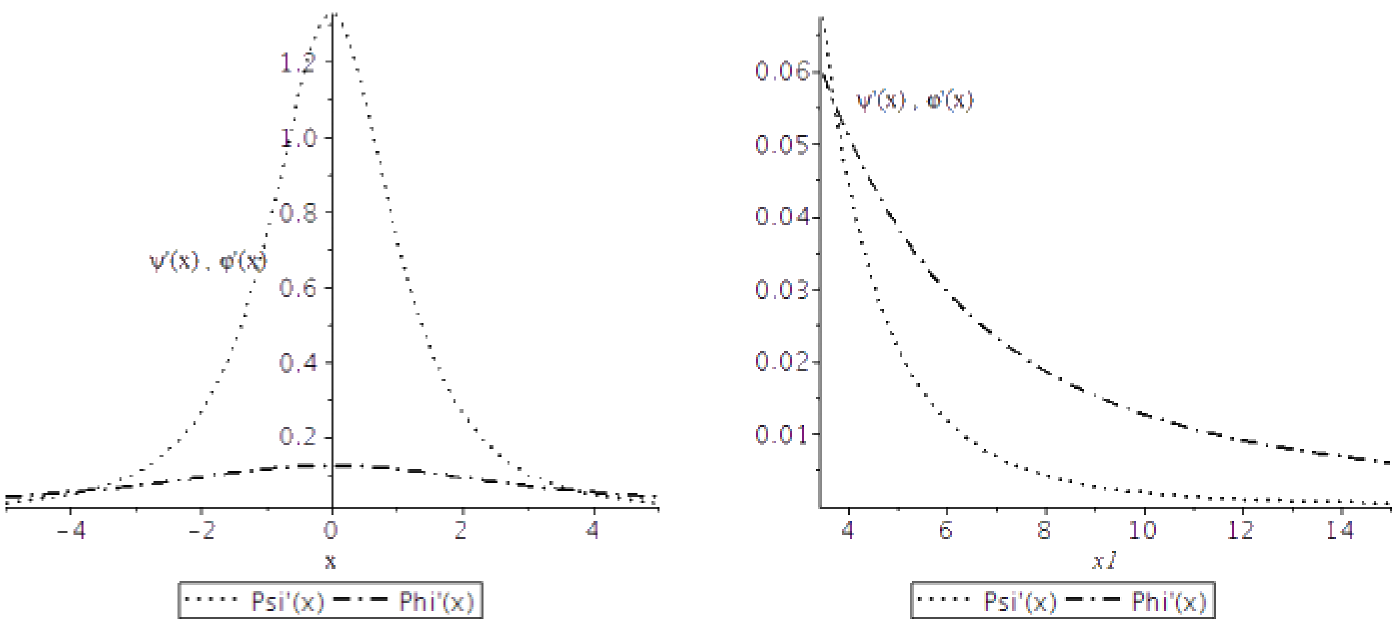
Then we obtain the same set of geometries as before. Different is the nature and behavior of the scalar fields.

Invisible Ghosts 2

For the scalar ϕ (using the arbitrariness) we assume

$$\phi(x) = K \arctan(Lx),$$

where K and L are adjustable constants. We choose K and L in such a way as to make the phantom field ψ decay at large x more rapidly than ϕ. Thus, for $n = 4$ we should take $K = 2/\sqrt{23}, \quad L = \sqrt{2/23}$, then $\psi' \sim x^{-4}$ while $\phi' \sim x^{-2}$ at large $|x|$.



The quantities ϕ' and ψ' in the strong and weak field regions

Invisible Ghosts 3

Obs. limits on global magnetic fields: $10^{-9} \geq |\vec{B}| \geq 10^{-18}$ Gauss.

Present scale factor $\approx 10^{28}$ cm, it corresponds to $r(u)$ in our metric.

We take the conservative estimate: let $|\vec{B}| \geq 10^{-18}$ Gauss now.
It evolves $\propto a^{-2} \quad \Rightarrow$
At recombination (since $a/a_0 \sim 10^{-3}$), $|\vec{B}| \sim 10^{-12}$ Gauss
At baryogenesis ($a/a_0 \sim 10^{-12}$), $|\vec{B}| \sim 10^{6}$ Gauss

Constraint on the energy density of a global magnetic field.
Degree of anisotropy at recombination (from CMB properties) $\sim 10^{-6}$.
$\rho_{\rm CMB}/\rho_{\rm magn} = {\rm const} \quad \Rightarrow \quad$ at present $\rho_{\rm magn} \lesssim 10^{-39}$ g cm^{-3}
$\Rightarrow \quad |B| \lesssim 10^{-8}$ Gauss
In reality it is much smaller (if any).

Theoretical stability limit of a classical magn. field in Weinberg-Salam theory: $B \lesssim 10^{24}$ Gauss. Hence, the corresponding min $r(u) \approx 10^{7}$ cm ~ 100 km.

Conclusion

Analytical exact solutions have been obtained in GR with an electromagnetic field and different kinds of phantom fields. These are globally regular static, spherically symmetric solutions describing traversable wormholes (with flat and AdS asymptotics) and regular black holes, in particular, black universes.

The configurations obtained are quite diverse and contain different numbers of Killing horizons, from zero to four. This substantially enriches the list of known structures of regular BH configurations.

Such models can be of interest both as descriptions of local objects (black holes and wormholes) and as a basis for building non-singular cosmological scenarios.

Phantom fields are not observed under usual conditions. This circumstance is accounted for by the concepts of trapped or (preferably) invisible ghosts.

Numerical estimates concerning a possible global magnetic field are compatible with a BU model.

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Thank You