Solar magnetorgam editing using discrete Morse theory.

Alexeev V. V., Yaroslavl State University
Makarenko N. G., Pulkovo Observatory, St. Petersburg
Park D. O., volunteer, St. Petersburg

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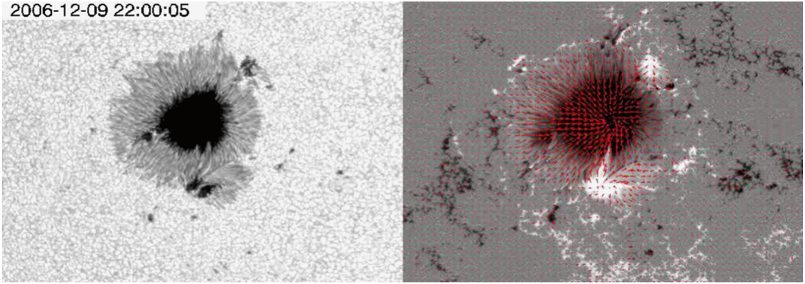
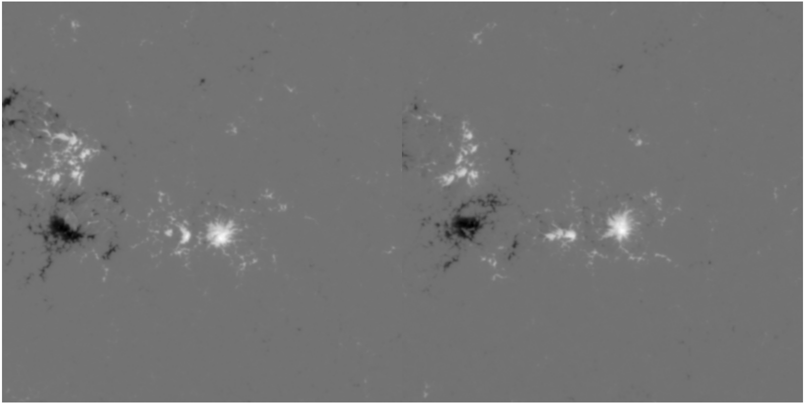
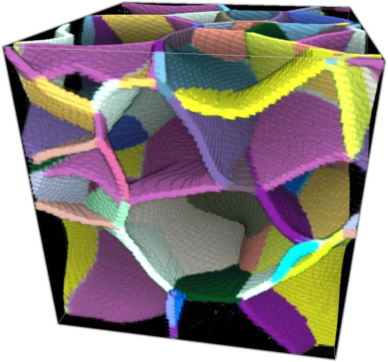
- Simplify complexity of the observed field
- Preserve main topological features

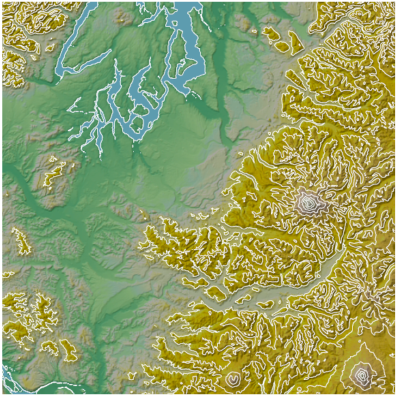


We use HMI / SDO magnetogram data.

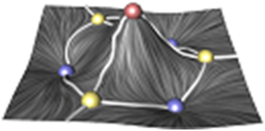


Astrophysics (voids, filaments and walls detection), geomorphology.



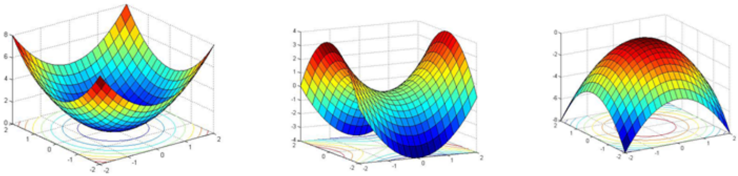


[Sousbie, 2010], [Bauer, 2011]



$f : \mathbb{R}^2 \to \mathbb{R}$ – a smooth function.
Point p is *critical* if $\nabla f(p) = 0$, and *non-degenerate* if $||\mathcal{H}_f(p)|| = ||d^2 f / dx_i dx_j(p)|| \neq 0$.
f is a *Morse function* if all its critical points are non-degenerate.

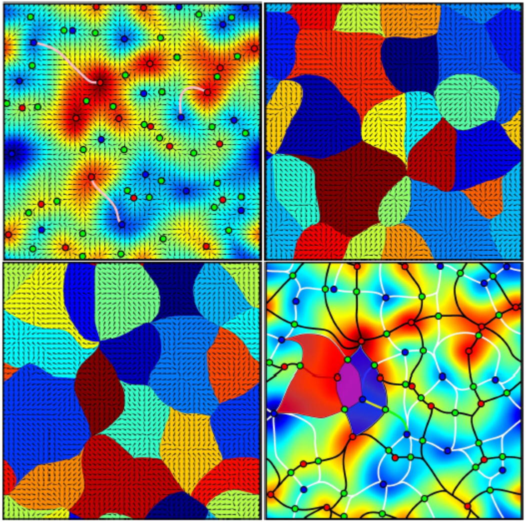
Morse index of a critical point – number of independent unstable directions. Neighbourhood of a critical point locally is $f(p) \pm x^2 \pm y^2$.



minimum (index 0) saddle (index 1) maximum (index 2)

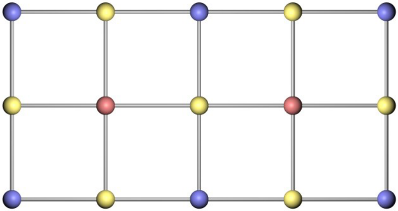
Critical points: *maxima*, *saddles*, and *minima*.

Morse-Smale complex obtained by intersecting of ascending and descending manifolds. [Sousbie, 2010]



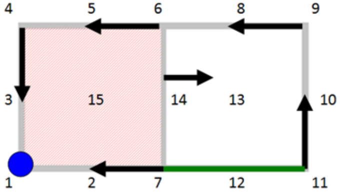
Magnetogram data is given on a discrete domain.
We need a generalization of Morse theory concepts for function $f : \mathbb{Z}^2 \to \mathbb{R}$. [Forman, 1998]

Construct a cell complex, assign values to cells.

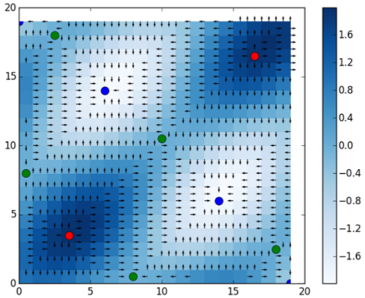


Maxima are quads, saddles are edges, minima are vertices.

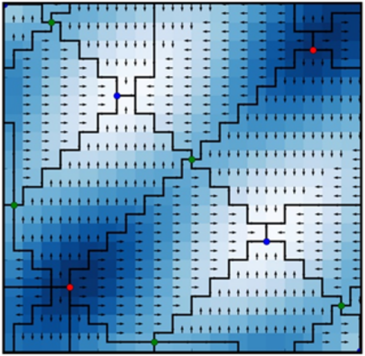
Two cells $\sigma < \tau$ form a *gradient arrow* $\langle \sigma, \tau \rangle$ if $f(\tau) \leq f(\sigma)$. All cells, except critical, are paired into gradient arrows.



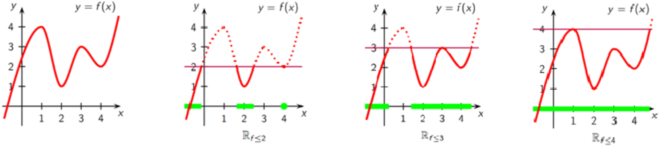
Cells 1, 12, and 15 are critical (minimum, saddle, and maximum, respectively).

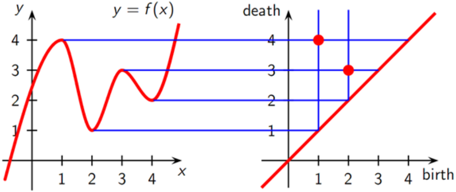


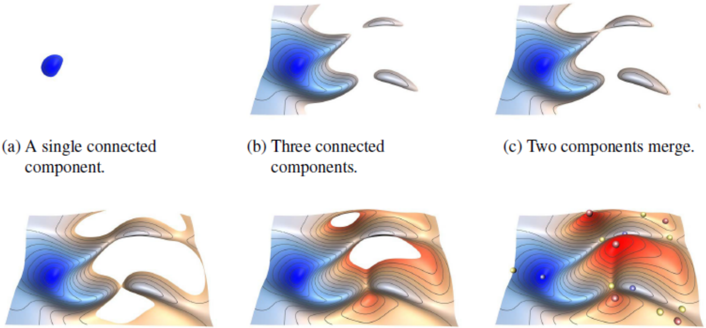
Discrete gradient field of function $f(x, y) = \sin \alpha x + \sin \beta y$ defined on evenly-spaced grid



Morse-Smale complex of function f


y = f(x)
$\mathbb{R}_{f\le 2}$
$\mathbb{R}_{f\le 3}$
$\mathbb{R}_{f\le 4}$


y = f(x)
death
birth



(a) A single connected component.

(b) Three connected components.

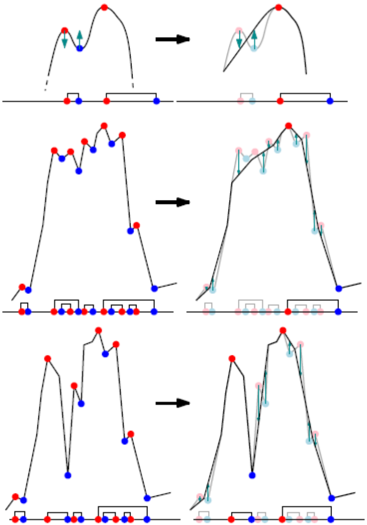
(c) Two components merge.

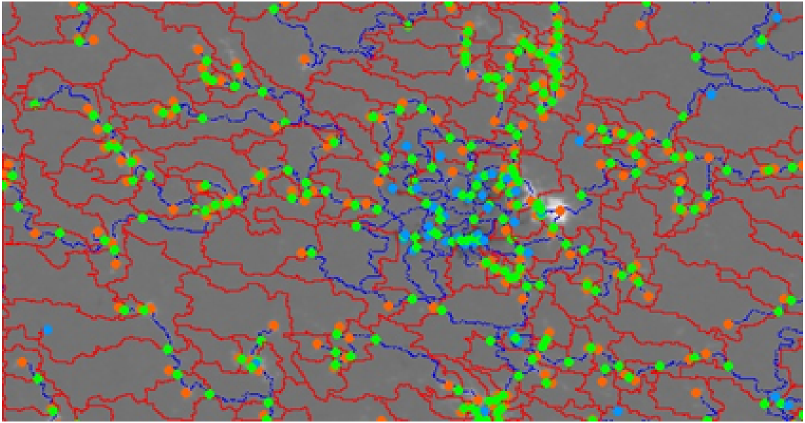
(d) Two tunnels are created.

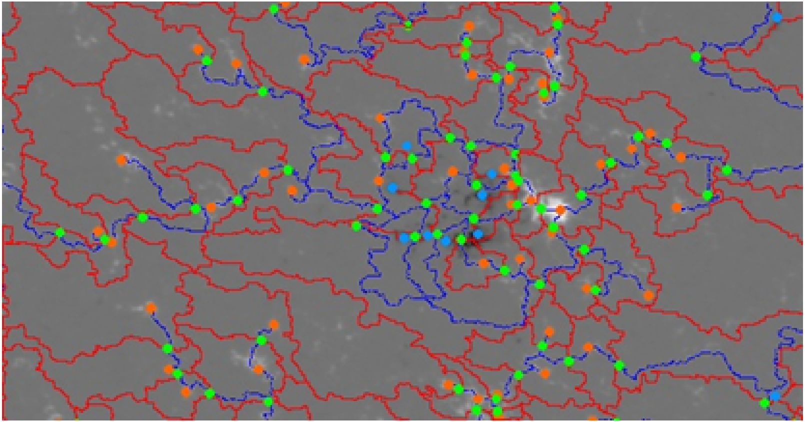
(e) A tunnel is closed, another is created.

(f) All tunnels are closed.

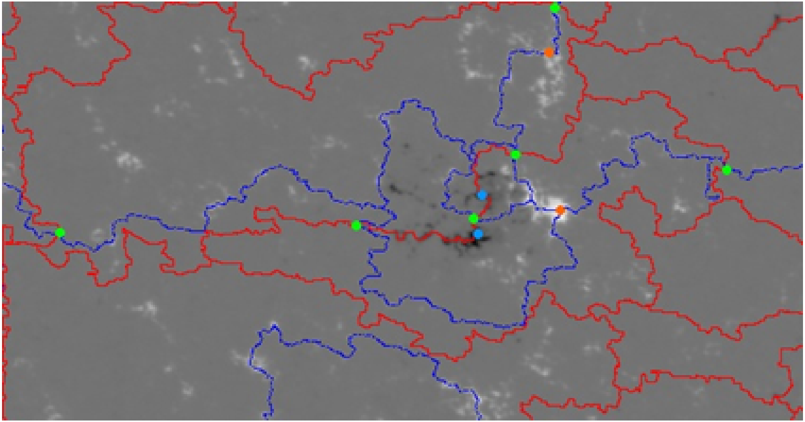
[Günther, 2012]

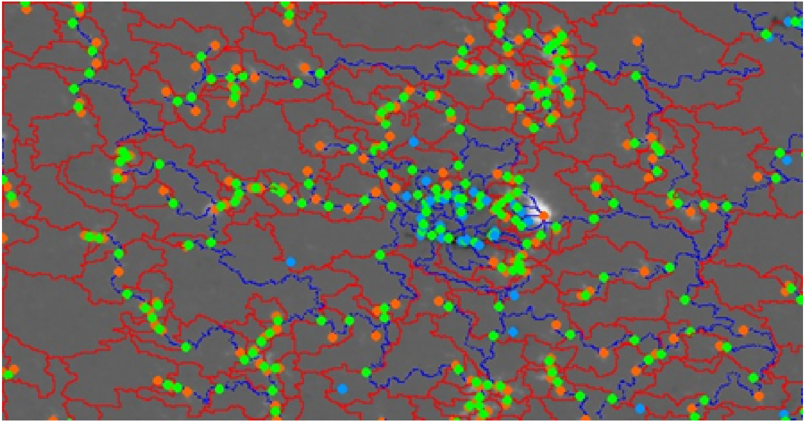




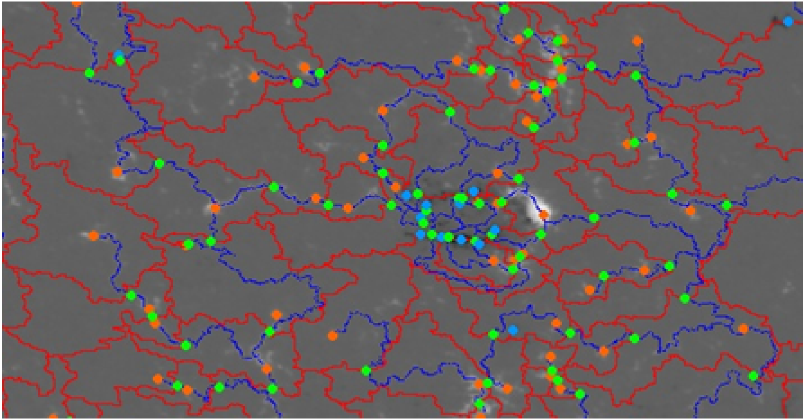


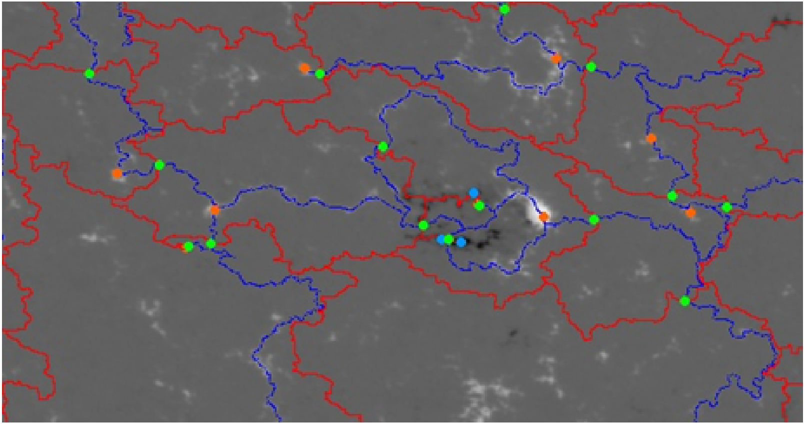
Morse-Smale Complexes of Active Regions I



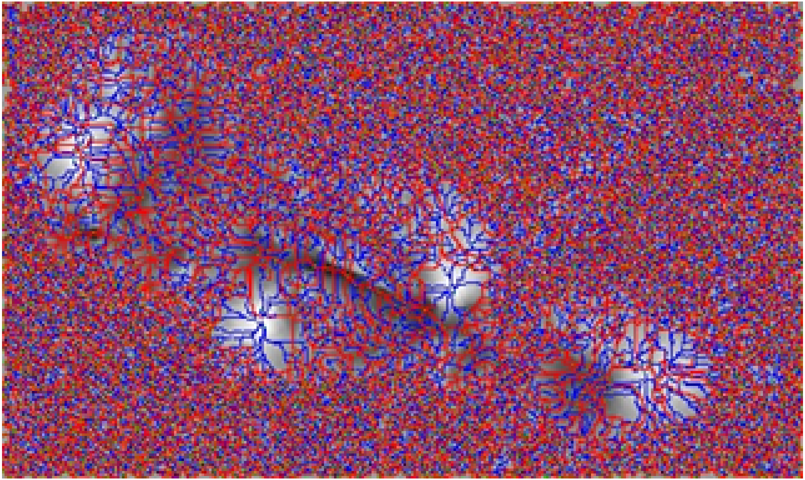


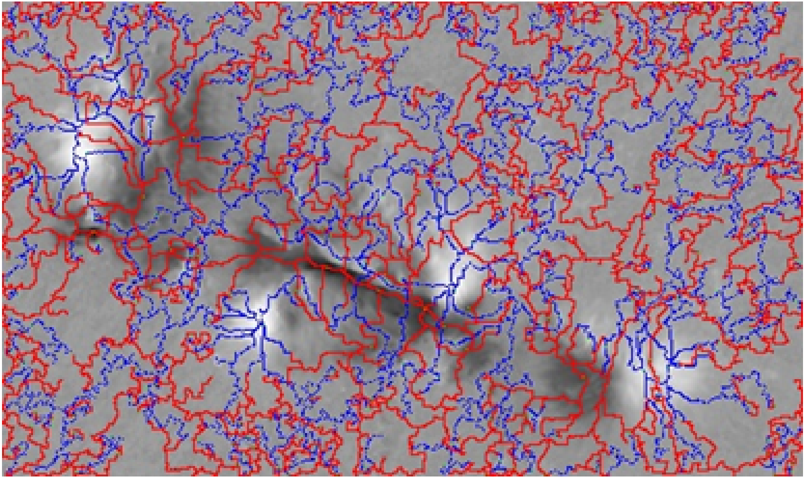
Morse-Smale Complexes of Active Regions II



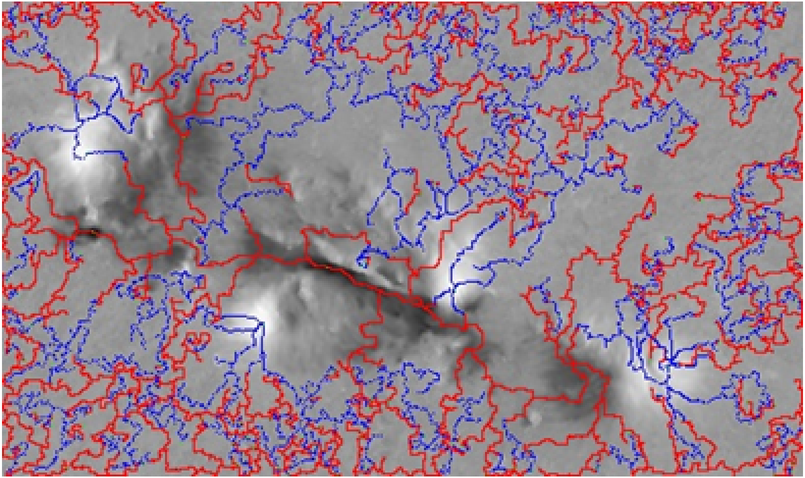


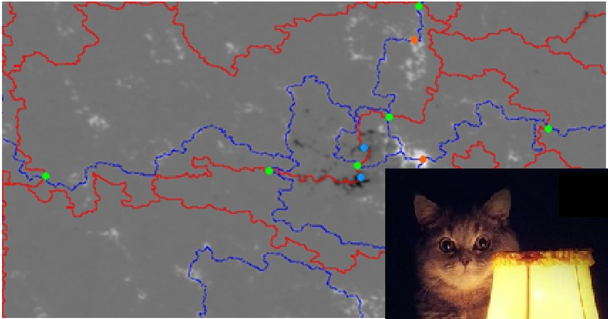
Morse-Smale Complexes of Active Regions III





Morse-Smale Complexes of Active Regions III





Thank you for attention!