Institute for Theoretical Physics

Quantum-induced trans-Planckian energy near horizon

or Could Firewalls be Realized in GR

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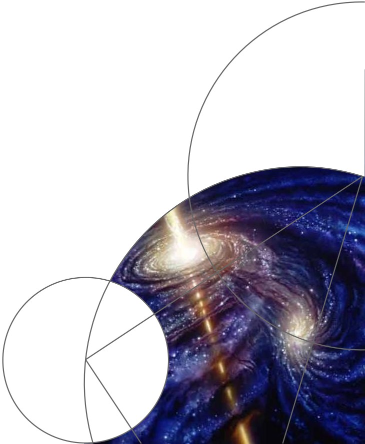


Points to discuss:

1. QM+GR $\sim$ Hawking's Paradox
2. Firewall as a way of resolving the information loss paradox
3. Realisation of Firewalls in models of BHs
4. Brief discussion of the results

Hawking's Paradox in BH Physics

IP: Simple Formulation

Standard QM/QFT:

$$|\Psi(-\infty)\rangle \longrightarrow |\Psi(+\infty)\rangle$$

$$|\Psi(+\infty)\rangle = S|\Psi(-\infty)\rangle, \qquad SS^\dagger = S^\dagger S = 1$$

Unitarity of S-matrix results in keeping the wave function probability distribution in the initial and final states:

$$\langle\Psi(+\infty)|\Psi(+\infty)\rangle = 1 = \langle\Psi(-\infty)|S^\dagger S|\Psi(-\infty)\rangle \equiv \langle\Psi(-\infty)|\Psi(-\infty)\rangle$$

and in Principle of Detailed Balance. The PDB in particular leads to invariance under $t \leftrightarrow -t$:

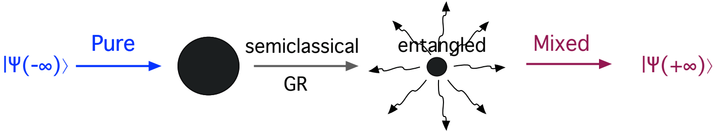
$$\langle\Psi(-\infty)|\Psi(+\infty)\rangle = \langle\Psi(+\infty)|S^\dagger S|\Psi(-\infty)\rangle = \langle\Psi(+\infty)|\Psi(-\infty)\rangle$$

Resumé:

Despite the probabilistic nature of the measurement process in QM, its evolution is completely determined: the system always goes from the final state to the initial state upon the time returning back.

IP: extended formulation

QFT with semi-quantum BH (Hawking 1975/76):



The evolution process of pure initial quatum states of semi-classical/semi-quantum theory of BHs into mixed final states during a BH evaporation is accompanied with violation of a fundamental principle of quantum physics – the principle of Unitarity or Quantum Determinism.

In more detail, to restore the initial pure states from the known final mixed states one needs sufficiently more information on quantum system. For instance, information on the entangled w.r.t. Hawking radiation states inside the BH, which are completely lost in the BH evaporation process, is required to this end.

This disagreement was called as the Information Paradox or, more exactly, the “Loss of Information” Paradox in BH Physics.

Entanglement

A part of the game is the entanglement of quantum states. (Schrödinger 1935)

What's the entanglement of quantum states?

Let's consider two QM sub-systems. Mixing/purity of a sub-system states depends directly on separability of the state vector of the complete QM system.

Pure/mixed non-separable states of the complete QM system are termed as entangled states. Important to note that the projection of entangled states onto sub-systems 1 and 2 is always described by the mixed states density matrix.

Entanglement

What's the entanglement of quantum states?

One may apply a specific criterion of the entangled state:

A state is the entangled state when one may establish observables $\mathcal{O}_{1,2}$ for the sub-systems 1 and 2, such that

$$\langle\Psi|\mathcal{O}_1 \otimes \mathcal{O}_2|\Psi\rangle \neq \langle\Psi|\mathcal{O}_1|\Psi\rangle\langle\Psi|\mathcal{O}_2|\Psi\rangle$$

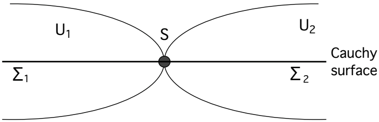
Therefore, the entanglement impacts even than parts of the common system are far remoted of each other, and any formal interaction between them is absent.

Since the entanglement is one of the key characteristics of QM it also becomes crucial in its relativistic version, i.e. in QFT.

Entanglement in QFT

In context of BH Physics it becomes important the “inverse” effect of creating entangled states upon partitioning a quantum system onto several sub-systems.

As an example we will consider the scalar QFT. Let’s divide the Cauchy surface Σ (i.e. the hypersurface of $t = \text{const}$) onto the pair of disjoint regions $\Sigma_{1,2}$ with the common boundary S: $\Sigma_1 \bigcup \Sigma_2 \bigcup S = \Sigma$ (Unruh&Wald, Rept.Prog.Phys. 80 (2017))



The domains $U_{1,2}$ are globally hyperbolic, with Cauchy surfaces $\Sigma_{1,2}$. (to define the scalar product between two solutions of the Klein-Gordon-Fock equation in a curved spacetime correctly.)

Now let’s evaluate the specific criterion of the entangled states

$$\langle\Psi|\mathcal{O}_1 \otimes \mathcal{O}_2|\Psi\rangle \neq \langle\Psi|\mathcal{O}_1|\Psi\rangle\langle\Psi|\mathcal{O}_2|\Psi\rangle$$

in the case.

Entanglement in scalar QFT

Observables for sub-systems 1 and 2 are given by operators in the domains $U_{1,2}$, while the correlator (2-point function) over an admissible state becomes (De Witt, 1965)

$$\langle\Psi|\phi(x_1)\phi(x_2)|\Psi\rangle \sim \frac{U(x_1,x_2)}{\sigma(x_1,x_2)}\,, \quad x_1\in\Sigma_1,\ \ x_2\in\Sigma_2$$

Here $U(x_1,x_2)$ is a smooth function; $\sigma(x_1,x_2)$ is the geodesic interval (1/2 of the square of the distance between points x_1 and x_2 along the geodesic).

Since

$$\lim_{x_{1,2}\to x}\sigma(x_1,x_2)=0,\ \ x\in S\,,$$

the 2-point function diverges in the coincidence limit:

$$\lim_{x_{1,2}\to x}\langle\Psi|\phi(x_1)\phi(x_2)|\Psi\rangle \rightsquigarrow \infty$$

On the other hand, for any admissible state holds

$$\langle\Psi|\phi(x_1)|\Psi\rangle\langle\Psi|\phi(x_2)|\Psi\rangle \stackrel{x_{1,2}\to x}{\longrightarrow} \left[\langle\Psi|\phi(x)|\Psi\rangle\right]^2 < \infty$$

Hence,

$$\langle\Psi|\phi(x_1)\phi(x_2)|\Psi\rangle \neq \langle\Psi|\phi(x_1)|\Psi\rangle\langle\Psi|\phi(x_2)|\Psi\rangle$$

Entanglement in scalar QFT

To sum up, we observed the entanglement of states effect after dividing the original system onto sub-systems, and this effect will be observed in any physically reasonable QFTs!

Remarkable example of the QFT states entanglement is the Unruh effect (Fulling 1973, Davies 1975, Unruh 1976):

Appearance of mixed states with non-zero temperature

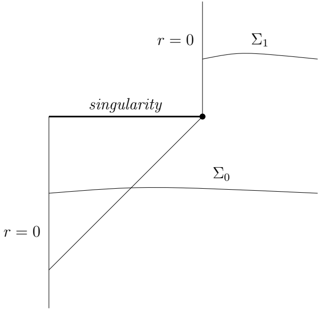
$$T_U = \frac{\hbar a}{2\pi c k_B},$$

which are detected by an accelerated observer after a formal splitting the Minkowski space onto four Rindler wedges can be explained as the manifestation of the QFT states entanglement in causally related domains. (Iso et al., PRD 96 (2017))

The Rindler space is a toy model to describe the space-time geometry near the BH event horizon. Hence, it is inevitable for any QFT the occurrence of standard entanglement of quantum states inside and outside of BH, the consequence of which is the Hawking radiation (mixed/thermal quantum states).

Entanglement in BH: Information Loss

The BHs evolution comes as follows (Unruh&Wald, Rept.Prog.Phys. 80 (2017)):



Quantum states on the Cauchy surface Σ_1 (after the BH evaporation) are still entangled with quantum states inside the BH at an early moment Σ_0, even if they are completely disappeared during the evaporation process! The initial Pure quantum state turns into the final Mixed state in the later time Σ_1.

According to that, the complete knowledge on the final state at the time Σ_1 turns out to be insufficient for determining the initial state. In this sense we talk on the information loss in BHs.

This conclusion holds true even in more sophisticated scenarios such as “Many-worlds”, “Baby Universe creation at the singularity point” etc., which do not go beyond the semi-classical approximation.

Firewall as a way of resolving the information loss paradox

Information Loss "Resolution": Firewalls

(Almheiri, Marolf, Polchinski, Sully, JHEP 1302 (2013))

No entanglement – no problem:

Perhaps, the Black Hole formed in the expected way further behaves radically different from the standard semi-classical picture by Hawking.

Specifically, at some point in time at the location of the event horizon it appears an object – a " firewall " – whose role is to " disentangle " the quantum states outside and inside the BH.

The proposed AMPS scenario employs the "no-cloning" theorem of quantum physics and the entropy subadditivity of complex systems.

Information Loss “Resolution”: Firewalls

The AMPS analysis of four complementarity postulates of BH Physics (Susskind, Thorlacius, Uglum 1993), based on the description of the BHs evolution within the standard QFT and semi-classical GR, results in inconsistency of two of them with the following one (AMPS 2013):

> A freely falling observer does not observe nothing extraordinary under the crossing the event horizon.

Keeping the standard description of the BHs evolution requires, at least for the “old” enough BHs, appearance of an extraordinary structure at the horizon, the role of which is to “disentangle” the quantum states outside and inside the BH.

Therefore, the BH horizon becomes the “firewall” location point; all objects falling inside the BH are burned in the firewall, any information on them disappears. The final quantum state after the BH complete evaporation is a pure state.

Information Loss "Resolution": Firewalls

Critics of the Firewall Proposal: (Unruh&Wald, 2017)

The proposal by AMPS is a radical one that determines by a sharp enough deviation from semi-classical description even in the small curvatures regime. In particular, to minimize the entanglement effect between the inside and outside of BH states requires a singularity of quantum fields on the horizon, which is converted into the "firewall".

The appearance of a local space-time peculiarity in some restricted space-time domain contradicts to the spirit of GR, where all points of the space-time continuum are supposed to be equivalent. The formation of a "firewall" is comparable to the effect of an unexpected materialization of the wall on the way of a uniformly accelerated observer.

In addition, the process of the "firewall" formation within the AMPS proposal manifestly violates causality, since to localize this object exactly on the event horizon, it is necessary to know in advance the entire subsequent evolution of the system.

Realisation of Firewalls in models of BHs

AJN & I.Y. Park, JHEP 05 (2018)

Realisation of Firewalls in models of BHs

Since the Firewall is intended to "disentangle" quantum states inside and outside of the BH, which is guaranteed to happen only if there are ultra-high energies/temperatures on the horizon. On account of the Tolman relation for the local temperature

$$T_{\mathrm{local}} = |g_{00}|^{-1/2} T_H$$

such an effect is principally achievable.

The more exact characteristic is the local energy density measured by a freely-falling observer (Lowe&Thorlacius 2013):

$$\rho \equiv T_{\mu\nu} U_K^\mu U_K^\nu$$

U_K^μ is the observer 4-velocity; $T_{\mu\nu} \equiv \langle K | T_{\mu\nu}^K | K \rangle$ is the v.e.v. of the energy-momentum operator over the Kruskal state $|K\rangle$ (the Hartle-Hawking vacuum); the subscript K is related to the Kruskal coordinates.

Within the semi-classical description of GR the local energy density does not undergo of strong changes on the even horizons of static charged BHs (Park I.Y. 2017).

Realisation of Firewalls in models of BHs

What about classical stress-energy tensor?

$$T_{\mu\nu} = -\frac{2}{\kappa^2}\Lambda g_{\mu\nu} + T_{\mu\nu}^{\text{matter}}$$

Within semi-classical description of GR one takes: $T_{\mu\nu} \to \hat{T}_{\mu\nu}$,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \langle \text{vac} | \hat{T}_{\mu\nu} | \text{vac} \rangle$$

A direct way beyond semi-classical approximation is to take into account quantum corrections to Einstein GR. For (one-)loop corrections we get

$$T_{\mu\nu} = -\frac{2}{\kappa^2}\Lambda g_{\mu\nu} + T_{\mu\nu}^{\text{matter}} + g_{\mu\nu}\left[c_1 R^2 - (4c_1 + c_2)\nabla^2 R + c_2 R_{\alpha\beta} R^{\alpha\beta}\right]$$

$$-2\left[2c_1 R R_{\mu\nu} - (2c_1 + c_2)\nabla_\mu \nabla_\nu R - 2c_2 R_{\alpha\mu\nu\beta} R^{\alpha\beta} + c_2 \nabla^2 R_{\mu\nu} + \ldots\right]$$

Realisation of Firewalls in models of BHs

For quantum-induced description of GR (at the one-loop corrections level) the local energy density at the horizons of static charged BHs in flat spacetime – Melvin-Schwarzschild (MS) solution (BH in an external magnetic field) and generalized MS solution (Preston&Poisson 2006) – does not change much.

A jump of the local energy density near the horizon can be realized for:

- a non-stationary BH solution with matter fields;
- space-times with non-trivial (negative) cosmological constant induced by quantum effects;
- special boundary conditions, specified in the way that the quantum matter fields modes take non-trivial values on the AdS boundary (the Dirichlet b.c.)

Once all these conditions are fulfilled (AJN & I.Y. Park 2018)

$$\rho \sim 1/\kappa^2 = 1/16\pi G \sim E_{\rm Pl}^2$$

Brief discussion of the results

Brief discussion of the results

Going beyond the semi-classical approximation (by taking into account quantum corrections to Einstein GR) does not implement the AMPS "firewall" prescription: to solve IP the existence of a "firewall" at the late stages of the BH evolution is required.

Nevertheless, we have shown (AJN & I.Y. Park 2018) a principle possibility of arising the quantum-induced trans-Plankian energy densities near the horizons of BHs at their birth and early stages of evolution.

This result seems to be important for astrophysics, since the SMBHs in the Galactic nuclei are described by non-stationary solutions to Einstein equations.

In addition, a well-known fact is the presence of ultra-hard radiation from the active Galactic nuclei, the mechanism of which is currently under debates. One of the explanations of this phenomenon could be quantum-induced trans-Planckian energy density at the SMBHs event horizons.