Cosmological attractors and inflationary scenarios with many scalar fields

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based on

M.N. Dubinin, E.Yu. Petrova, E.O. Pozdeeva, M.V. Sumin and S.Yu. Vernov, JHEP 1712 (2017) 036, arxiv:1705.09624

M.N. Dubinin, E.Yu. Petrova, E.O. Pozdeeva, and S.Yu. Vernov, IJGMMP 15 (2018) 1840001, arXiv:1712.03072

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The Strong Coupling Approximation

D. Roest, *J. Cosmol. Astropart. Phys.* **1401** (2014) 007 [1309.1285 [hep-th]];
M. Galante, R. Kallosh, A. Linde and D. Roest, *Phys. Rev. Lett.* **112** (2014) 011303[1310.3950 [hep-th]];
R. Kallosh, A. Linde and D. Roest, *J. High Energy Phys.* **1409** (2014) 062 [1407.4471 [hep-th]];
M. Galante, R. Kallosh, A. Linde and D. Roest, *Phys. Rev. Lett.* **114** (2015) 141302[1412.3797 [hep-th]];
P. Binetruy, E. Kiritsis, J. Mabillard, M. Pieroni and C. Rosset, *J. Cosmol. Astropart. Phys.* **1504** (2015) 033[1407.0820 [astro-ph.CO]]

- The idea of a cosmological attractor is based on an observation that the kinetic term in Jordan frame practically does not affect the slow-roll parameters if the 'strong coupling regime' is respected during inflation.
- It allows to get approximate values of $n_{\rm s}$ and r in the analytic form.

Conformal Transformation

- Generic action which is dependent on N scalar fields ϕ^I, $I = 1, ..., N$ with the standard kinetic term and nonminimal coupling to gravity

$$S_J = \int d^4x\sqrt{-\tilde{g}}\left[f(\phi^I)\tilde{R} - \frac{1}{2}\delta_{IJ}\tilde{g}^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]. \qquad (1)$$

tilde denotes the metric tensor and curvature in the Jordan frame.

- This action can be transformed to the action of the following chiral cosmological model[1]:

$$S_E = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{1}{2}\mathcal{G}_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - W\right], \qquad (2)$$

$$\mathcal{G}_{IJ} = \frac{M_{Pl}^2}{2f(\phi^K)}\left[\delta_{IJ} + \frac{3f_{,I}f_{,J}}{f(\phi^K)}\right], \quad W = M_{Pl}^4\frac{V}{4f^2}, \quad M_{Pl} \equiv \frac{1}{\sqrt{8\pi G}},$$

$f_{,I} = \partial f/\partial\phi^I$.

- Metric tensors in the Jordan and the Einstein frames are related as

$$g_{\mu\nu} = \frac{2}{M_{Pl}^2}f(\phi^I(x))\tilde{g}_{\mu\nu}(x).$$

[1] R.N. Greenwood, D.I. Kaiser, E.I. Sfakianakis, Phys. Rev. D **87** (2013) 064021

• By definition the strong coupling regime of the field system takes place if the following inequality is respected

$$\delta_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J \ll \frac{3}{f(\phi^K)}f_{,I}f_{,J}\partial_\mu\phi^I\partial_\nu\phi^J. \quad (3)$$

• Using this approximation, we get

$$S_E = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{g^{\mu\nu}}{2}\partial_\mu\Theta\partial_\nu\Theta - \frac{M_{Pl}^4 V}{4f^2}\right].$$

• The role of inflaton in the strong coupling approximation is performed by the "effective field"

$$\Theta = \sqrt{\frac{3}{2}}M_{Pl}\ln\left(\frac{f}{f_0}\right) \quad (4)$$

where f_0 is a positive constant with the same dimension as f. If we choose $f_0 = M_{Pl}^2/2$, then $\Theta = 0$ corresponds to zero values of all scalar fields.

• In terms of which the action S_E includes the standard kinetic term of Θ and does not include kinetic terms of any other scalar fields which can be interpreted as model parameters.

• This circumstance allows one to calculate the inflationary parameters in the SC approximation using the single-field model.

OBSERVATION DATA

There are a few main parameters that can be obtained by the observation data:
(Planck 2015 results. XX. Constraints on inflation, Astron.Astrophys. **594** (2016) A20)

- The joint BICEP2/Keck Array and Planck constraint on tensor-to-scalar power ratio r: $r < 0.12$ (95% c.l.).
- The scalar spectral index $n_{\rm s}$.

$$n_{\rm s} = 0.968 \pm 0.006.$$

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- The scalar spectral index n_{s}.

$$n_{\mathrm{s}} = 0.968 \pm 0.006.$$

- The constraints on local non-Gaussianity are

$$f_{NL}^{local} = 0.8 \pm 5.0. \qquad (68\% CL).$$

- The upper bound on the Hubble parameter during inflation of

$$H < 3.6 \times 10^{-5} M_{Pl}.$$

Similar bound has been calculated in the GUT motivated inflationary model A.O. Barvinsky and A.Yu. Kamenshchik, *Phys. Lett. B* **332** (1994) 270 [gr-qc/9404062].

INFLATION MOTIVATED BY THE QFT

A fundamental step towards the unification of physics at all energy scales could be the possibility to describe the inflation using particle physics models.
A number of advantages of simplified SUSY GUTs in comparison with nonsupersymmetric GUTs such as naturally longer period of exponential expansion and better stability of the effective Higgs potential with respect to radiative corrections due to cancelation of loop diagrams have been noted quite long ago:
J. Ellis, D. V. Nanopoulos, K. A. Olive and K. Tamvakis, *Primordial supersymmetric inflation*, *Nucl. Phys.* **B221** (1983) 524.
B. A. Ovrut and P. J. Steinhardt, *Supersymmetric inflation, baryon asymmetry and the gravitino problem*, *Phys. Lett.* **B147** (1984) 263.

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B. A. Ovrut and P. J. Steinhardt, *Supersymmetric inflation, baryon asymmetry and the gravitino problem*, *Phys. Lett.* **B147** (1984) 263.

There are a lot of papers that describe inflation using supersymmetry:

G. R. Dvali, *Phys. Lett.* **B387** (1996) 471 [hep-ph/9605445].

K. Enqvist and A. Mazumdar, *Phys. Rept.* **380** (2003) 99, [hep-ph/0209244].

L. E. Ibanez, F. Marchesano and I. Valenzuela, J. High Energy Phys. **1501** (2015) 128 [arXiv:1411.5380 [hep-th]].

I. Antoniadis, A. Chatrabhuti, H. Isono and R. Knoops, Eur. Phys. J. C **77** (2017) 724 [arXiv:1706.04133 [hep-th]].

THE HIGGS-DRIVEN INFLATION

There are models of inflation, where the role of the inflaton is played by the Higgs field nonminimally coupled to gravity. (F.L. Bezrukov and M. Shaposhnikov, *Phys. Lett.* B **659** (2008) 703–706, arXiv:0710.3755). They add $W^{(0)}(\phi)$ to the standard GR term and get the following action:

$$S = \int d^4x\sqrt{-g}\left[\left(\frac{M_{PL}^2}{2} + \xi\phi^2\right)R - \frac{1}{2}(\partial\phi)^2 - \lambda\left(\phi^2 - \phi_0^2\right)^2\right].$$

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This model have been actively studied

A.O. Barvinsky, A.Y. Kamenshchik, and A.A. Starobinsky, J. Cosmol. Astropart. Phys. **0811** (2008) 021 (arXiv:0809.2104);
F. Bezrukov, D. Gorbunov and M. Shaposhnikov, J. Cosmol. Astropart. Phys. **0906** (2009) 029, arXiv:0812.3622;
A. De Simone, M.P. Hertzberg and F. Wilczek, Phys. Lett. B **678** (2009) 1 (arXiv:0812.4946);
A.O. Barvinsky, A.Y. Kamenshchik, C. Kiefer, A.A. Starobinsky, and C.F. Steinwachs, J. Cosmol. Astropart. Phys. **0912** (2009) 003 (arXiv:0904.1698);
J. Garcia-Bellido, D.G. Figueroa, and J. Rubio, Phys. Rev. D **79** (2009) 063531 (arXiv:0812.4624);
F. Bezrukov, Class. Quant. Grav. **30** (2013) 214001 (arXiv:1307.0708)

The MSSM-inspired model with non-minimal interaction

- We start with the non-minimal coupling model:

$$S=\int d^4x\sqrt{-g}[f(\Phi_1,\Phi_2)R-\delta^{ab}g^{\mu\nu}\partial_\mu\Phi_a^\dagger\partial_\nu\Phi_b-V(\Phi_1,\Phi_2)], \qquad (5)$$

where g is the determinant of metric tensor $g_{\mu\nu}$, and R is the scalar curvature.

- In the single-field Higgs-driven inflation the function f has been chosen as a sum of the Hilbert–Einstein term and the induced gravity term. We choose the function f in an analogous form:

$$f(\Phi_1,\Phi_2)=\frac{M_{Pl}^2}{2}+\xi_1\Phi_1^\dagger\Phi_1+\xi_2\Phi_2^\dagger\Phi_2 \qquad (6)$$

where ξ_1 and ξ_2 are positive dimensionless constants.

- The potential V is MSSM effective potential

• The Minimal Supersymmetric Standard Model two-doublet effective potential is

$$V(\Phi_1,\Phi_2) = V_2(\Phi_1,\Phi_2) + V_4(\Phi_1,\Phi_2), \tag{7}$$

where

$$V_2 = -\,\mu_1^2(\Phi_1^\dagger\Phi_1) - \,\mu_2^2(\Phi_2^\dagger\Phi_2) - [\mu_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.],$$

$$\begin{aligned} V_4 \;&=\; \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+\; \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + h.c.\right]. \end{aligned}$$

It has been proposed in
P. Fayet, Nucl. Phys. **D90** (1975) 104;
K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **68** (1982) 927

• The dimensionless factors λ_i ($i = 1, ..., 7$) at the tree level are expressed, using the $SU(2)$ and $U(1)$ gauge couplings g_2 and g_1, in the form:

$$\lambda_1^{\texttt{tree}} = \lambda_2^{\texttt{tree}} = \frac{g_1^2 + g_2^2}{8}, \qquad \lambda_3^{\texttt{tree}} = \frac{g_2^2 - g_1^2}{4},$$
$$\lambda_4^{\texttt{tree}} = -\frac{g_2^2}{2}, \qquad \lambda_{5,6,7}^{\texttt{tree}} = 0.$$

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• At the superpartners mass scale M_{SUSY} renormalization group evolved tree-level quartic couplings λ_i can be evaluated, using the collider data for g_1 and g_2 at m_{top} scale.
The gauge boson masses at tree level

$$m_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2}, \quad m_W = \frac{v}{2} g_2, \quad v = \sqrt{v_1^2 + v_2^2} = (G_F\sqrt{2})^{-1/2},$$

where G_F is the Fermi constant.
Substituting pole masses
$m_Z = 91.2$ GeV, $m_W = 80.4$ GeV, and $v = 246$ GeV,
we obtain

$$g_1 = 0.36 \quad \text{and} \quad g_2 = 0.65$$

at the m_{top} scale.

Transformation to Scalar Fields Model

• Two Higgs doublets of the MSSM can be parameterized using the $SU(2)$ states

$$\Phi_1 = \left(\begin{array}{c} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{array} \right), \quad \Phi_2 = \left(\begin{array}{c} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{array} \right), \qquad (8)$$

• The mass basis of scalars is constructed in a standard way. The $SU(2)$ eigenstates ($\omega_a^\pm, \eta_a$ and χ_a, $a = 1, 2$) are expressed through mass eigenstates of the Higgs bosons h, H_0, A and $H^\pm$ and the Goldstone bosons G^0, $G^\pm$ by means of two orthogonal rotations

$$\left(\begin{array}{c} \eta_1 \\ \eta_2 \end{array} \right) = \mathcal{O}_\alpha \left(\begin{array}{c} H_0 \\ h \end{array} \right), \left(\begin{array}{c} \chi_1 \\ \chi_2 \end{array} \right) = \mathcal{O}_\beta \left(\begin{array}{c} G^0 \\ A \end{array} \right), \left(\begin{array}{c} \omega_1^\pm \\ \omega_2^\pm \end{array} \right) = \mathcal{O}_\beta \left(\begin{array}{c} G^\pm \\ H^\pm \end{array} \right),$$

where the rotation matrix

$$\mathcal{O}_X = \left(\begin{array}{cc} \cos X & -\sin X \\ \sin X & \cos X \end{array} \right), \qquad X = \alpha, \beta. \qquad (9)$$

$$t_\beta \equiv \tan(\beta) = \frac{v_2}{v_1}.$$

We use the unitary gauge $G^0 = G^{\pm} = 0$, therefore,

$$\eta_1 = \cos(\alpha)H_0 - \sin(\alpha)h, \qquad \eta_2 = \sin(\alpha)H_0 + \cos(\alpha)h,$$

$$\chi_1 = -\sin(\beta)A, \quad \chi_2 = \cos(\beta)A, \quad \omega_1^{\pm} = -\sin(\beta)H^{\pm}, \quad \omega_2^{\pm} = \cos(\beta)H^{\pm}.$$

The h-boson is identified as the observed $125.09 \pm 0.24\, GeV$ scalar state.
The 'alignment limit' of the MSSM is used:

$$\beta - \alpha \approx \frac{\pi}{2}.$$

We assume $\beta = \frac{\pi}{2} + \alpha$.
The limits on $\tan\beta$ coming from models of low-energy supersymmetry are assumed to be $1 < t_\beta \leqslant m_{top}/m_b \simeq 35$.
The latest results of ATLAS and CMS collaborations show that regions of large $\tan\beta$ at the 95% confidence level (CL) should be excluded.
At the same time $t_\beta = \infty$ is interesting as a toy model.

In the alignment limit with $\beta = \pi/2 + \alpha$ the potential V_4 can be written in terms of the scalar fields:

$$
\begin{aligned}
V_4 \;=\; & \frac{1}{32\left[t_\beta^2+1\right]^2}\Big\{\left(g_1^2+g_2^2\right)\left(t_\beta^2-1\right)^2 h_v^4 - 8\left(g_1^2+g_2^2\right)t_\beta\left(t_\beta^2-1\right)H_0 h_v^3 \\
+ \; & \left[\left(2(2H^-H^+ - A^2 - H_0^2)g_2^2 - 2(A^2+H_0^2+2H^-H^+)g_1^2\right)t_\beta^4\right. \\
+ \; & \left((4A^2+20H_0^2+8H^-H^+)g_1^2 + 4(A^2+5H_0^2+6H^-H^+)g_2^2\right)t_\beta^2 \\
+ \; & \left. 2(2H^-H^+ - A^2 - H_0^2)g_2^2 - 2(A^2+H_0^2+2H^-H^+)g_1^2\right] h_v^2 \\
+ \; & 8(g_1^2+g_2^2)t_\beta\left(t_\beta^2-1\right)(A^2+H_0^2+2H^-H^+)H_0 h_v \\
+ \; & \left(g_1^2+g_2^2\right)\left(t_\beta^2-1\right)^2 (A^2+H_0^2+2H^-H^+)^2\Big\},
\end{aligned}
$$

where $h_v = h + v$.

The function f in terms of the scalar fields is

$$
\begin{aligned}
f = \frac{M_{Pl}^2}{2} + \frac{\xi_1}{2(t_\beta^2+1)}\left[(A^2+H_0^2+2H^-H^+)t_\beta^2 + 2H_0 h_v t_\beta + h_v^2\right] \\
+ \frac{\xi_2}{2(t_\beta^2+1)}\left[h_v^2 t_\beta^2 - 2H_0 h_v t_\beta + A^2 + H_0^2 + 2H^-H^+\right].
\end{aligned}
$$

The potential V depends on five real scalar fields

$$\phi^1 = \frac{H^+ + H^-}{\sqrt{2}},\ \phi^2 = \frac{H^+ - H^-}{\sqrt{2}\mathrm{i}},\ \phi^3 = A,\ \phi^4 = H_0,\ \phi^5 = h_v. \qquad (10)$$

- If we set $\phi^5 = h_v = 0$ and $V_2 \ll V_4$ during inflation, then

$$f = \frac{M_{Pl}^2}{2} + \frac{(\xi_1 t_\beta^2 + \xi_2)(A^2 + H_0^2 + 2H^+H^-)}{2\left(t_\beta^2 + 1\right)}. \qquad (11)$$

and the potential $V \approx V_4$ can be expressed as a function of f:

$$V_4 = \frac{1}{32}(g_1^2 + g_2^2)\left(t_\beta^2 - 1\right)^2 \left(\frac{2f - M_{Pl}^2}{\xi_1 t_\beta^2 + \xi_2}\right)^2. \qquad (12)$$

In the Einstein frame the potential is

$$W = \frac{M_{Pl}^4 V_4}{4f^2} = \frac{\left(g_1^2 + g_2^2\right) M_{Pl}^4}{32\left(\xi_1 t_\beta^2 + \xi_2\right)^2}\left(t_\beta^2 - 1\right)^2 \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\Theta}{M_{Pl}}}\right)^2. \qquad (13)$$

In the spatially flat Friedman–Lemaitre–Robertson–Walker (FLRW) metric with the interval

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right),$$

where $a(t)$ is the scale factor, the slow-roll parameters are

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{W'_\Theta}{W}\right)^2 = \frac{4\,\mathrm{e}^{-\frac{2\sqrt{6}\Theta}{3M_{\rm Pl}}}}{3\left(1-\mathrm{e}^{-\frac{\sqrt{6}\Theta}{3M_{\rm Pl}}}\right)^2},$$

$$\eta = M_{\rm Pl}^2\frac{W''_{\Theta\Theta}}{W} = \frac{4\left(2-\mathrm{e}^{\frac{\sqrt{6}\Theta}{3M_{\rm Pl}}}\right)}{3\left(1-\mathrm{e}^{\frac{\sqrt{6}\Theta}{3M_{\rm Pl}}}\right)^2},$$

where primes denote derivatives with respect to Θ.
The inflationary parameters are

$$n_{\rm s} = 1 - \frac{8M_{Pl}^2\left(M_{Pl}^2+2f\right)}{3\left(M_{Pl}^2-2f\right)^2}, \qquad r = \frac{64M_{Pl}^4}{3\left(M_{Pl}^2-2f\right)^2}. \qquad (14)$$

Note that these expressions for $n_{\rm s}(f)$ and $r(f)$ do not depend on t_β and coincide with the corresponding formulae for $\beta=\pi/2$ (when $t_\beta=\infty$). At the same time it is not correct to say that the corresponding inflationary scenarios do not depend on β, the direction to potential minima in the $(v_1,\, v_2)$-plane, because the Hubble parameter is expressed through f as follows:

$$H_f^2 \approx \frac{M_{Pl}^2\left({g_1}^2+{g_2}^2\right)\left(t_\beta^2-1\right)^2\left(M_{Pl}^2-2f\right)^2}{384\left(\xi_1\, t_\beta^2+\xi_2\right)^2 f^2}. \qquad (15)$$

One can see that inflationary scenarios are excluded at $\beta=\pi/4$.

The temperature data of the Planck full mission and first release of the polarization data at large angular scales constrain the spectral index of curvature perturbations to

$$n_{\mathrm{s}} = 0.968 \pm 0.006 \quad (68\%\,\mathrm{CL}), \tag{16}$$

and restrict the tensor-to-scalar ratio from above

$$r < 0.11 \quad (95\%\,\mathrm{CL}). \tag{17}$$

Using the observable value of $n_{\mathrm{s}} = 0.968$, we obtain from Eq. (14) the corresponding value of $f = 43.14 M_{Pl}^2$, so the Hubble parameter is expressed in a compact form

$$H_f^2 \approx 0.01 \, \frac{\left(t_\beta^2 - 1\right)^2 M_{Pl}^2 \left(g_1^2 + g_2^2\right)}{\left(\xi_1 \, t_\beta^2 + \xi_2\right)^2}. \tag{18}$$

Numerical calculations

- Let us consider a spatially flat FLRW universe.
- Varying the action S_E, we get

$$\begin{aligned} H^2 &= \frac{1}{3M_{Pl}^2}\left(\frac{\dot{\sigma}^2}{2} + W\right), \\ \dot{H} &= -\frac{1}{2M_{Pl}^2}\dot{\sigma}^2, \end{aligned} \tag{19}$$

where the Hubble parameter $H = \dot{a}/a$,

$$\dot{\sigma}^2 \equiv \mathcal{G}_{IJ}\dot{\phi}^I\dot{\phi}^J,$$

and dots mean the time derivatives. • Field equations have the following form:

$$\ddot{\phi}^I + 3H\dot{\phi}^I + \Gamma^I{}_{JK}\dot{\phi}^J\dot{\phi}^K + \mathcal{G}^{IK}W'_{,K} = 0\,, \tag{20}$$

where $\Gamma^I{}_{JK}$ is the Christoffel symbol for the field-space manifold, calculated in terms of $\mathcal{G}_{IJ}$,
$W'{}_{,K} = \partial W/\partial\phi^K$.
Hereafter, primes denote derivatives with respect to the fields.

• During inflation the scalar factor a is a monotonically increasing function. To describe the evolution of scalar fields during inflation we use the number of e-foldings $N_e = \ln(a/a_e)$, as a new measure of time. Using $d/dt = H\, d/dN_e$ one can write Eqs. (19) and (20) in the form

$$
\begin{aligned}
H^2 &= \frac{2W}{6M_{Pl}^2 - (\sigma')^2},\\
\frac{d\ln H}{dN_e} &= -\,\frac{1}{2M_{Pl}^2}\left(\sigma'\right)^2,\\
\frac{d\phi^I}{dN_e} &= \psi^I,\\
\frac{d\psi^I}{dN_e} &= -\left(3+\frac{d\ln H}{dN_e}\right)\psi^I - \Gamma^I{}_{JK}\psi^J\psi^K - \frac{1}{H^2}\mathcal{G}^{IK}W^{'}_{,K},
\end{aligned}
$$

So, we get the following system of ten first order equations

$$
\begin{cases}
\dfrac{d\phi^I}{dN_e} = \psi^I,\\[2ex]
\dfrac{d\psi^I}{dN_e} = -\left(3-\dfrac{\left(\sigma'\right)^2}{2M_{Pl}^2}\right)\psi^I - \Gamma^I{}_{JK}\psi^J\psi^K - \dfrac{6M_{Pl}^2-(\sigma')^2}{2W}\mathcal{G}^{IK}W^{'}_{,K}.
\end{cases}
$$

Parameters of an inflationary model

• In order to calculate the observables, spectral index $n_{\rm s}$ and tensor-to-scalar ratio r, slow-roll parameters are introduced analogously to the single-field inflation

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta_{\sigma\sigma} = M_{Pl}^2 \frac{\mathcal{M}_{\sigma\sigma}}{W},$$

where

$$\mathcal{M}_{\sigma\sigma} \equiv \hat{\sigma}^K \hat{\sigma}^J (\mathcal{D}_K \mathcal{D}_J W), \tag{21}$$

$\sigma^I = \dot{\phi}^I/\dot{\sigma}$ is the unit vector in the field space and $\mathcal{D}$ denotes a covariant derivative with respect to the field-space metric, $\mathcal{D}_I \phi^J = \partial_I \phi^J + \Gamma^J_{IK} \phi^K$.

Numerical calculations show that all scalar fields monotonically decrease before and during inflation, so the function f is also a monotonically decreasing function. For this reason, such initial values of scalar fields are chosen that the corresponding values of $f > 43.14 M_{Pl}^2$. Thus, the strong coupling approximation simplifies the choice of initial conditions for numerical calculations.

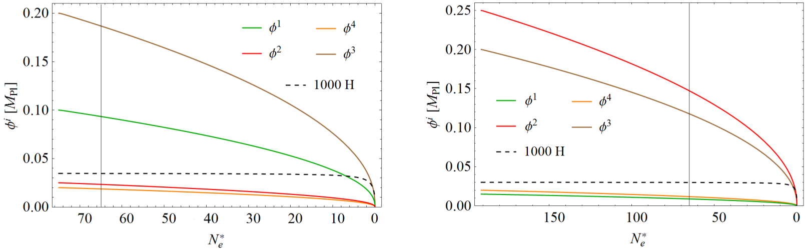

Figure: Evolution of the fields and the Hubble parameter as functions of the number of e-foldings $N_e^* = -N_e$ during inflation in scenarios A_1 (left) and C_1 (right), see Table 1. The vertical line corresponds to $N_e^* = 65$.

Numerical solutions of the equations of motion

Table: The parameters of the model and the initial field values for numerical calculations.

Scenario	t_β	ξ_1	ξ_2	ϕ_1/M_{Pl}	ϕ_2/M_{Pl}	ϕ_3/M_{Pl}	ϕ_4/M_{Pl}
A_1	5	2000	2000	0.1	0.025	0.2	0.02
A_2	5	2000	2000	0.2	0.05	0.1	0.15
B_1	10	2500	2500	0.15	0.15	0.1	0.1
B_2	10	2000	1000	0.15	0.15	0.1	0.2
C_1	20	2500	1000	0.015	0.25	0.2	0.02
C_2	20	2500	500	0.015	0.25	0.2	0.02
D_1	40	2000	2000	0.01	0.025	0.2	0.02
D_2	40	2000	2000	0.12	0.12	0.12	0.12

Table: The values of function f and the Hubble parameter H at $N_e = -65$, together with the tensor-to-scalar ratio r and spectral index n_s for successful inflationary scenarios.

Scenario	f/M_{Pl}^2	H/M_{Pl} $[10^{-5}]$	r	n_s
A_1	45.042	3.461	0.002661	0.9694
A_2	45.247	3.462	0.002637	0.9695
B_1	44.807	2.941	0.002649	0.9695
B_2	45.048	3.694	0.002660	0.9694
C_1	44.910	2.989	0.002677	0.9695
C_2	45.084	2.991	0.002656	0.9694
D_1	44.908	3.461	0.002661	0.9694
D_2	45.387	3.462	0.002621	0.9696

The case of $\beta = \pi/2$

The case $t_\beta = \infty$ ($\beta = \pi/2$ and $\alpha = 0$) has been consider in detail in M.N. Dubinin, E.Yu. Petrova, E.O. Pozdeeva, M.V. Sumin and S.Yu. Vernov, *JHEP* 1712 (2017) 036, arxiv:1705.09624

We have made numerical calculations without any approximations and consider the case of non-zero h_v as well.

We found **two types of inflationary scenarios:**

- A) $h_v = 0$, other fields roll slowly down to the potential minimum
- B) h_v and other nonzero fields demonstrate rapidly damped oscillations

for the number of e-foldings before the end of inflation $N_e^* \gg 65$.

Scenario	ξ_1	ξ_2	ϕ_0^1	ϕ_0^2	ϕ_0^3	ϕ_0^4	ϕ_0^5
A_1	2500	any	0.2	0.24	0.3	0.1	0
A_2	2500	any	2×10^{-3}	0	0.45	0.2	0
A_3	2500	any	0.2	0.26	0.5	0.6	0
B_1	1100	500	0.3	0.2	0	0	0.1
B_2	1100	500	0.4	0.6	0	0	0.3
B_3	2200	1000	0.3	0.2	0	0	0.1
B_4	2200	2200	0.1	0.1	0	0	0.155

Table: Initial conditions (in units of M_{Pl}), $m_A = 200\ GeV$, $\phi^5 = h_v$.

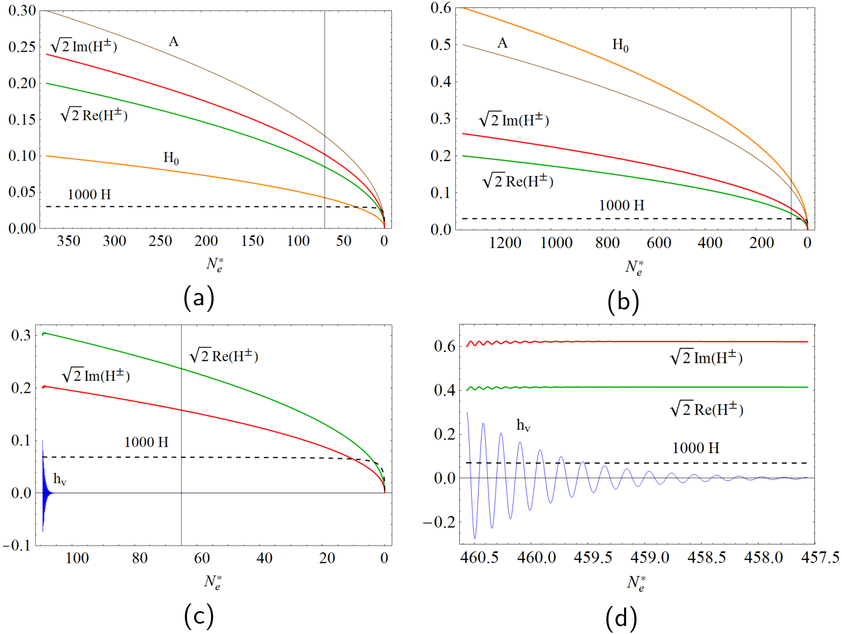

Figure: Evolution for the scenarios A_1 (a), A_3 (b), B_1 (c) and B_2 (d).

Scenario	H $[10^{-5}]$	r	$n_{\rm s}$
A_1	2.99983	0.00266259	0.969398
A_2	2.99983	0.00266259	0.969398
A_3	2.99983	0.00266255	0.969399
B_1	6.81778	0.00266322	0.969396
B_2	6.81778	0.00266325	0.969396
B_3	3.40892	0.00265832	0.969424
B_4	2.98224	0.00263611	0.969555

Table: The Hubble parameter H, tensor-to-scalar ratio r and spectral index $n_{\rm s}$ for successful inflationary scenarios at $N_e^* = 65$, $m_A = 200\ GeV$.

• For the Hubble parameter $H \sim 10^{-5} M_{Pl}$ the values of $n_{\rm s}$ and r coincide up to five and three digits, correspondingly. Such "attractor behavior" when over a wide range of initial conditions the system evolves along the same trajectory in the course of inflation is known for single-field models, but it is not an obvious observation, generally speaking, for multifield models.

Conclusions

1. We constuct the inflationary scenarios which could be induced by the two-Higgs-doublet potential of the Minimal Supersymmetric Standard Model (MSSM) where five scalar fields have non-minimal couplings to gravity.
2. Observables following from such MSSM-inspired multifield inflation are calculated and a number of consistent inflationary scenarios are constructed.
3. Cosmological evolution with different initial conditions for the multifield system leads to consequences fully compatible with observational data on the spectral index and the tensor-to-scalar ratio.
4. It is demonstrated that the strong coupling approximation is precise enough to describe such inflationary scenarios.
5. Inflationary scenarios have been constructed for different values of t_β.

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Thank for your attention