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Late-time power-law stages of cosmological evolution in teleparallel gravity with nonminimal coupling

Skugoreva M. A.

Kazan (Volga region) Federal University

Introduction

- Teleparallel gravity (TG) is based on geometry of absolute parallelism (**Einstein, 1920-th**) – using a field of orthonormal bases (tetrads) e^{μ}_{A} for tangent space-times at each point of space-time.
- The Weitzenböck connection (**1923**) is applied in this theory instead of the Levi-Civita one, that leads to zero curvature ***R = 0*** and non-zero torsion ***T***.
- Equations of motion of TG coincide exactly with those of General Relativity (GR). However, modifications of these theories are not equivalents and their field equations differ from each other. Therefore, generalizations of TG (for example, *f(T)* gravity, scalar-torsion gravity) give rise to new cosmological dynamics and can describe **late-time cosmic acceleration of the Universe**

Introduction

The Weitzenböck connection is used in teleparallel gravity

$$\overset{W}{\Gamma}{}^{\lambda}_{\nu\mu} = e^{\lambda}_{A}\partial_{\mu} e^{A}_{\nu} \quad .$$

Then torsion tensor and torsion scalar are

$$T^{\lambda}_{\mu\nu} \equiv \overset{W}{\Gamma}{}^{\lambda}_{\nu\mu} - \overset{W}{\Gamma}{}^{\lambda}_{\mu\nu} = e^{\lambda}_{A}(\partial_{\mu} e^{A}_{\nu} - \partial_{\nu} e^{A}_{\mu}) \quad ,$$

$$T \equiv \frac{1}{4} T^{\rho\nu\mu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\nu\mu} T_{\nu\mu\rho} - T^{\rho}_{\rho\mu} T^{\nu\mu}_{\nu} \quad .$$

The relation between of quantities, which are calculated applying the Levi-Civita connection (L) and the Weitzenböck one (W):

$$\overset{L}{R} = -\overset{W}{T} - 2\overset{L}{\nabla}{}^{\mu}\left(T^{\nu}_{\mu\nu}\right)$$

Aim of the work

The aim of this work was the investigation of the late-time cosmological evolution in teleparallel gravity with nonminimal coupling with the following Lagrangian

$$L = \frac{1}{2}\sqrt{-g}\left(\frac{T}{8\pi G} + \partial_\mu \varphi\, \partial^\mu \varphi + \xi\, T\, B(\varphi) - 2V(\varphi)\right) + L_m,$$

where

$\xi < 0,\ B(\varphi) = \varphi^N$, $N > 2$ and $V(\varphi) = V_0 \varphi^n$,

$p = \omega\rho$, $\omega \in [-1;1]$ $V_0 \geq 0$, $n < 0$

The tetrad $e^A{}_\mu = diag(1, a(t), a(t), a(t))$ ($ds^2 = dt^2 - a^2(t) dl^2$) and Planck units are used $c = \hbar = 1$

Methods of the investigation

Numerical integration,

methods of theory of differential equations,

algebraic methods

are applyed in this work.

Main equations

Equations of gravitation and scalar fields are derived by varying the action with the considered Lagrangian

$$3H^2 = K\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi) - 3\xi H^2 B(\varphi) + \rho\right), \qquad (1)$$

$$2\dot{H} = -K\left(\dot{\varphi}^2 + 2\xi H\dot{\varphi}B'(\varphi) + 2\xi \dot{H} B(\varphi) + \rho(1+\omega)\right), \qquad (2)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + 3\xi H^2 B'(\varphi) + V'(\varphi) = 0, \qquad (3)$$

here $K = 8\pi G$, $\quad ' = \dfrac{d}{d\varphi}$.

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The vacuum case ($\rho=0$).

The asymptotic solution ($\varphi \to \infty$) for *N* > 2,

$$V(\varphi)=V_0\varphi^n \text{, } n\leqslant -N.$$

$$H(t)=H_0 t^{-\frac{2N}{N+2}}, \quad a(t)=a_0 e^{H_0\frac{N+2}{2-N}t^{\frac{2-N}{N+2}}},$$

$$\varphi(t)=\varphi_0 t^{\frac{2}{N+2}}, \quad t\to+\infty$$

This solution is **stable** with respect to homogeneous variations of the initilal data (numerical result).The Universe expands with the deceleration at late times and approaches asymptotically a static state. It is rather surprising since not so many stable stationary cosmological models are known.

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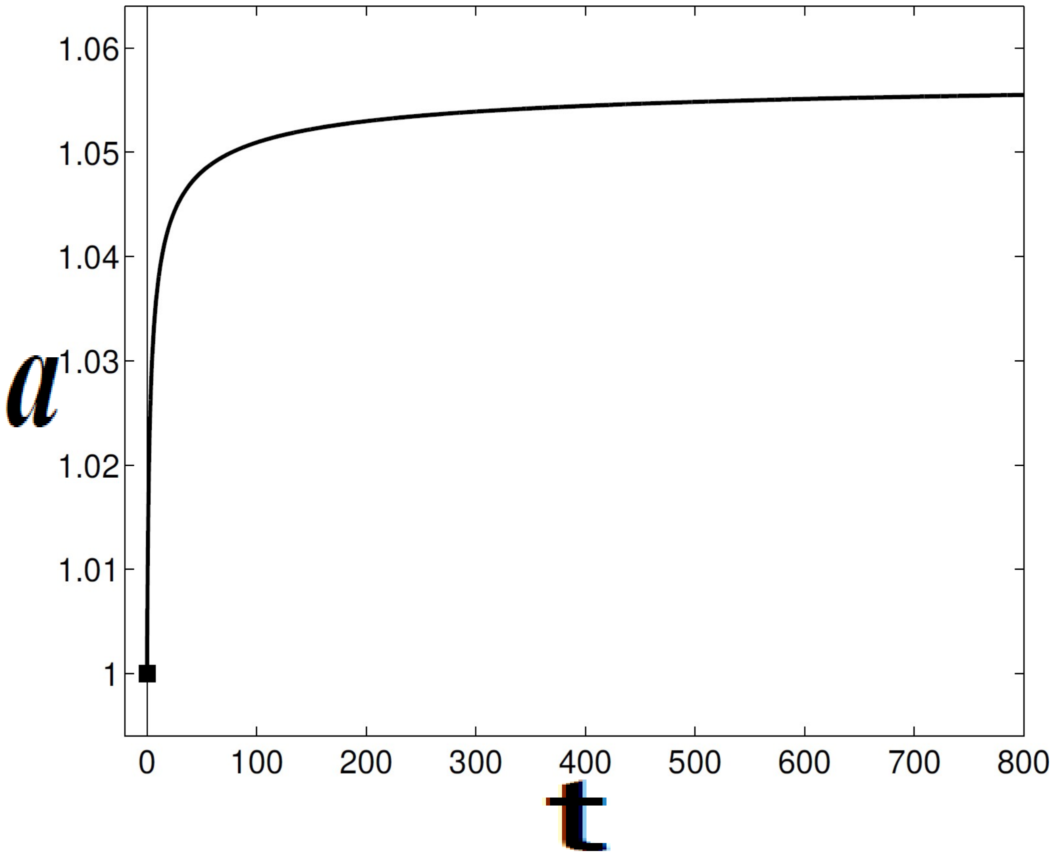
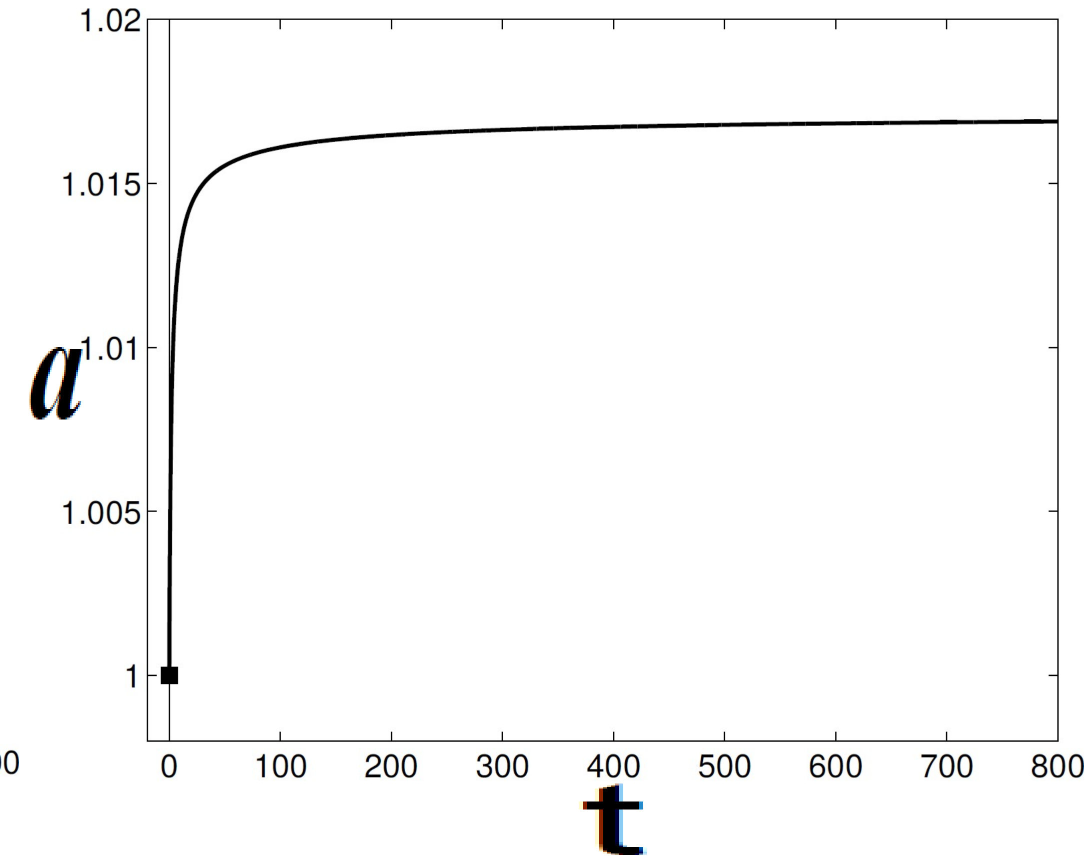
Fig. 1. The evolution of the scale factor *a(t)* for *N* > 2, $V(\varphi)=V_0\varphi^n, n\leqslant -N, \rho=0$

$$\xi=1 \quad , \quad V_0=1 \quad , \quad K=1$$





$N=-n=6$

$N=8, n=-10$

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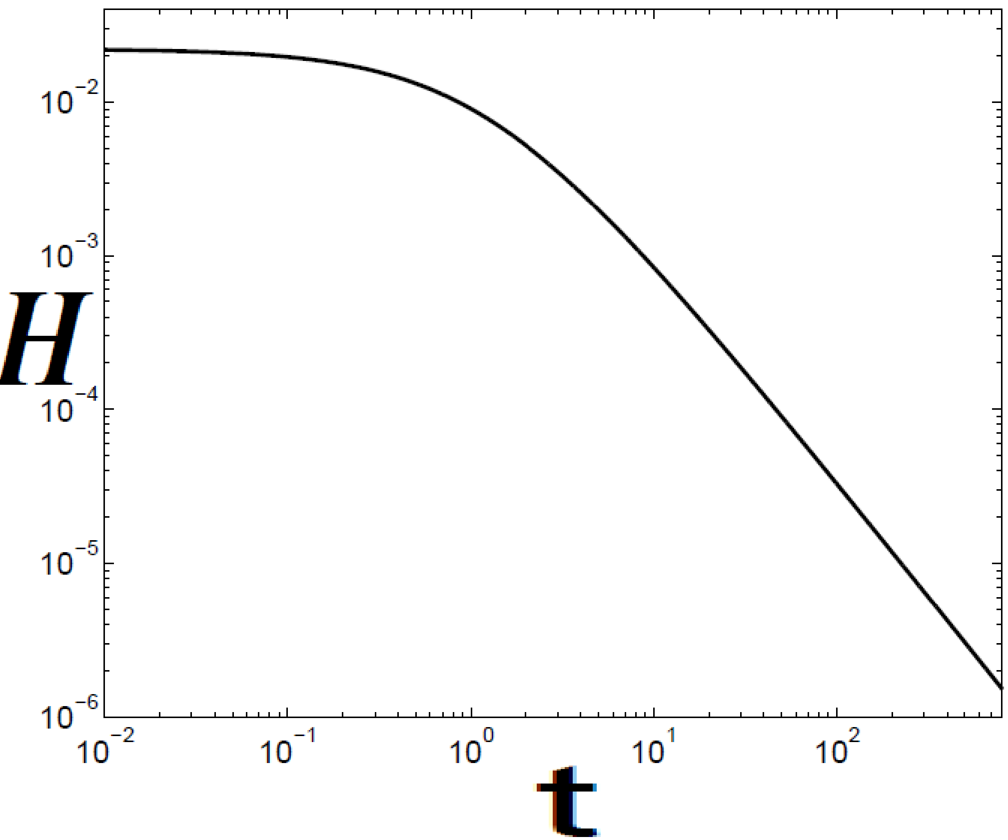
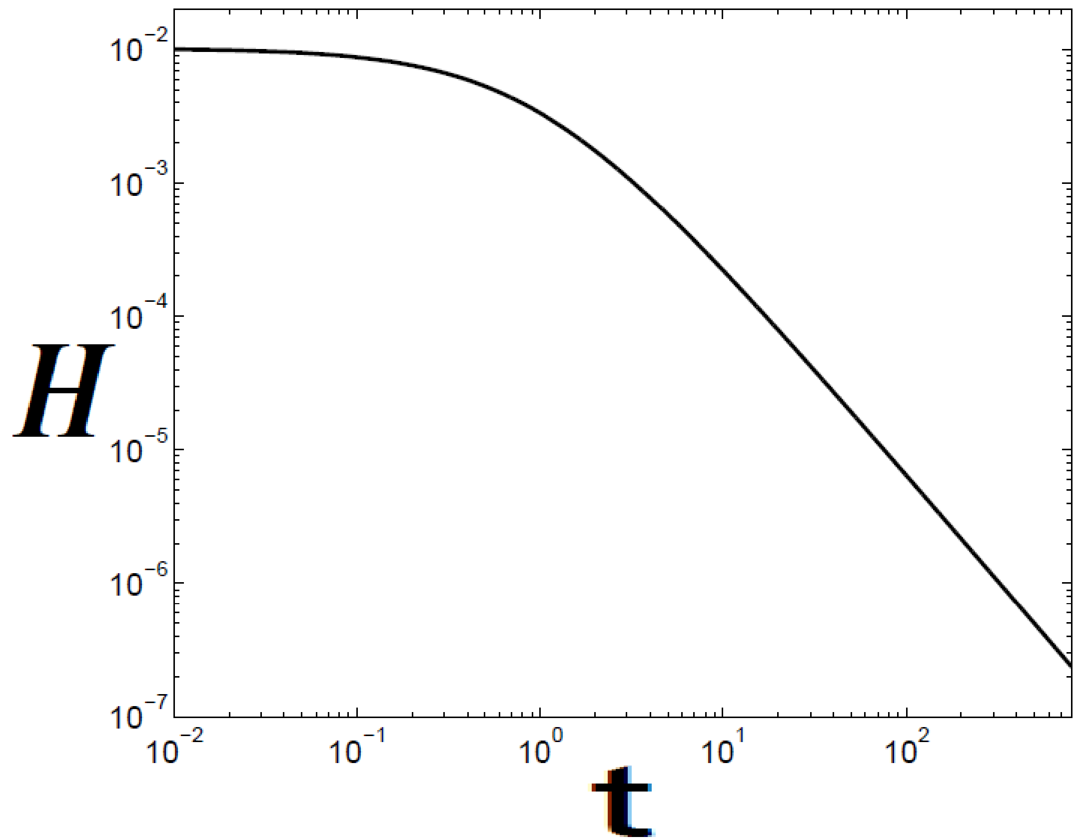
Fig. 2. The evolution of Hubble parameter *H(t)* for *N* > 2, $V(\varphi)=V_0\varphi^n$, $n\leqslant -N$, $\rho=0$

$$\xi=1 \quad , \quad V_0=1 \quad , \quad K=1$$



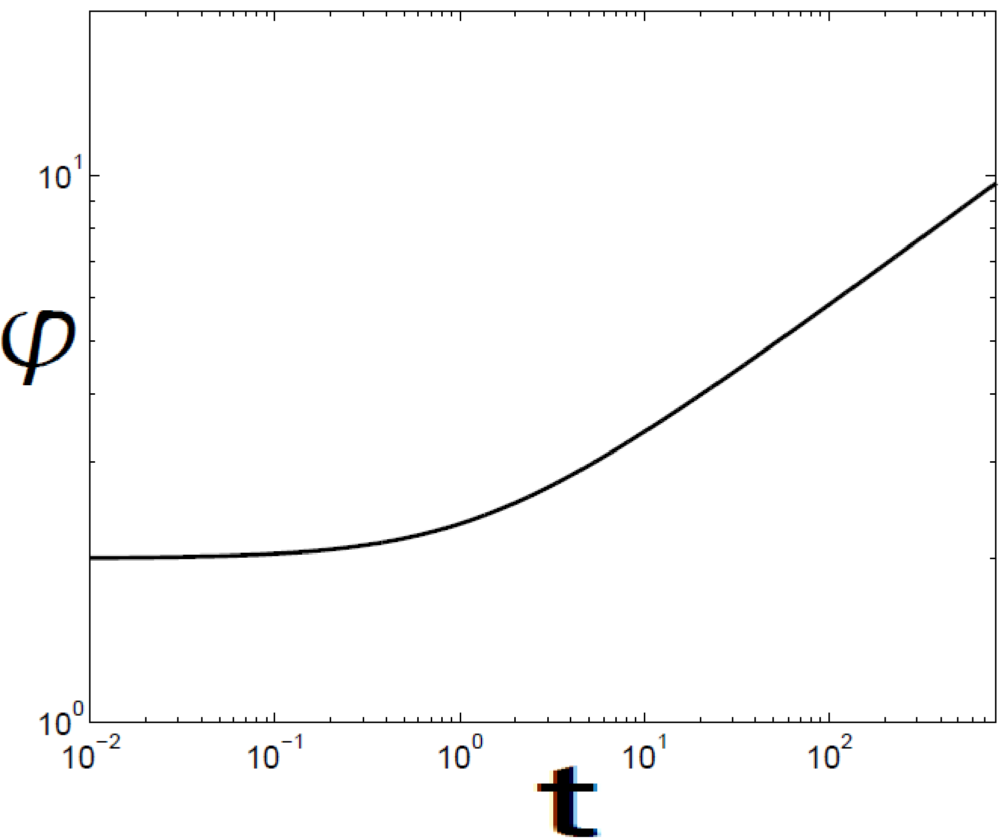
$N=-n=6$



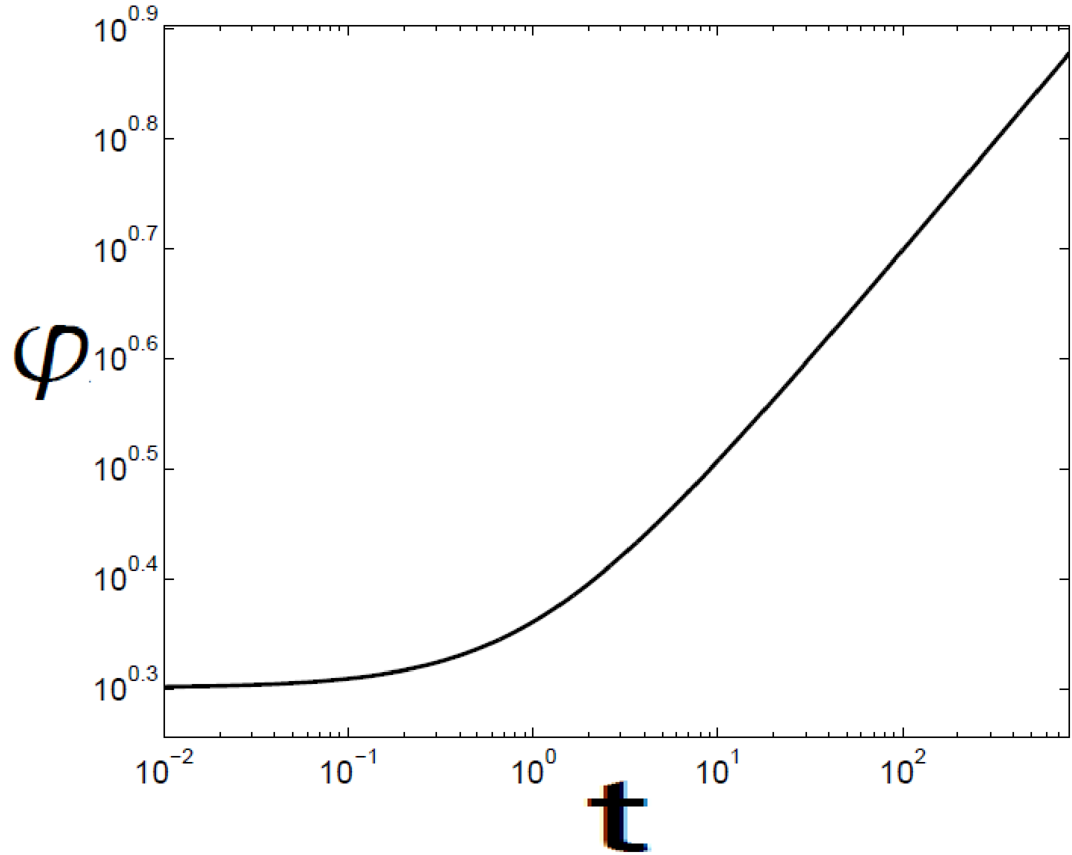
$N=8, n=-10$

Fig. 3. The evolution of the scalar field $\varphi(t)$ for $N > 2$, $V(\varphi)=V_0\varphi^n$, $n \leqslant -N$, $\rho=0$

$\xi=1$, $V_0=1$, $K=1$



$N=-n=6$



$N=8, n=-10$

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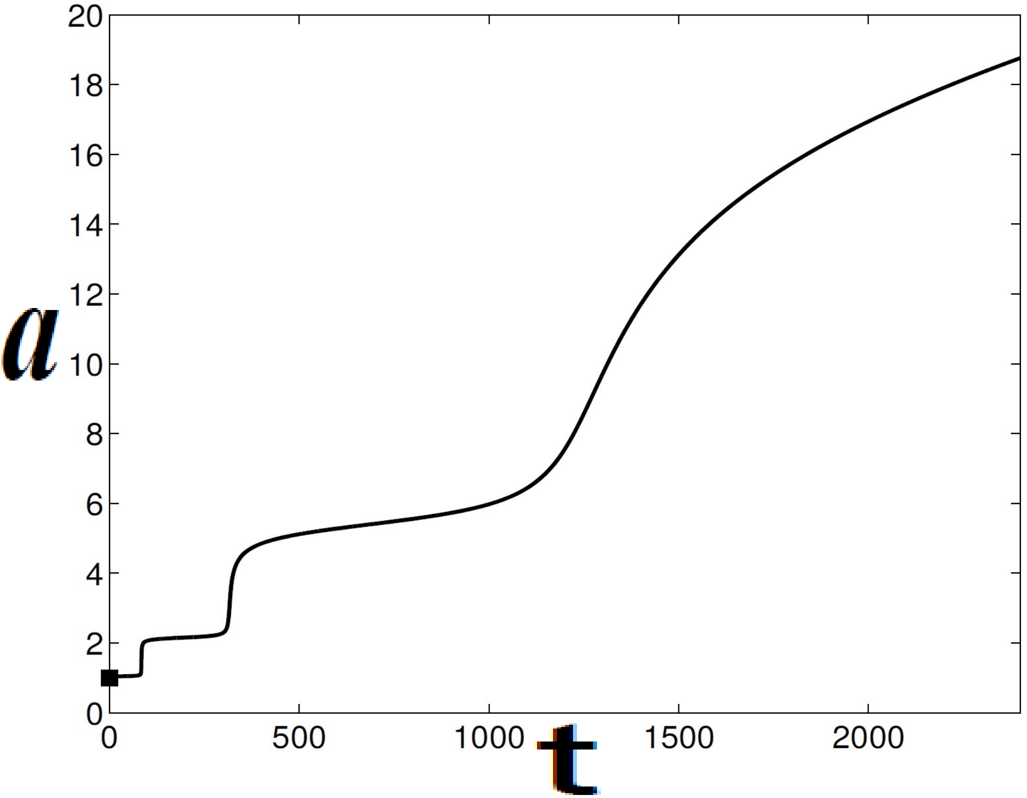
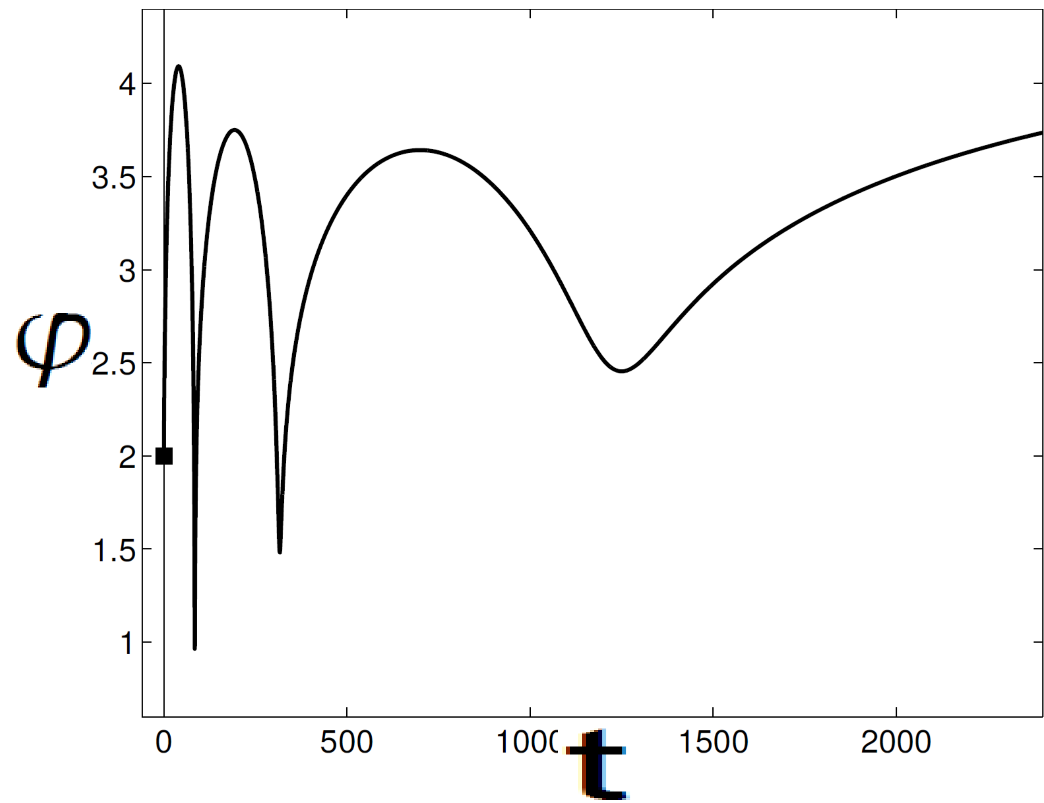
The matter case ($\rho \neq 0$).

Fig. 4. Evolution of *a(t)*, $\varphi(t)$

for *N* > 2, $V(\varphi)=V_0\varphi^n$ ***n* = – *N*,** $\rho \neq 0 \ll 1$

$$N=-n=6, \quad \xi=1 \quad , \quad \omega=0 \quad , \quad V_0=1 \quad , \quad K=1$$





If the cosmological evolution starts from small ρ, then it pass through several transient quasistatic stages with $a \approx const$ similarly to «loitering universe» in GR.

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The matter case ($\rho \neq 0$).
The asymptotic solution ($\varphi \to \infty$) for *N* > 2, $V(\varphi) = V_0 \varphi^n$, *n* < – *N*,

$$a(t) = a_0 t^{\frac{2n}{3(1+\omega)(n-N)}}, \quad \varphi(t) = \varphi_0 t^{\frac{2}{N-n}},$$

$$\rho(t) = \rho_0 t^{\frac{2n}{N-n}}, \quad t \to +\infty$$

This asymptotic regime is **scaling** one as $\omega = \omega_{\varphi}$. It is the generalization of the solution for N = 2, which has been found earlier in the paper **M.A.Skugoreva, A.V.Toporensky**, Eur. Phys. J. C **76**, 340 (2016). It can suitable for description of radiation- and matter-dominated stages of the Universe.

Fig. 5. The evolution of the scale factor *a(t)* for *N* > 2, $V(\varphi)=V_0\varphi^n$, *n* < – *N*, $\rho\neq 0$

$N=4, n=-6, \quad \xi=4 \ , \ \omega=0 \ , \ V_0=1 \ , \ K=1$

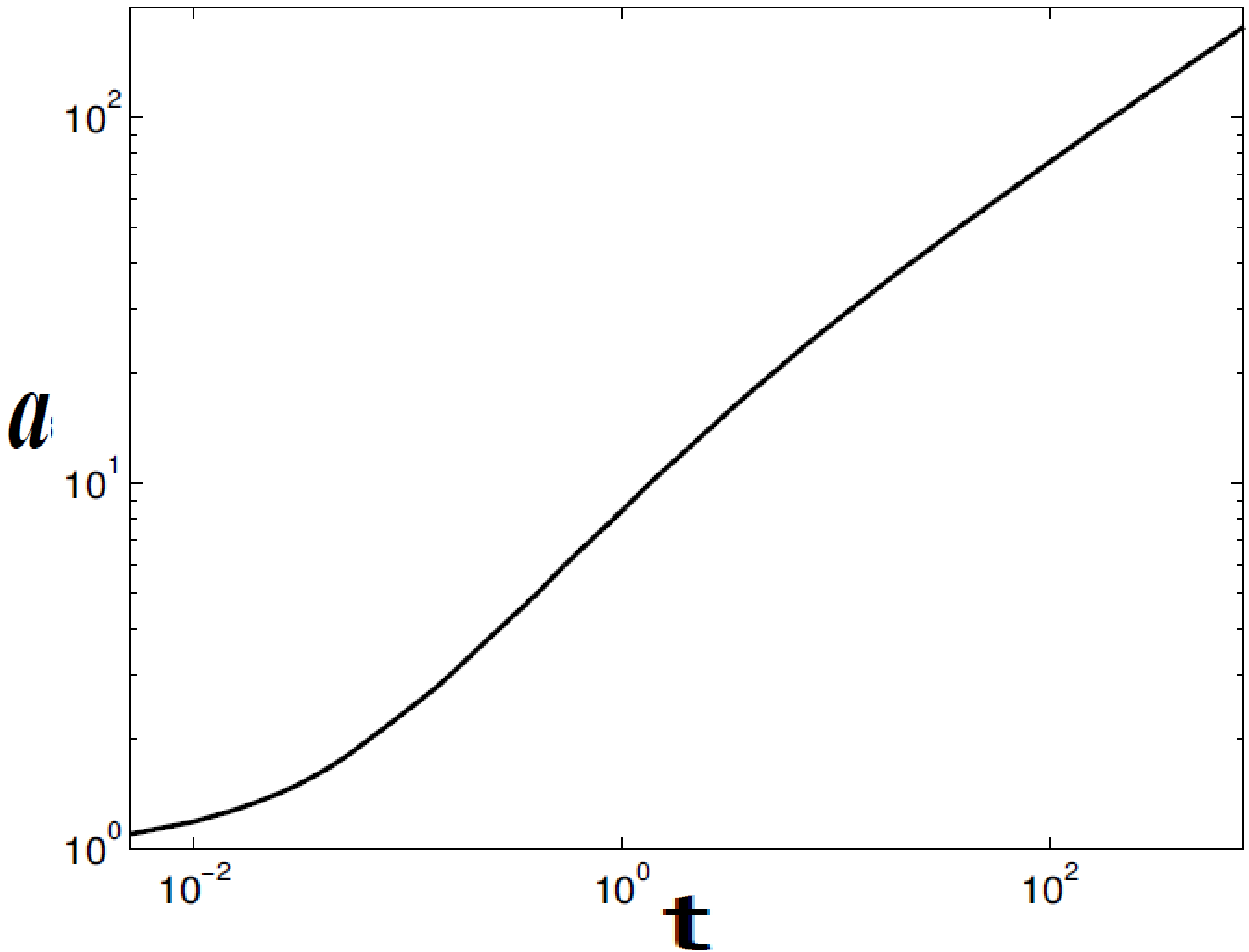
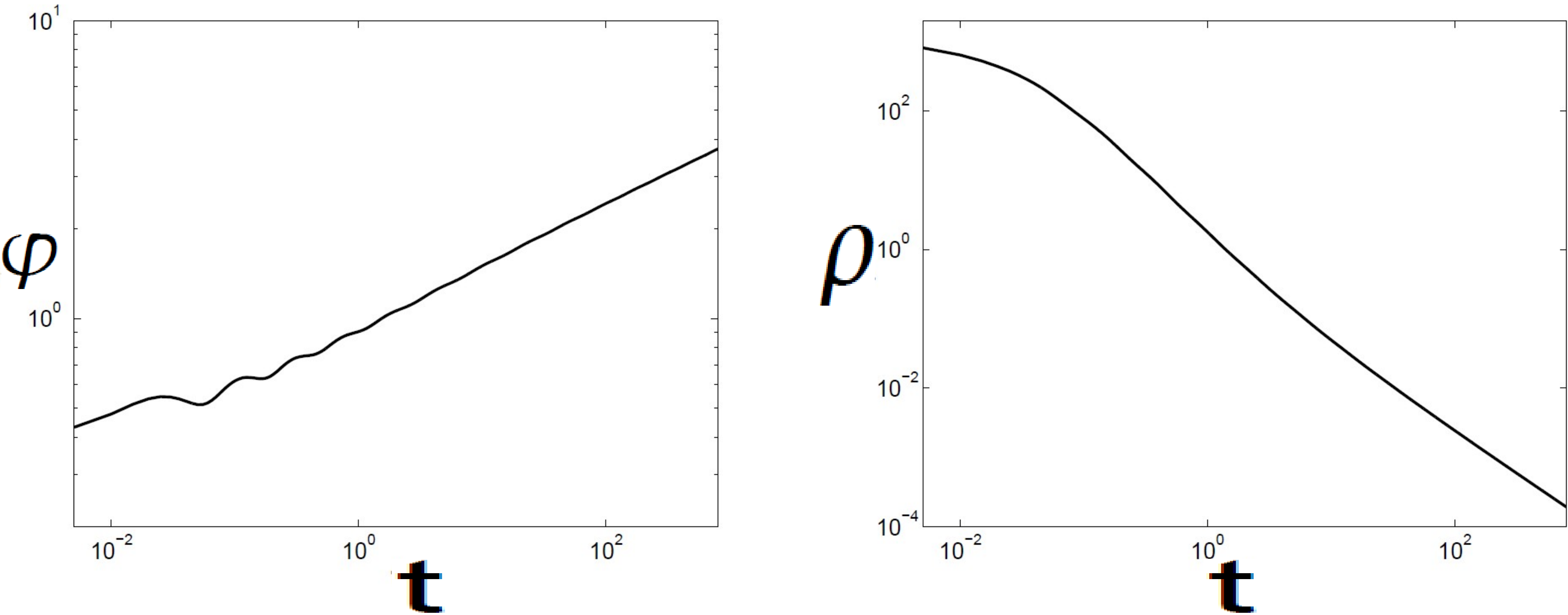


Fig. 6. The evolution of $\varphi(t)$, $\rho(t)$ for $N > 2$, $V(\varphi) = V_0 \varphi^n$, $n < -N$, $\rho \neq 0$

$$N = 4, n = -6, \quad \xi = 4, \quad \omega = 0, \quad V_0 = 1, \quad K = 1$$



Conclusion

1. In the model of teleparallel gravity with nonminimal coupling function $B(\varphi)=\varphi^N$, $N > 2$ and the potential $V(\varphi)=V_0\varphi^n$, $n<0$ **two asymptotic solutions** have been found at late times, which are **stable** with respect to variations of initial data: 1). the vacuum solution with $a(t)\to const$ 2). the **scaling solution** – in the matter case.

2. Strongly decreasing potentials ($n\leqslant -N$) lead to the late-time behaviour of cosmological quantities differing from those at the cosmic acceleration stage of the real Universe.

3. Models with decreasing potentials and a perfect fluid might be intresting due to the **scaling solution**, which can be used for constructing realistic cosmological models.

Thanks for attantion!

Proposals of a work send to
Maria A. Skugoreva

masha-sk@mail.ru