Bound orbits near black holes with scalar hair

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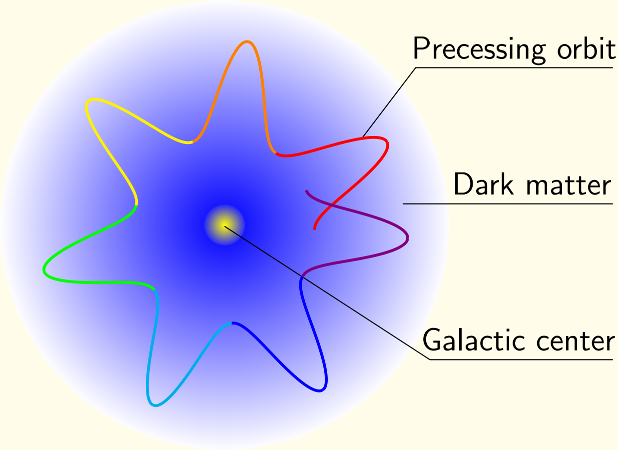
Dark matter around a galactic center

$$\Sigma = \frac{1}{8\pi}\int\left\{-S/2 + \langle d\phi, d\phi\rangle - 2V(\phi)\right\}\sqrt{|g|}\,d^4x \qquad (1)$$



The inverse problem method

The method was developed in the following papers:

[1] O Bechmann O and O Lechtenfeld

Class. Quant. Grav., **12**, pp. 1473 – 1482, 1995

[2] K A Bronnikov and G N Shikin

Gravitation & Cosmology, **8**, pp. 107 – 116, 2002

A variant of the method: the quadratures formulas in

[3] D A Solovyev and A N Tsirulev

Class. Quant. Grav., **29**, 055013, 17pp, 2012

The metric and quadratures

$$ds^2 = A dt^2 - \frac{dr^2}{f} - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{2}$$

$$F = -\int_r^\infty \phi'^2 r dr\,, \qquad \xi = r + \int_r^\infty \big(1 - \mathrm{e}^F\big) dr\,, \tag{3}$$

$$A = 2r^2 \int_r^\infty \frac{\xi - 3m}{r^4}\,\mathrm{e}^F dr\,, \qquad f = \mathrm{e}^{-2F} A\,, \tag{4}$$

$$\widetilde{V}(r) = \frac{1}{2r^2}\left(1 - 3f + r^2\phi'^2 f + 2\,\mathrm{e}^{-F}\,\frac{\xi - 3m}{r}\right), \tag{5}$$

$$V(\phi) = \widetilde{V}(r(\phi)) \tag{6}$$

General properties of the solution

$$F = -\int_r^\infty \phi'^2 r\,dr, \quad \xi = r + \int_r^\infty (1 - \mathrm{e}^F)\,dr, \quad A = 2r^2 \int_r^\infty \frac{\xi - 3m}{r^4}\,\mathrm{e}^F dr$$

F, e^F, and ξ are monotonically increasing functions, and ξ is convex downwards:

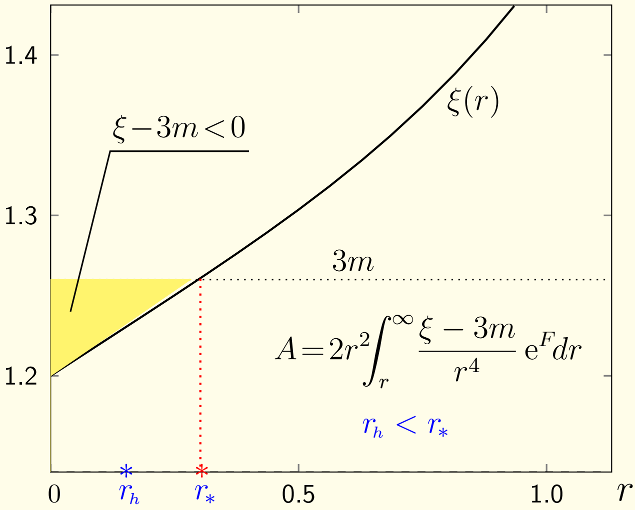
$$F' = \phi'^2 r \geqslant 0, \qquad \xi' = \mathrm{e}^F > 0, \qquad \xi'' = \mathrm{e}^F \phi'^2 r > 0, \tag{7}$$

$$0 < \mathrm{e}^{F(0)} \leqslant 1,\ \xi(0) > 0; \qquad \mathrm{e}^F \to 1,\ \xi \sim r,\ r \to \infty \tag{8}$$

This solution is invariant under the scale transformations

$$r \to r/a\,, \quad m \to m/a\,, \quad V \to a^2 V \tag{9}$$

Compact black holes



$r_h \to 0$ as $3m \to \xi(0) + 0$

A two-parameter family of solutions

$$e^{F(r)} = \begin{cases} 1-\alpha+2\alpha r^3/7, & 0 \leqslant r \leqslant 1 \\ 1-2\alpha/r^3+9\alpha/(7r^4), & 1 \leqslant r < \infty \end{cases} \tag{10}$$

$$A = \begin{cases} -\dfrac{(1-\alpha)(2m-\alpha)}{r} + (1-\alpha)^2 + O(r^2), & 0 \leqslant r \leqslant 1 \\ 1 - \dfrac{2m}{r} - \dfrac{2\alpha}{5r^3} + O(r^{-4}), & 1 \leqslant r < \infty \end{cases}$$

The parameter m is the Schwarzschild mass, and $\alpha \in [0,1)$ is the parameter of 'intensity' of the scalar field.

The radius of the horizon $r_h \to 0$ as $2m - \alpha \to 0$

Precessing orbits

We define *an oscillation angle* φ_{osc}
as the angle between any two nearest periapsis points.
It equals the angle taken for one oscillation:

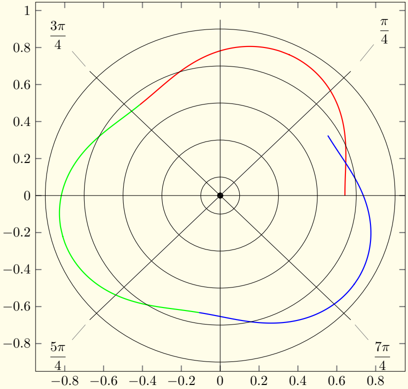
$$\varphi_{osc} = \frac{2}{J} \int_{r_{min}}^{r_{max}} \frac{r^2 \mathrm{e}^{2F}}{E - V_{eff}} \, dr \,, \quad V_{eff} = A\left(1 + \frac{J^2}{r^2}\right) \tag{11}$$

A precession angle per orbit is defined as

$$\Delta\varphi \, = \, \varphi_{osc} - 2\pi \tag{12}$$

This orbit is close to the circular one

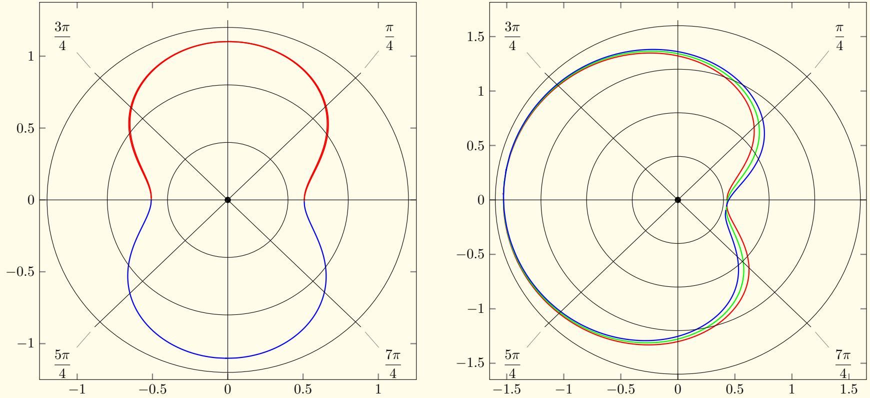
$$\alpha = 0.8, \quad 2m = 0.42 = r_{Sch}, \quad r_h = 0.16 \approx 0.38\, r_{Sch}$$



$$J = 1.35, \quad E = 0.79, \quad \Delta\varphi = -4.0 \quad \text{(2.74 osc)}$$

Increasing E implies increasing $\Delta\varphi$

$$\alpha = 0.8, \quad 2m = 0.42 = r_{Sch}, \quad r_h = 0.16 \approx 0.38\, r_{Sch}$$

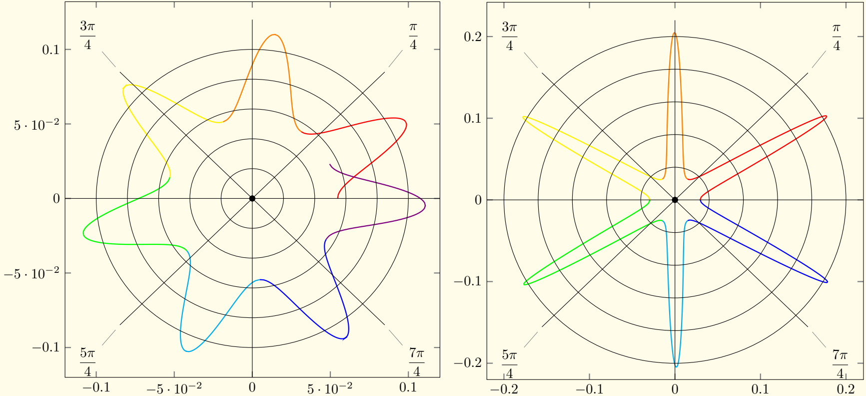


Left panel: $J = 1.35,\ E = 0.812,\ \Delta\varphi = -3.1416$ (2.0 osc)

Right panel: $J = 1.35,\ E = 0.836,\ \Delta\varphi = -0.07$ (1.02 osc)

An extremely compact black hole

$$\alpha = 0.8, \quad 2m = 0.401 = r_{Sch}, \quad r_h = 0.01 \approx 0.025 r_{Sch}$$



Left panel: $J = 0.03$, $E = 0.044$, $\Delta\varphi = -5.32$ (6.63 osc)

Right panel: $J = 0.03$, $E = 0.054$, $\Delta\varphi = -5.23$ (7.0 osc)

Distinguishing characteristics – I

The radius of the horizon and the radius of the ISCO are less (maybe, much less) than the ones for the vacuum black hole of the same mass. Boson stars have no ISCO. Scalar field wormholes and naked singularities (of the same positive(!) mass) have marginal (or degenerated) stable circular orbits with the zero angular momentum.

Distinguishing characteristics – II

The precession angle per orbit is negative for orbits sufficiently close to ISCO and becomes zero with increasing the specific energy of a test particle. This neutral non-circular orbit has parameters $r_{min} \gtrsim r_{Sch}$ and $r_{max} \gtrsim 3r_{Sch}$.

An orbit with a deficit of the precession angle per orbit ($\Delta\varphi < 0$) does not envelop the center during one oscillation, that is, during the movement between any two nearest periapsis points.

Prospects of the observations: Sgr A*

S2: the pericenter distance $= 1400\, r_{Sch}$
S14: the pericenter distance $= 560\, r_{Sch}$

To detect star orbits in the region near the Schwarzschild radius, the joint spatial resolution of infrared- and radio-telescopes must be a hundred times greater than the current value.

It will be achieved, highly likely, in the next fifteen years:

[4] R Abuter, A Amorim, N Anugu et al
Detection of the gravitational redshift in the orbit of the star S2 near the Galactic centre massive black hole.
A&A **615**, L15, 10pp, 2018