Study of viscous-convection instabilities of thin laminar accretion flows

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Plan

- Accretion disks in the Universe
- Instabilities in accretion flows
- Vertical structure of accretion flow with microscopic viscosity in two cases of heat conductivity: electron/molecular and radiative

Disk accretion

Accretion on supermassive black hole

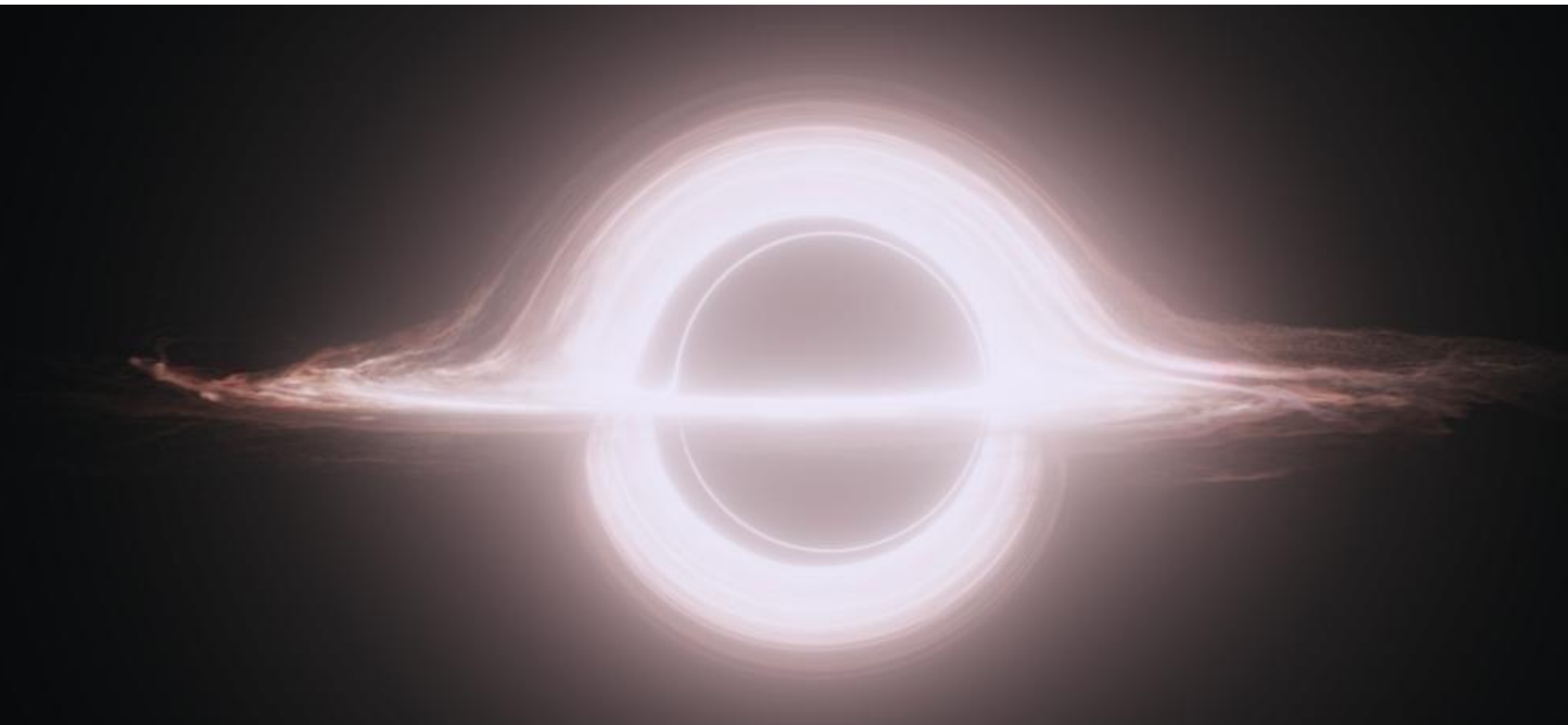
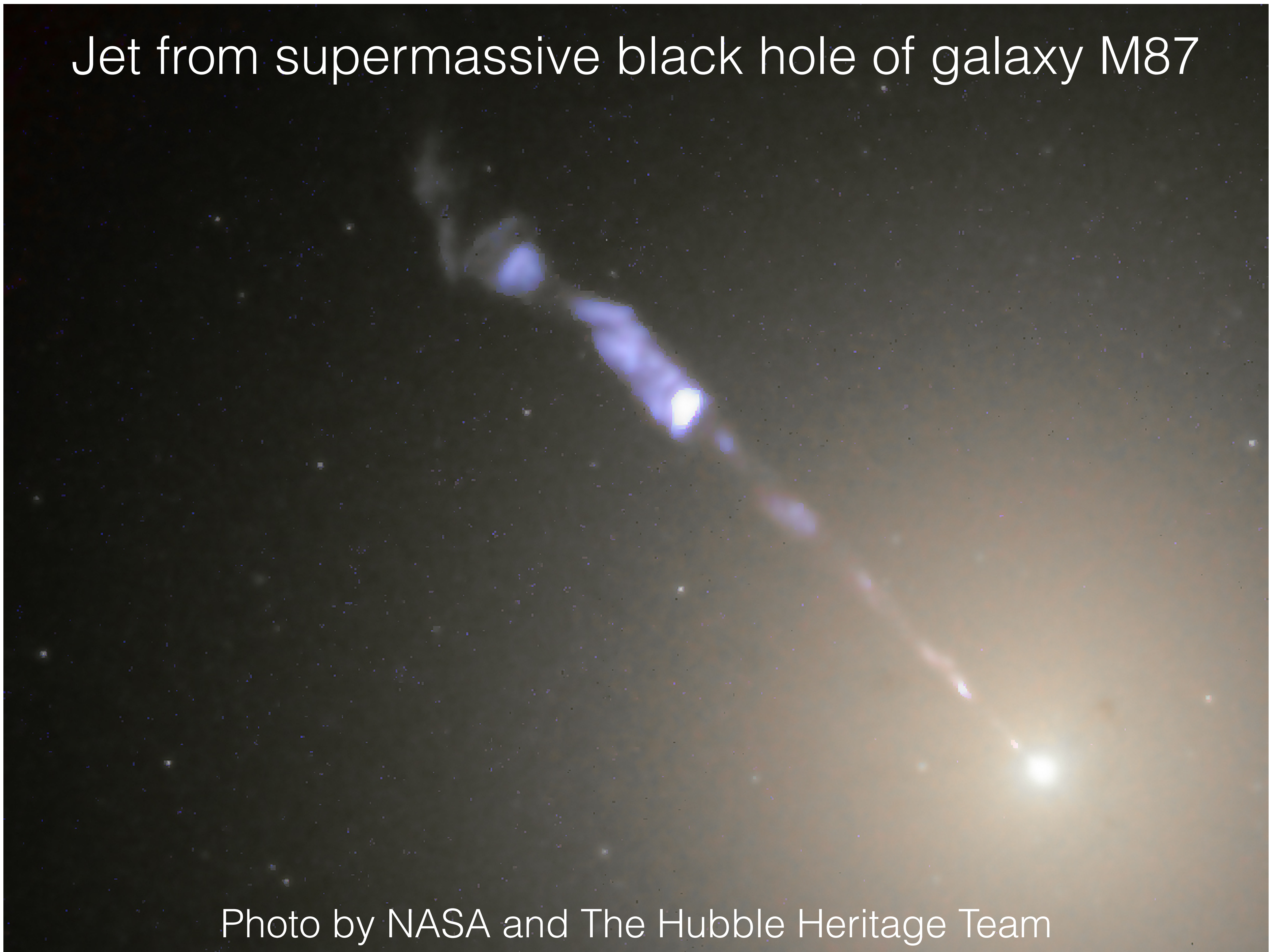
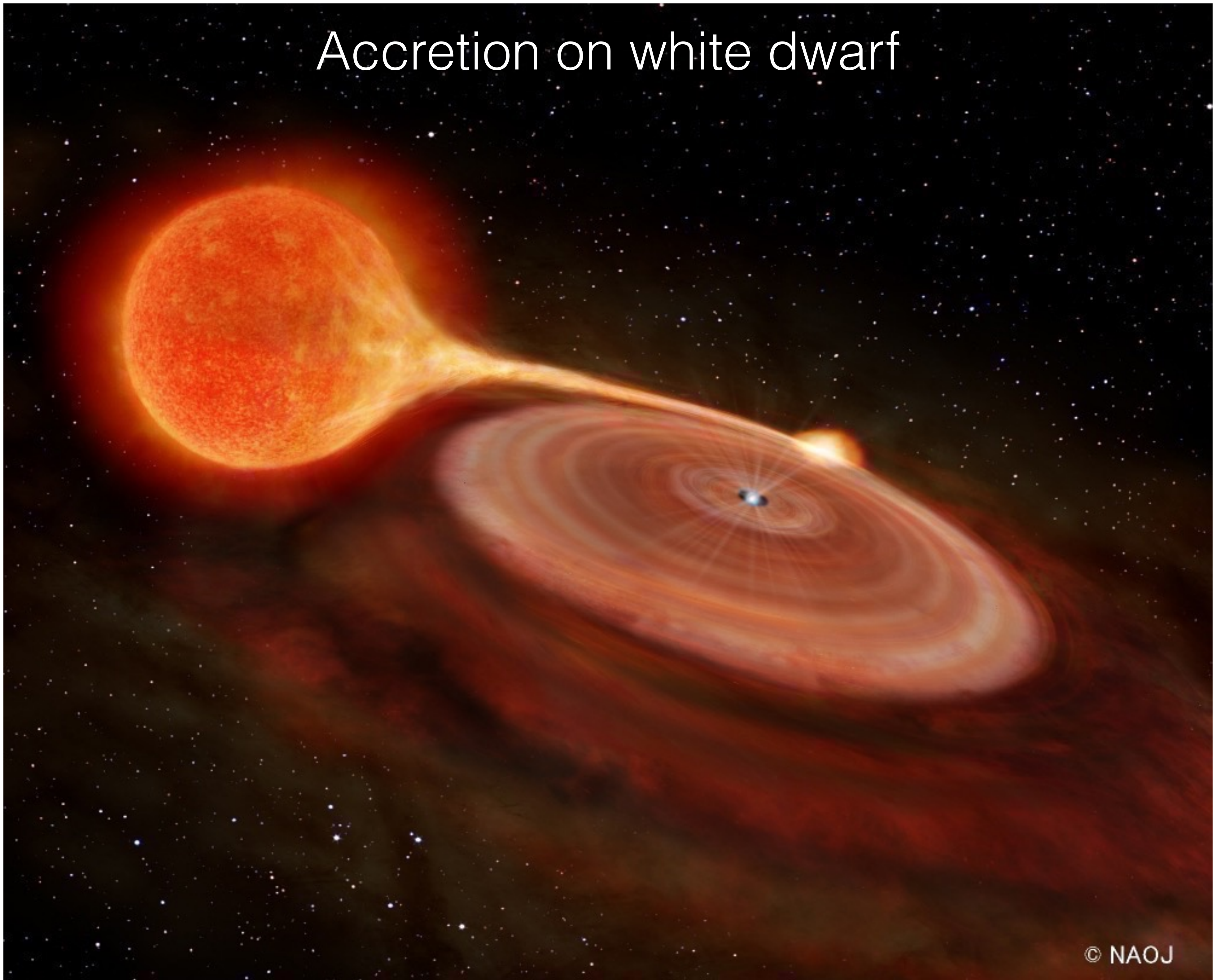


Image by Double Negative, Warner Bros.

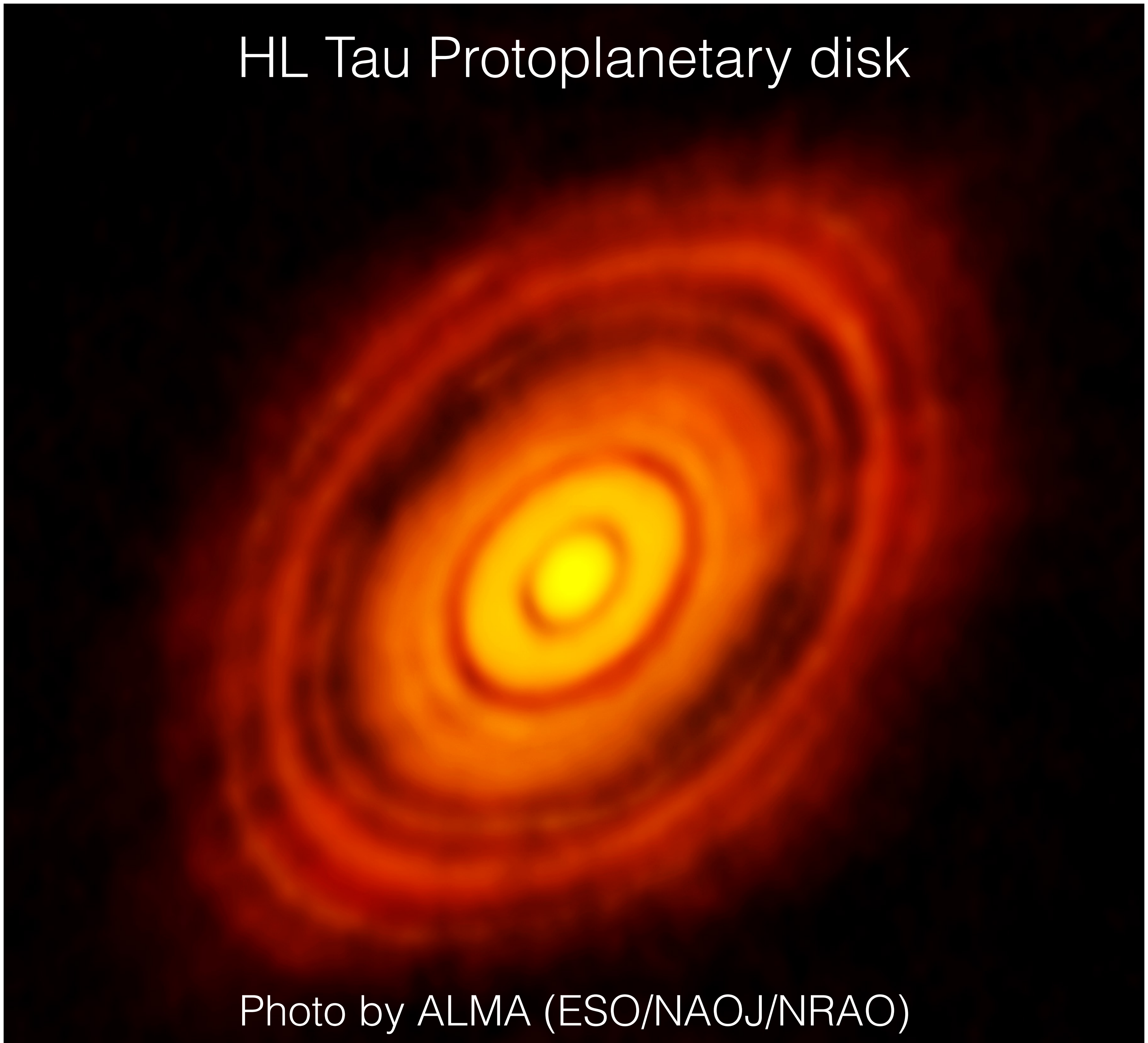
Disk accretion



Disk accretion



Disk accretion



Standard theory of disk accretion

- 1969, Lynden-Bell D. *Galactic Nuclei as Collapsed Old Quasars.*

- 1973, Shakura N. I. & Sunyaev R. A. *Black holes in binary systems. Observational appearance.* (The most cited paper on theoretical astrophysics)

- 1973, Novikov I. D. & Thorne K. S. *Astrophysics of black holes*

Motivation

The Keplerian flow is highly stably due the classic Rayleigh criterion. So the physics of turbulence appearance in thin laminar Keplerian accretion flows is still under discussion.

- Magnetorotational instability (MRI) — Balbus, S. A. & Hawley, J. F. (1991, 1998); originally was discovered by Velikhov (1959) and Chandrasekar (1960)
- Subcritical turbulence — Lominadze et al. (1988), Razdoburdin & Zhuravlev (2012, 2014, 2015, 2017, 2018)
- Unstable modes in viscous sheared flows — Shakura & Postnov (2015), Malanchev et al. (2016)
- …

Structure of the disk with ionic viscosity

System of viscous hydrodynamics equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \text{ — continuity equation,}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\nabla) \cdot u = -\frac{1}{\rho}\nabla p - \nabla\Phi + \boldsymbol{N} \text{ Navier–Stokes equation,}$$

$$\frac{\rho \mathcal{R} T}{\mu}\left[\frac{\partial s}{\partial t} + (\boldsymbol{u}\nabla) \cdot s\right] = Q_{\text{visc}} - \nabla \boldsymbol{F} \text{ — energy equation.}$$

Assumptions:

- Thin axisymmetric accretion disk
- Viscosity is described by microscopic processes

Vertical temperature distribution for electron heat conductivity

$$\frac{\rho \mathcal{R} T}{\mu}\left[\frac{\partial s}{\partial t} + (\mathbf{v}\nabla)\cdot s\right] = \frac{\mathrm{d}E_{\mathrm{visc}}}{\mathrm{d}t\,\mathrm{d}V} - \nabla\cdot Q\,.$$

Axisymmetric case:

$$P\frac{\partial s}{\partial t} = \frac{\partial}{\partial z}\left(\kappa(T)\,\frac{\partial T}{\partial z}\right) + \eta(T)\,r^2\,\left(\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right)^2\,.$$

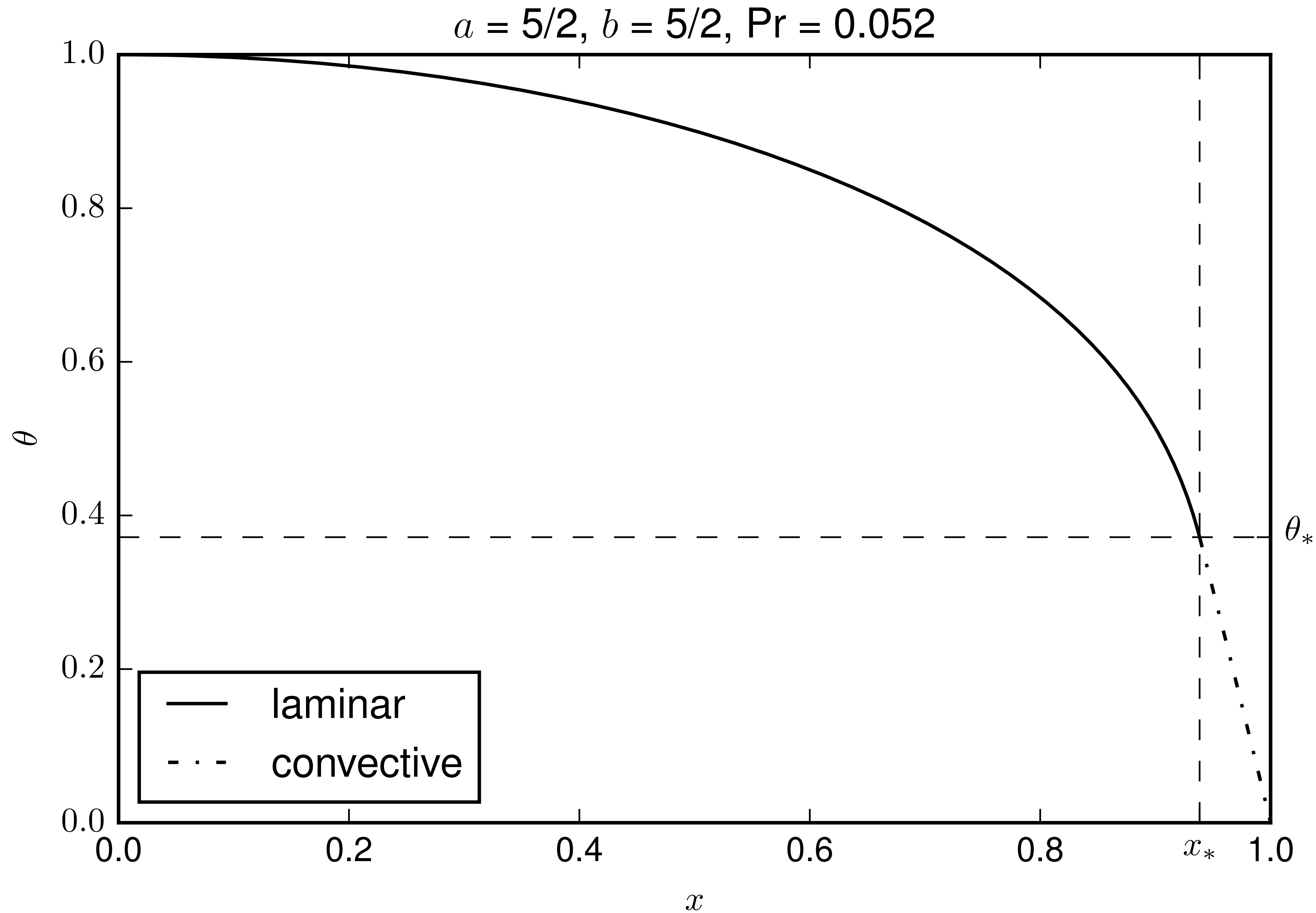
In stationary case and assuming $\kappa(T)\sim T^a$ & $\eta(T)\sim T^b$:

$$\frac{\partial^2\theta}{\partial x^2} + \frac{a}{\theta}\left(\frac{\partial\theta}{\partial x}\right)^2 + B\theta^{b-a} = 0\,.$$

The solution is

$$x(\theta) = 1 - \theta^{a+1}\,\frac{{}_2F_1\left(\frac{1}{2},\frac{a+1}{a+b+1};\frac{2a+b+2}{a+b+1};\theta^{a+b+1}\right)}{{}_2F_1\left(\frac{1}{2},\frac{a+1}{a+b+1};\frac{2a+b+2}{a+b+1};1\right)}\,.$$

Vertical temperature distribution for electron heat conductivity



Vertical structure for radiation heat conductivity

$$\varkappa = \varkappa_c \frac{p^{\varsigma}}{\theta^{\varsigma+\psi}}\,, \eta = \eta_c\, \theta^b\, p^d$$

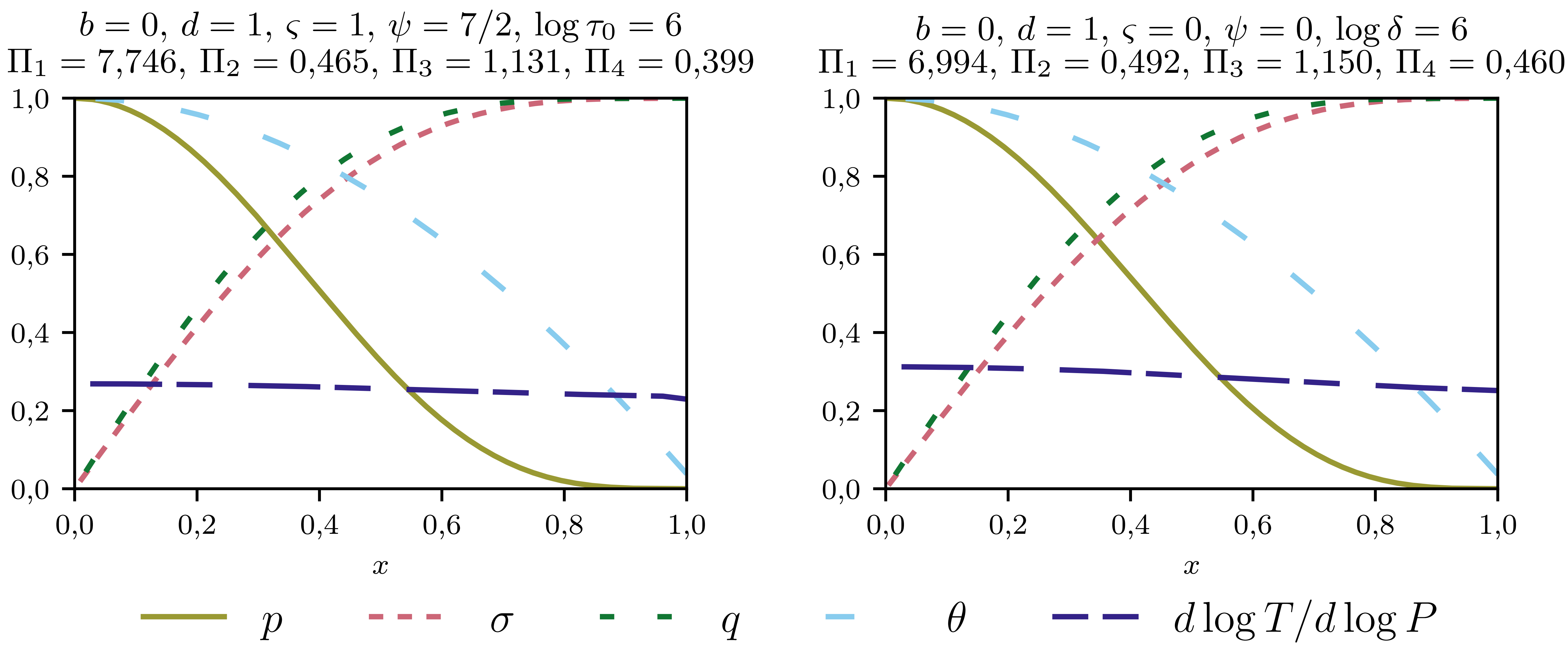
$$\frac{\partial p}{\partial \sigma} = -\Pi_1\, \Pi_2\, x\,, \quad \Pi_1 \equiv \frac{\Omega^2\, z_0^2\, \mu}{\mathcal{R}\, T_c} = \frac{\xi^2}{\gamma}\,;$$

$$\frac{\partial x}{\partial \sigma} = \Pi_2\, \frac{\theta}{p}\,, \quad \Pi_2 \equiv \frac{\Sigma}{z_0\, \rho_c}\,;$$

$$\frac{\partial q}{\partial \sigma} = \Pi_3\, \theta^{b+1} p^{d-1}\,, \quad \Pi_3 \equiv \eta_c \frac{\Sigma}{Q_0\, \rho_c} \left(r \frac{\mathrm{d}\Omega}{\mathrm{d}r} \right)^2\,;$$

$$\frac{\partial \theta}{\partial \sigma} = -\Pi_4\, q \frac{p^{\varsigma}}{\theta^{\psi+\varsigma+3}}\,, \quad \Pi_4 \equiv \frac{3}{16}\, \varkappa_c\, \Sigma \left(\frac{T_{eff}}{T_c} \right)^4\,.$$

Vertical structure for radiation heat conductivity



Conclusions

- Radiation heat conductivity disks are highly convectively stable
- Electron (and molecular) heat conductivity disks are convectively unstable
  - ✦ Ionized accretion disks have convective upper layer
  - ✦ Molecular accretion disks should be fully convective unstable

Thank you
for your attention