Cylindrical wormholes

without exotic matter

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K..B., J.P.S. Lemos, PRD 79, 104089 (2009); arXiv: 0902.2360;
K.B., V. G. Krechet, J.P.S. Lemos, Phys. Rev. D 87, 084060 (2013); arXiv: 1303.2993
K.B., V. G. Krechet, arXiv: 1807.03641

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Plan of the talk

1. Notion of a wormhole: spherical vs. cylindrical
   Throat : different versions of its definition
   Flat or string asymptotics

2. Static cylindrical wormholes
   A no-go theorem: $\rho < 0$ for twice as. regular wormholes
   Examples.

3. Rotation. Structure of the equations.
   Examples: vacuum, scalar-vacuum solutions, anisotr. fluid
   Bad asymptotic behavior

4. Thin shells: attempt to construct a realistic wormhole by joining the throat region with flat space regions

Wormholes (spherical vs. cylindrical)

Static, spherically symmetric space-times:

$$ds^2 = A(u)dt^2 - B(u)du^2 - r^2(u)(d\theta^2 + \sin^2\theta d\phi^2)$$

A wormhole throat: a regular minimum of ***r(u):*** $r = r_{th}$

A wormhole: a regular configuration where ***A(u)*>0** and ***B(u)*>0** everywhere (*no horizons*), and, far from the throat, on its both sides, $r \gg r_{th}$.

The Universe may also contain structures **infinitely extended** along a certain direction, like cosmic strings. While starlike structures are, in the simplest case, described by spherical symmetry, the simplest stringlike configurations are **cylindrically symmetric.**

Static, spherically symmetric wormholes: basic facts

$$ds^2 = A(u)dt^2 - B(u)du^2 - r^2(u)(d\theta^2 + \sin^2\theta d\phi^2)$$

At the throat: $r' = 0$, $r'' > 0$ =>

for matter of general form compatible with the symmetry, $T_\mu^\nu = \mathrm{diag}(\rho, -p_r, -p_\perp, -p_\perp)$,

these conditions lead to

$$\rho + p_r < 0, \qquad p_r < 0.$$

("Exotic" matter, violation of the Null Energy Condition. But no restriction on sign ρ)

Flat asymptotic: at large r approx.Schwarzschild, with mass m.

Mass function: $$m(r) = 4\pi G \int_{r_0}^{r} \rho r^2 dr,$$ r_0 = integration constant.

At the throat, $2m(r) = r$. Integrating from $r_0 = r_{th}$, we obtain $$r_{\rm th} = 2m - \varkappa \int_{r_{\rm th}}^{\infty} \rho r^2 dr,$$

<u>This means:</u> if $\rho > 0$, then $r_{th} < 2m = r$ (Schwarzschild).

a wormhole with a throat of a few meters will have **huge gravity** of, say, Jupiter !!

To avoid that, negative densities are necessary.

Wormholes (spherical vs. cylindrical)

Static, cylindrically symmetric space-times: their general metric can be taken in the form

$$ds^2 = e^{2\gamma(u)}dt^2 - e^{2\alpha(u)}du^2 - e^{2\xi(u)}dz^2 - e^{2\beta(u)}d\phi^2,$$

where u = arbitrary admissible cylindrical radial coordinate,
z = longitudinal, ϕ = angular coordinate.

Definition:

A wormhole throat: a regular minimum of ***$r(u) \equiv e^{\beta}$: $r = r_{th}$***

A wormhole: a regular configuration where, far from the throat, on its both sides, ***$r >> r_{th}$***.\

Alternative definition:

the same, but using, instead of ***$r(u)$*** [the circular radius],
$a(u) \equiv \exp(\beta+\xi)$ [the area function].

$$ds^2 = e^{2\gamma(u)}dt^2 - e^{2\alpha(u)}du^2 - e^{2\xi(u)}dz^2 - e^{2\beta(u)}d\phi^2,$$

Boundary conditions for cylindrical wormholes

Consider the most natural situation that the wormhole **is observed as a stringlike source of gravity** from an otherwise very weakly curved or even flat environment.

We require: there is a spatial infinity, i.e., at some $u = u_\infty$, $r \equiv e^\beta \to \infty$, the metric is either flat or corresponds to the gravitational field of a cosmic string.

This means: (1) $\gamma \to \text{const}, \qquad \xi \to \text{const} \quad \text{as } u \to u_\infty$

(2) at large *r*, $|\beta'|e^{\beta-\alpha} \to 1 - \mu, \qquad \mu = \text{const} < 1 \quad \text{as } u \to u_\infty$

(the parameter **μ** is an **angular defect**). A ***flat asymptotic:*** **μ =0**.

We say ***"a regular asymptotic"*** in the sense ***"a flat or string asymptotic."***

$$ds^2 = e^{2\gamma(u)}dt^2 - e^{2\alpha(u)}du^2 - e^{2\xi(u)}dz^2 - e^{2\beta(u)}d\phi^2,$$

Einstein equations: $$G_\mu^\nu = -\varkappa T_\mu^\nu, \qquad \varkappa = 8\pi G,$$

or $$R_\mu^\nu = -\varkappa \tilde{T}_\mu^\nu, \qquad \tilde{T}_\mu^\nu = T_\mu^\nu - \frac{1}{2}\delta_\mu^\nu T_\alpha^\alpha$$

We have

$$R_0^0 = -e^{-2\alpha}[\gamma'' + \gamma'(\gamma' - \alpha' + \beta' + \xi')],$$

$$R_1^1 = -e^{-2\alpha}[\gamma'' + \xi'' + \beta'' + \gamma'^2 + \xi'^2 + \beta'^2 - \alpha'(\gamma' + \xi' + \beta')],$$

$$R_2^2 = -e^{-2\alpha}[\xi'' + \xi'(\gamma' - \alpha' + \beta' + \xi')],$$

$$R_3^3 = -e^{-2\alpha}[\beta'' + \beta'(\gamma' - \alpha' + \beta' + \xi')],$$

$$G_1^1 = e^{-2\alpha}(\gamma'\xi' + \beta'\gamma' + \beta'\xi').$$

The most general form of the stress-energy tensor: $$T_\mu^\nu = \mathrm{diag}(\rho, -p_r, -p_z, -p_\phi),$$

where ρ = energy density,

p_i = pressures of any physical origin in the respective directions.

$$ds^2 = e^{2\gamma(u)}dt^2 - e^{2\alpha(u)}du^2 - e^{2\xi(u)}dz^2 - e^{2\beta(u)}d\phi^2,$$

Conditions on the throat

Harmonic radial coordinate is used: $\alpha = \beta + \gamma + \xi$

1. **At a minimum of circular radius *r (u)***, due to **$\beta' = 0$ and $\beta'' > 0$**, we have ***$R_3^3 < 0$***, and from the corresponding component of the Einstein eqs it follows that

$$T_0^0 + T_1^1 + T_2^2 - T_3^3 = \rho - p_r - p_z + p_\phi < 0.$$

If $T_2^2 = T_3^3$, that is, $p_z = p_\phi$, in particular, for **Pascal isotropic fluids** we obtain $p_r > \rho$, violation of ***Dominant Energy Condition*** (if, as usual, $\rho > 0$).

In the general case of anisotropic pressures, **none of the standard energy conditions are necessarily violated.**

2. However, **if the throat is defined through** ***the area function*** ***$a(u) \equiv \exp(\beta+\xi)$***, we have there $\beta' + \xi' = 0$, $\beta'' + \xi'' > 0$, whence $R_2^2 + R_3^3 < 0 \Rightarrow T_0^0 + T_1^1 = \rho - p_r < 0.$

In addition, substituting $\beta' + \xi' = 0$ into the Einstein equation $G_1^1 = -\varkappa T_1^1$, we find

$-T_1^1 = p_r \leq 0.$ Combining these two conditions, we see that

$$\rho < p_r \leq 0$$

on the throat, i.e., **there is necessarily a region with negative energy density !!!**

$$ds^2 = e^{2\gamma(u)}dt^2 - e^{2\alpha(u)}du^2 - e^{2\xi(u)}dz^2 - e^{2\beta(u)}d\phi^2,$$

Asymptotic regularity and a no-go theorem

At a regular asymptotic, both *r (u)* and *a(u)* tend to infinity. If there are two such asymptotics, **both functions have minima** at some finite *u*, i.e., there occur both **a throat as a minimum of** *r (u)* and **a throat as a minimum of** *a(u)* (they do not necessarily coincide if there is no mirror symmetry).

This leads to the following result (**no-go theorem**) :

In general relativity, any static, cylindrically symmetric wormhole with two regular asymptotics contains a region where the energy density is negative.

Another formulation:

In general relativity, a static, cylindrically symmetric, twice asymptotically regular wormhole cannot exist if the energy density $\rho = T_0^0$ is everywhere nonnegative.

$$ds^2 = e^{2\gamma(u)}dt^2 - e^{2\alpha(u)}du^2 - e^{2\xi(u)}dz^2 - e^{2\beta(u)}d\phi^2,$$

Example: Einstein-Maxwell fields and nonlinear electrodynamics (NED)

Electromagnetic fields with cyl. symmetry:

Radial (R): $F_{01}(u)\ (E^2 = F_{01}F^{10})$, $F_{23}(u)\ (B^2 = F_{23}F^{23})$.

Azimuthal (A): $F_{03}(u)\ (E^2 = F_{03}F^{30})$, $F_{12}(u)\ (B^2 = F_{12}F^{12})$.

Longitudinal (L): $F_{02}(u)\ (E^2 = F_{02}F^{20})$, $F_{13}(u)\ (B^2 = F_{13}F^{13})$.

(*E* and *B* = abs. values of electric field strength and magnetic induction, resp.)

$L_e = -\Phi(F)/(16\pi)$, $F := F^{\mu\nu}F_{\mu\nu}$ Maxwell: $\Phi(F) \equiv F$.

Solutions are known: for Maxwell fields [K.B., 1979]
and NED [K. B, G. N. Shikin, and E. N. Sibileva, 2003].

Wormhole solutions: easily obtained with **A-fields only** (Maxwell ED)
or **preferably** (NED) . Example (Maxwell ED):

$$ds^2 = \frac{\cosh^2(hu)}{Kh^2}[e^{2au}dt^2 - e^{2(a+b)u}du^2 - e^{2bu}d\varphi^2] - \frac{Kh^2}{\cosh^2(hu)}dz^2,$$

$$F_{03} = i_m = \text{const}; \qquad F^{12} = i_e e^{-2\alpha}, \qquad i_e = \text{const},$$

$$K = [G(i_e^2 + i_m^2)]^{-1},\ h^2 = ab,\ a, b = \text{const} \qquad a > 0, \qquad b > 0$$

CONCLUSION on static cyl. wormholes

- Cyl. wormhole geometries can exist without WEC or NEC violation
- A number of explicit examples of cyl. wormholes with non-phantom matter are known
- Main difficulty (*as always with cyl. symmetry*): obtaining flat or string (regular) asymptotics.
- It is necessary to have **negative density** to obtain twice asympt. regular wormholes.

The latter problem is still more important if we try to apply cylindrically symmetric solutions as an approximate description of toroidal systems. This approximation must work well if a torus containing matter and significant curvature is thin, like a circular string.

Can rotation help?

Rotation

A vortex gravitational field in terms of tetrads:

$$\omega^{\mu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} e_{a\nu} e^{a}_{\rho;\sigma},$$

Cylindrical metric with rotation:

$$ds^2 = \mathrm{e}^{2\gamma(x)}[dt - E(x)\,\mathrm{e}^{-2\gamma(x)}\,d\varphi]^2 - \mathrm{e}^{2\alpha(x)}dx^2 - \mathrm{e}^{2\mu(x)}dz^2 - \mathrm{e}^{2\beta(x)}d\varphi^2,$$

Angular velocity of a congruence of timelike curves:

$$\omega = \sqrt{\omega_\alpha \omega^\alpha} \qquad \omega = \tfrac{1}{2}(E\,\mathrm{e}^{-2\gamma})'\,\mathrm{e}^{\gamma-\beta-\alpha}$$

under an arbitrary choice of the radial coordinate *x* («gauge»).

Ricci tensor component (~ matter flux) $R^3_0 = 0$

=> in the comoving reference frame

$$\omega = \omega_0\,\mathrm{e}^{-\mu-2\gamma}, \qquad \omega_0 = \mathrm{const}.$$

$$ds^2 = e^{2\gamma(x)}[dt - E(x)\, e^{-2\gamma(x)}\, d\varphi]^2 - e^{2\alpha(x)}dx^2 - e^{2\mu(x)}dz^2 - e^{2\beta(x)}d\varphi^2,$$

In the comoving reference frame (arbitrary gauge):

$$R^1_1 = -e^{-2\alpha}[\beta'' + \gamma'' + \mu'' + \beta'^2 + \gamma'^2 + \mu'^2 - \alpha'(\beta' + \gamma' + \mu')] + 2\omega^2;$$
$$R^2_2 = \Box_1 \mu;$$
$$R^3_3 = \Box_1 \beta + 2\omega^2$$
$$R^4_4 = \Box_1 \gamma - 2\omega^2$$

where $\Box_1 f = -g^{-1/2}[\sqrt{g} g^{11} f']' = -e^{-2\alpha}[f'' + f'(\beta' + \gamma' + \mu' - \alpha')].$

so that $R^\nu_\mu = {}_s R^\nu_\mu + {}_\omega R^\nu_\mu, \quad {}_\omega R^\nu_\mu = \omega^2\, \mathrm{diag}(-2, 2, 0, 2),$

$$G^\nu_\mu = {}_s G^\nu_\mu + {}_\omega G^\nu_\mu, \quad {}_\omega G^\nu_\mu = \omega^2\, \mathrm{diag}(-3, 1, -1, 1)$$

$\underset{s}{G^\nu_\mu}$ and $\underset{\omega}{G^\nu_\mu}$ (each separately) satisfy the "conservation law"

$\nabla_\alpha G^\alpha_\mu = 0$ with respect to the static metric with $E \equiv 0$

The rotational part of the Einstein tensor behaves in the Einstein equations as **an additional SET with very exotic properties:**
the energy density is $-3\omega^2/\varkappa < 0$

$$ds^2 = e^{2\gamma(x)}[dt - E(x)\, e^{-2\gamma(x)}\, d\varphi]^2 - e^{2\alpha(x)} dx^2 - e^{2\mu(x)} dz^2 - e^{2\beta(x)} d\varphi^2,$$

Definitions of throats: the same as in the static case.

r-throat: a regular minimum of the **circular radius** $r(x) = \exp(\beta)$
r-wormhole: a regular configuration with $r \gg r_{min}$ on both sides

a-throat: a regular minimum of the **area function** $a(\underline{u}) = \exp(\beta + \mu)$
a-wormhole: a regular configuration with $a \gg a_{min}$ on both sides

Due to rotation, it is much easier to obtain wormholes than with $\omega = 0$.

Main problem: bad asymptotic behavior,
e.g., we do not have $\omega \to 0$ where γ, $\mu \to$ **const** since

$$\omega = \omega_0\, e^{-\mu - 2\gamma}, \qquad \omega_0 = \text{const.}$$

at least in the present (comoving) reference frame.

$$ds^2 = e^{2\gamma(x)}[dt - E(x)\, e^{-2\gamma(x)}\, d\varphi]^2 - e^{2\alpha(x)}dx^2 - e^{2\mu(x)}dz^2 - e^{2\beta(x)}d\varphi^2,$$

Example 1:

vacuum or a massless scalar field,

$$L_s = \frac{1}{2}\varepsilon\partial_\alpha\phi\partial^\alpha\phi$$

Harmonic radial coordinate: $\alpha = \beta + \gamma + \mu$

$$R_2^2 = 0 \quad\Rightarrow\quad \mu'' = 0,$$
$$R_3^3 = 0 \quad\Rightarrow\quad \beta'' - 2\omega^2\, e^{2\alpha} = 0,$$
$$R_4^4 = 0 \quad\Rightarrow\quad \gamma'' + 2\omega^2\, e^{2\alpha} = 0,$$

$$\mu = -mu \qquad \text{[with a certain choice of } z \text{ scale]},$$
$$\beta + \gamma = 2hu \qquad \text{[with a certain choice of } t \text{ scale]},$$
$$\beta'' - \gamma'' = 4\omega_0^2\, e^{2\beta-2\gamma}.$$

Solution:

$$\phi = Cu$$

$$\omega = \frac{e^{mu-2hu}}{2s(k,u)},$$

$$e^{2\beta} = \frac{e^{2hu}}{2\omega_0 s(k,u)},$$
$$e^{2\gamma} = 2\omega_0 s(k,u)\, e^{2hu},$$
$$e^{2\mu} = e^{-2mu},$$

$$E = e^{2hu} s(k,u) \int du\, \frac{e^{2hu}}{s(k,u)}.$$

$$s(k,u) = \begin{cases} k^{-1}\sinh ku, & k>0, \quad u \in \mathbb{R}_+; \\ u, & k=0, \quad u \in \mathbb{R}_+; \\ k^{-1}\sin ku, & k<0, \quad 0<u<\pi/|k|. \end{cases}$$

$$k^2\,\mathrm{sign}\,k = 4(h^2 - 2hm) - 2\varkappa\varepsilon C^2.$$

Parameters: ω_0, C, h, m, k. (All inessential constants have been absorbed.)

Wormhole solutions: **all solutions with $k < 0$, and many with $k \geq 0$.**

Asymptotic behavior: **in all cases, either $\exp(\gamma) \to 0$, or $\exp(\gamma) \to \infty$.**

$$ds^2 = e^{2\gamma(x)}[dt - E(x)\, e^{-2\gamma(x)}\, d\varphi]^2 - e^{2\alpha(x)} dx^2 - e^{2\mu(x)} dz^2 - e^{2\beta(x)} d\varphi^2,$$

Example 2:

Anisotropic fluid with the SET

$$T^0_0 = -T^1_1 = T^2_2 = -T^3_3 = \rho(x), \quad T^0_3 = -2\rho E\, e^{-2\gamma}$$

$$\rho = \rho_0\, e^{-2\gamma-2\mu}, \qquad \rho_0 = \text{const} > 0,$$

Solution for the metric:

$$r^2 \equiv e^{2\beta} = \frac{r_0^2}{Q^2(x_0^2 - x^2)}, \qquad e^{2\gamma} = Q^2(x_0^2 - x^2),$$

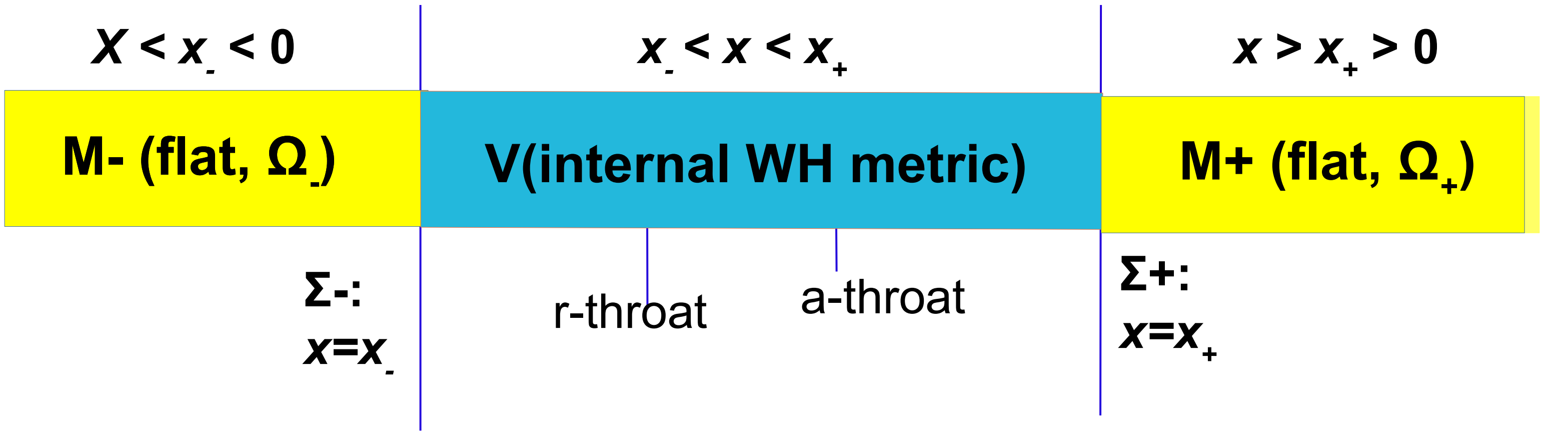
$$x_0 := \frac{\omega_0}{\varkappa\rho_0 r_0}, \qquad Q^2 := \varkappa\rho_0 r_0^2,$$

$$e^{2\mu} = e^{2mx}(x_0 - x)^{1-x/x_0}(x_0 + x)^{1+x/x_0},$$

Integration constants: $\omega_0,\ \rho_0,\ r_0$ and m.

Wormhole nature of space-time, singularities at $x = x_0$ and $x = -x_0$, where $r \to \infty$ and $\exp(\gamma) \to 0$.

Trying to build a wormhole model with two flat asymptotics



Metrics in M+ and M-: $$ds^2_{\rm M} = dt^2 - dX^2 - dz^2 - X^2(d\varphi + \Omega dt)^2.$$

Relevant quantities:

$$e^{2\gamma} = 1 - \Omega^2 X^2, \qquad e^{2\beta} = \frac{X^2}{1 - \Omega^2 X^2},$$

$$E = \Omega X^2, \qquad \omega = \frac{\Omega}{1 - \Omega^2 X^2}.$$

(arbitrary scales along the *z* and *t* axes can be chosen)

Matching:

$$[\beta] = 0, \qquad [\mu] = 0, \qquad [\gamma] = 0, \qquad [E] = 0,$$

Next step:

find the surface stress-energy tensors on Σ+ and Σ-

This is done in Darmois-Israel formalism in terms of the extrinsic curvature:

$$S_a^b = -\frac{1}{8\pi}[\tilde{K}_a^b], \qquad \tilde{K}_a^b := K_a^b - \delta_a^b K, \qquad K = K_a^a, \qquad \textbf{[f] := f(+) - f(-).}$$

With natural parametrization of **Σ+** and **Σ-** $K_{ab} = -\,\mathrm{e}^{\alpha(u)}\Gamma^1_{ab} = \frac{1}{2}\mathrm{e}^{-\alpha(u)}\frac{\partial g_{ab}}{\partial x^1}.$

⇨

$$\tilde{K}_2^2 = -\,\mathrm{e}^{-\alpha}(\beta' + \gamma'),$$
$$\tilde{K}_3^3 = -\,\mathrm{e}^{-\alpha}(\mu' + \gamma') - E\omega\,\mathrm{e}^{-\beta-\gamma},$$
$$\tilde{K}_4^4 = -\,\mathrm{e}^{-\alpha}(\beta' + \mu') + E\omega\,\mathrm{e}^{-\beta-\gamma},$$
$$\tilde{K}_4^3 = \omega\,\mathrm{e}^{\gamma-\beta}.$$

$-S_4^4$ is the surface density while $S_2^2 = p_z$ and $S_3^3 = p_\varphi$ are pressures in the respective directions.

(No need to adjust coordinates in different regions since all relevant quantities are reparametrization-independent.)

Can both surface stress-energy tensors be physically plausible and non-exotic at some values of the system parameters?

Criterion: the WEC (including the NEC)

$$\frac{S_{00}}{g_{00}} = \sigma \geq 0, \quad S_{ab}\xi^a\xi^b \geq 0,$$

ξ^a is any null vector ($\xi^a\xi_a = 0$) on $\Sigma = \Sigma_\pm$

We choose the null vectors in z and ϕ directions as

$$\xi^a_{(1)} = (\mathrm{e}^{-\gamma},\ \mathrm{e}^{-\mu},\ 0), \qquad \xi^a_{(2)} = (\mathrm{e}^{-\gamma} + E\mathrm{e}^{-\beta-2\gamma},\ 0,\ \mathrm{e}^{-\beta}),$$

Our requirements read:

$$[\mathrm{e}^{-\alpha}(\beta' + \mu')] \leq 0,$$

$$[\mathrm{e}^{-\alpha}(\mu' - \gamma')] \leq 0,$$

$$[\mathrm{e}^{-\alpha}(\beta' - \gamma') + 2\omega] \leq 0,$$

No-go theorem

Many internal wormhole solutions are unable to meet these requirements, In particular, those in which

$$T_t^t = T_\varphi^\varphi$$

This includes, for example, field systems with

$$L = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi) - P(\phi)F^{\mu\nu}F_{\mu\nu},$$

i.e., **scalar fields** with an arbitrary potential ***V*(*ϕ*)** and an arbitrary function ***P*(*ϕ*)** characterizing the scalar-electromagnetic interaction, assuming that **$\phi = \phi(x)$** and that the **Maxwell tensor** describes a stationary azimuthal magnetic field ($F_{21} = -F_{12}$) or its electric analog.

It has been proven that for all such systems the NEC is inevitably violated either on Σ+ or on Σ- or on both

[K.B., arXiv: 1509.06924].

The above no-go theorem does not apply to our example with an anisotropic fluid with

$$T^0_0 = -T^1_1 = T^2_2 = -T^3_3 = \rho(x), \quad T^0_3 = -2\rho E\, \mathrm{e}^{-2\gamma}$$

$$\rho = \rho_0\, \mathrm{e}^{-2\gamma-2\mu}, \qquad \rho_0 = \mathrm{const} > 0,$$

The internal solution reads

$$r^2 \equiv \mathrm{e}^{2\beta} = \frac{r_0^2}{Q^2(x_0^2 - x^2)}, \qquad \mathrm{e}^{2\gamma} = Q^2(x_0^2 - x^2),$$

$$x_0 := \frac{\omega_0}{\varkappa\rho_0 r_0}, \qquad Q^2 := \varkappa\rho_0 r_0^2,$$

$$\mathrm{e}^{2\mu} = \mathrm{e}^{2mx}(x_0 - x)^{1-x/x_0}(x_0 + x)^{1+x/x_0},$$

Integration constants: $\omega_0,\ \rho_0,\ r_0$ and m.

Singularities at $x = x_0$ and $x = -x_0$, where $r \to \infty$ and $\exp(\gamma) \to 0$.

Assume m=0 and $\boldsymbol{x_+ = -x_-}$ The matching conditions at $x_+ < x_0$ and $x_- > -x_0$ are

$$r_0^2[Q^2(x_0^2 - x^2)]^{-1} = X^2[1 - \Omega^2 X^2]^{-1},$$

$$Q^2(x_0^2 - x^2) = 1 - \Omega^2 X^2,$$

Here $X = r_0$ or $-r_0$ are coordinates of the junctions in the two Minkowski regions.

WEC on both junctions can be written as

$$\mathrm{W1} = \mathrm{e}^{-\mu_+}[x_+ + (x_0^2 - x_+^2)Y] - 1/Q^2 \geq 0,$$

$$\mathrm{W2} = \mathrm{e}^{-\mu_+}[x_+ + (x_0^2 - x_+^2)Y] + \Omega^2 r_0^2/Q^2 \geq 0,$$

$$\mathrm{W3}\pm = 2\,\mathrm{e}^{-\mu_+}[\omega_0 r_0 \pm Q^2 x_+] - (1 + \Omega_\pm r_0)^2 \geq 0,$$

$$\mathrm{e}^{\mu_+} = \mathrm{e}^{\mu(x_+)} = \mathrm{e}^{\mu(x_-)}$$

$$Y = \frac{1}{2x_0} \ln \frac{x_o + x_+}{x_0 - x_+}$$

We also have $$r_0 \Omega_\pm = \pm\sqrt{1 - Q^2(x_0^2 - x_+^2)}.$$

The **remaining junction condition [*E*] =0** implies for the internal region

$$E(x_+) - E(x_-) = 2\,\mathrm{e}^{2\gamma}\omega_0 \int_{x_-}^{x_+} \frac{dx}{(x_0^2 - x^2)^2} = \frac{r_0}{x_0^2}\left[2x_0 x_+ + (x_0^2 - x_+^2) \ln \frac{x_0 + x_+}{x_0 - x_+}\right]$$

whence $$2x_0 x_+ + (x_0^2 - x_+^2) \ln \frac{x_0 + x_+}{x_0 - x_+} = 2x_0^2 \sqrt{1 - Q^2(x_0^2 - x_+^2)}.$$

This gives Q^2 as a function of x_0 and x_+

Now we can try to choose such values of the dimensionless parameters Q, x_0, $x_+ < x_0$ that all **W1, W2, W3 > 0.**

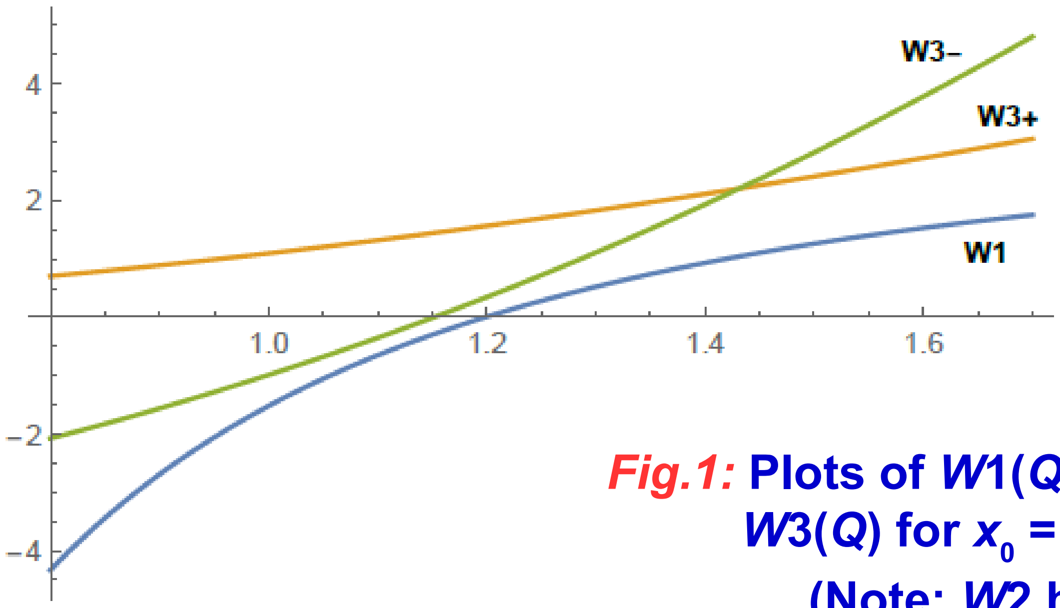


Fig.1: **Plots of $W1(Q)$ (multiplied by 5) and $W3(Q)$ for $x_0 = 0.5$ and $x_+ = 0.2$. (Note: $W2$ holds if $W1$ does.)**

Example: let $x_0 = 0.5$ and $x_+ = 0.2$. Then we obtain $Q \approx 1.429$, such a value that all WEC requirements are satisfied according to Fig. 1. Other such examples are also easily found.

It is also easy to verify that by construction $g_{33} < 0$ in the whole space, which excludes the possible existence of closed timelike curves.

This completes the construction of regular cylindrical wormhole models in GR without exotic matter.

Conclusion on rotating cylindrical wormholes in GR

Stationary cylindrical configurations:

The vortex grav. field is singled out and behaves like matter with exotic properties. Exact solutions have been found for scalar fields and some kinds of anisotropic perfect fluid.

Is it possible to have WH geometry without exotic matter?

YES **(vortex gravitational field instead of WEC violating matter)**

Can they have two flat (or string) asymptotic regions?

NO **if we consider pure solutions with rotation**

YES **with Minkowski regions and rotating thin shells, under a proper choice of matter in the internal region**

K.B., J.P.S. Lemos, *Cylindrical wormholes*, PRD 79, 104089 (2009); arXiv: 0902.2360

K.B., V. G. Krechet, J.P.S. Lemos, *Rotating cylindrical wormholes,* Phys. Rev. D 87, 084060 (2013); arXiv: 1303.2993

K.B., V. G. Krechet, *Asymptotically flat cylindrical wormholes without exotic matter in GR,* arXiv: 1807.03641

THANK YOU!