Cosmological perturbations during the kinetic inflation in the Horndeski theory

Sergey Sushkov

KAZAN FEDERAL UNIVERSITY

ICPPG 2018

4th International Conference on Particle Physics and Astrophysics

22 - 26 October 2018, Moscow, Russia

Plan of the talk

- Scalar fields in gravitational physics
- Horndeski model
- Cosmological models with nonminimal derivative coupling
  - No potential
  - Cosmological constant
  - Power-law potential
- The screening Horndeski cosmologies
- Perturbations
- Summary

Scalar fields in gravitational physics:

- gravitational potential in Newtonian gravity
- variation of "fundamental" constants
- Brans-Dicke theory initially elaborated to solve the Mach problem
- various compactification schemes
- the low-energy limit of the superstring theory
- scalar field as inflaton
- scalar field as dark energy and/or dark matter
- fundamental Higgs bosons, neutrinos, axions, . . .
- etc. . .

Horndeski theory

In 1974, Horndeski derived the action of the most general scalar-tensor theories with second-order equations of motion
[G.Horndeski, *Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space*, IJTP **10**, 363 (1974)]

Horndeski Lagrangian:

$$L_{\rm H} = \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(X,\phi)\,,\\
\mathcal{L}_3 &= G_3(X,\phi)\,\Box\phi\,,\\
\mathcal{L}_4 &= G_4(X,\phi)\,R + \partial_X G_4(X,\phi)\,\delta^{\mu\nu}_{\alpha\beta}\,\nabla^\alpha_\mu\phi\nabla^\beta_\nu\phi\,,\\
\mathcal{L}_5 &= G_5(X,\phi)\,G_{\mu\nu}\nabla^{\mu\nu}\phi - \tfrac{1}{6}\,\partial_X G_5(X,\phi)\,\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\,\nabla^\alpha_\mu\phi\nabla^\beta_\nu\phi\nabla^\gamma_\rho\phi\,,
\end{aligned}$$

where $X = -\frac{1}{2}(\nabla\phi)^2$, and $G_k(X,\phi)$ are arbitrary functions,
and $\delta^{\lambda\rho}_{\nu\alpha} = 2!\,\delta^\lambda_{[\nu}\delta^\rho_{\alpha]}$, $\delta^{\lambda\rho\sigma}_{\nu\alpha\beta} = 3!\,\delta^\lambda_{[\nu}\delta^\rho_\alpha\delta^\sigma_{\beta]}$

Fab Four subclass of the Horndeski theory

There is a special subclass of the theory, sometimes called Fab Four (F4), for which the coefficients are chosen such that the Lagrangian becomes

$$L_{\mathrm{F4}} = \sqrt{-g}\,(\mathcal{L}_J + \mathcal{L}_P + \mathcal{L}_G + \mathcal{L}_R - 2\Lambda)$$

with

$$\begin{aligned}
\mathcal{L}_J &= V_J(\phi)\, G_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi\,, \\
\mathcal{L}_P &= V_P(\phi)\, P_{\mu\nu\rho\sigma} \nabla^\mu \phi \nabla^\rho \phi \nabla^{\nu\sigma} \phi\,, \\
\mathcal{L}_G &= V_G(\phi)\, R\,, \\
\mathcal{L}_R &= V_R(\phi)\, (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2).
\end{aligned}$$

Here the double dual of the Riemann tensor is

$$P^{\mu\nu}{}_{\alpha\beta} = -\frac{1}{4}\,\delta^{\mu\nu\gamma\delta}_{\sigma\lambda\alpha\beta}\, R^{\sigma\lambda}{}_{\gamma\delta} = -R^{\mu\nu}{}_{\alpha\beta} + 2R^{\mu}_{[\alpha}\delta^{\nu}_{\beta]} - 2R^{\nu}_{[\alpha}\delta^{\mu}_{\beta]} - R\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta]}\,,$$

whose contraction is the Einstein tensor, $P^{\mu\alpha}{}_{\nu\alpha} = G^{\mu}{}_{\nu}$.

Fab Four subclass of the Horndeski theory

Fab Four Lagrangian:

$$L_{\mathrm{F4}} = \sqrt{-g}\,(\mathcal{L}_J + \mathcal{L}_P + \mathcal{L}_G + \mathcal{L}_R - 2\Lambda)$$

- The Fab Four model is distinguished by the *screening property* – it is the most general subclass of the Horndeski theory in which flat space is a solution, despite the presence of the cosmological term Λ.
- This property suggests that Λ is actually irrelevant and hence there is no need to explain its value.
- Indeed, however large Λ is, Minkowski space is always a solution and so one may hope that a slowly accelerating universe will be a solution as well.

Theory with nonminimal kinetic coupling

Action:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - (\varepsilon\, g_{\mu\nu} + \eta\, G_{\mu\nu})\nabla^\mu\phi\nabla^\nu\phi - 2V(\phi)\right] + S_{\rm m}$$

Field equations:

$$M_{\rm Pl}^2 G_{\mu\nu} = T_{\mu\nu}^{(\phi)} + \eta\,\Theta_{\mu\nu} + T_{\mu\nu}^{(\rm m)}$$

$$[\epsilon g^{\mu\nu} + \eta G^{\mu\nu}]\nabla_\mu\nabla_\nu\phi = V'_\phi$$

$$\begin{aligned}
T_{\mu\nu}^{(\phi)} &= \epsilon\left[\nabla_\mu\phi\nabla_\nu\phi - \tfrac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right] - g_{\mu\nu}V(\phi),\\
\Theta_{\mu\nu} &= -\tfrac{1}{2}\nabla_\mu\phi\,\nabla_\nu\phi\,R + 2\nabla_\alpha\phi\,\nabla_{(\mu}\phi R^\alpha_{\nu)} - \tfrac{1}{2}(\nabla\phi)^2 G_{\mu\nu} + \nabla^\alpha\phi\,\nabla^\beta\phi\,R_{\mu\alpha\nu\beta}\\
&\quad + \nabla_\mu\nabla^\alpha\phi\,\nabla_\nu\nabla_\alpha\phi - \nabla_\mu\nabla_\nu\phi\,\Box\phi + g_{\mu\nu}\Big[-\tfrac{1}{2}\nabla^\alpha\nabla^\beta\phi\,\nabla_\alpha\nabla_\beta\phi + \tfrac{1}{2}(\Box\phi)^2\\
&\quad - \nabla_\alpha\phi\,\nabla_\beta\phi\,R^{\alpha\beta}\Big]\\
T_{\mu\nu}^{(\rm m)} &= (\rho + p)U_\mu U_\mu + p g_{\mu\nu}\,,
\end{aligned}$$

Notice: *The field equations are of second order!*

Ansatz:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2,$$
$$\phi = \phi(t)$$

$a(t)$ *cosmological factor*, $H = \dot{a}/a$ *Hubble parameter*

Field equations:

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2\left(\epsilon - 9\eta H^2\right) + V(\phi),$$
$$M_{\rm Pl}^2(2\dot{H} + 3H^2) = -\frac{1}{2}\dot{\phi}^2\left[\epsilon + \eta\left(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}\right)\right] + V(\phi),$$
$$\frac{d}{dt}\left[(\epsilon - 3\eta H^2)a^3\dot{\phi}\right] = -a^3\frac{dV(\phi)}{d\phi}$$

Cosmological models: General formulas

Ansatz:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2,$$
$$\phi = \phi(t)$$

$a(t)$ *cosmological factor*, $H = \dot{a}/a$ *Hubble parameter*

Field equations:

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2\left(\epsilon - 9\eta H^2\right) + V(\phi),$$
$$M_{\rm Pl}^2(2\dot{H} + 3H^2) = -\frac{1}{2}\dot{\phi}^2\left[\epsilon + \eta\left(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}\right)\right] + V(\phi),$$
$$\frac{d}{dt}\left[(\epsilon - 3\eta H^2)a^3\dot{\phi}\right] = -a^3\frac{dV(\phi)}{d\phi}$$

$V(\phi) \equiv const \quad \Longrightarrow \quad \dot{\phi} = \dfrac{Q}{a^3(\epsilon - 3\eta H^2)}$ $\quad Q$ is a scalar charge

Trivial model without kinetic coupling, i.e. $\eta = 0$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - (\nabla\phi)^2\right]$$

Trivial model without kinetic coupling, i.e. $\eta = 0$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - (\nabla\phi)^2\right]$$

Solution:

$$a_0(t) = t^{1/3}; \quad \phi_0(t) = \frac{1}{2\sqrt{3\pi}}\ln t$$

$$ds_0^2 = -dt^2 + t^{2/3}d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

Model without free kinetic term, i.e. $\epsilon = 0$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - \eta G^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]$$

Cosmological models: II. No potential $V(\phi) \equiv 0$

Model without free kinetic term, i.e. $\epsilon = 0$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - \eta G^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]$$

Solution:

$$a(t) = t^{2/3}; \quad \phi(t) = \frac{t}{2\sqrt{3\pi|\eta|}}, \quad \eta < 0$$

$$ds_0^2 = -dt^2 + t^{4/3}d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

Model for an ordinary scalar field ($\epsilon = 1$) with nonminimal kinetic coupling $\eta \neq 0$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - (g^{\mu\nu} + \eta G^{\mu\nu})\phi_{,\mu}\phi_{,\nu}\right]$$

Model for an ordinary scalar field ($\epsilon = 1$) with nonminimal kinetic coupling $\eta \neq 0$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[M_{\rm Pl}^2 R - (g^{\mu\nu} + \eta G^{\mu\nu}) \phi_{,\mu} \phi_{,\nu} \right]$$

Asymptotic for $t \to \infty$:

$$a(t) \sim a_0(t) = t^{1/3}; \quad \phi(t) \sim \phi_0(t) = \frac{1}{2\sqrt{3\pi}} \ln t$$

Notice: *At large times the model with $\eta \neq 0$ has the same behavior like that with $\eta = 0$*

Cosmological models: III. No potential $V(\phi) \equiv 0$

Asymptotics for early times

The case $\eta < 0$:

$$a_{t\to 0} \approx t^{2/3}; \quad \phi_{t\to 0} \approx \frac{t}{2\sqrt{3\pi|\eta|}}$$

$$ds^2_{t\to 0} = -dt^2 + t^{4/3}d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

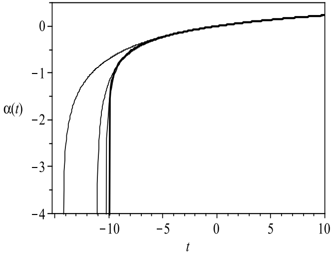
The case $\eta > 0$:

$$a_{t\to -\infty} \approx e^{H_\eta t}; \quad \phi_{t\to -\infty} \approx Ce^{-t/\sqrt{\eta}}$$

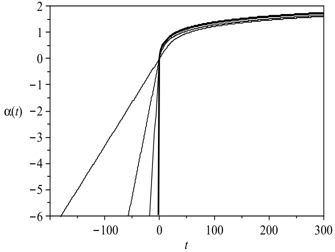
$$ds^2_{t\to -\infty} = -dt^2 + e^{2H_\eta t}d\mathbf{x}^2$$

de Sitter asymptotic with $H_\eta = 1/\sqrt{9\eta}$

Plots of $\alpha = \ln a$ in case $\eta \neq 0$, $\epsilon = 1$, $V = 0$.



(a) $\eta < 0$;
$\eta = 0; -1; -10; -100$



(b) $\eta > 0$;
$\eta = 0; 1; 10; 100$

De Sitter asymptotics: $\alpha(t) = \dfrac{t}{\sqrt{9\eta}} \quad \Rightarrow \quad H = \dfrac{1}{\sqrt{9\eta}}$

Notice: *In the model with nonmnimal kinetic coupling one get de Sitter phase without any potential!*

Cosmological models: IV. Cosmological constant

Models with the constant potential $V(\phi) = M_{\rm Pl}^2\Lambda = const$

$$S = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2(R-2\Lambda) - [\epsilon g^{\mu\nu} + \eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right]$$

Models with the constant potential $V(\phi) = M_{\rm Pl}^2\Lambda = const$

$$S = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2(R - 2\Lambda) - [\epsilon g^{\mu\nu} + \eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right]$$

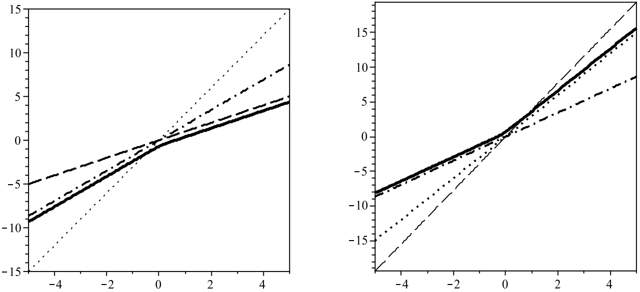
There are two exact de Sitter solutions:

I. $\alpha(t) = H_\Lambda t, \quad \phi(t) = \phi_0 = const,$

II. $\alpha(t) = \dfrac{t}{\sqrt{3|\eta|}}, \quad \phi(t) = M_{\rm Pl}\left|\dfrac{3\eta H_\Lambda^2 - 1}{\eta}\right|^{1/2} t,$

$$H_\Lambda = \sqrt{\Lambda/3}$$

Plots of $\alpha(t)$ in case $\eta > 0$, $\epsilon = 1$, $V = M_{\rm Pl}^2\Lambda$



(a) $H_\Lambda^2 < \dot\alpha^2 < 1/9\eta$ (b) $1/9\eta < \dot\alpha^2 < 1/3\eta < H_\Lambda^2$

De Sitter asymptotics:
$\alpha_1(t) = H_\Lambda t$ *(dashed)*,
$\alpha_2(t) = t/\sqrt{9\eta}$ *(dash-dotted)*,
$\alpha_3(t) = t/\sqrt{3\eta}$ *(dotted)*.

$$S = \int d^4x\sqrt{-g}\left\{M_{\rm Pl}^2 R - [g^{\mu\nu} + \eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu} - 2V(\phi)\right\}$$

$\nearrow$

What a role does a potential play in cosmological models with the nonminimal kinetic coupling?

Power-law potential $V(\phi) = V_0\phi^N$

Skugoreva, Sushkov, Toporensky, PRD 88, 083539 (2013)

Models with the quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$

Primary (early-time) "kinetic" inflation:

$$H_{t\to-\infty} \approx \frac{1}{\sqrt{9\eta}}(1 + \tfrac{1}{2}\eta m^2)$$

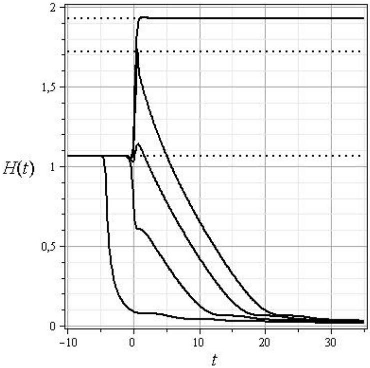
Late-time cosmological scenarios:

Oscillatory asymptotic or "graceful" exit from inflation

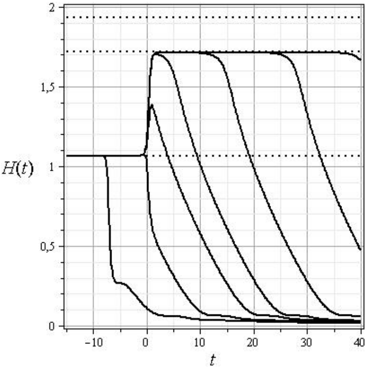
$$H_{t\to\infty} \approx \frac{2}{3t}\left[1 - \frac{\sin 2mt}{2mt}\right]$$

quasi-de Sitter asymptotic or secondary inflation

$$H_{t\to\infty} \approx \frac{1}{\sqrt{3\eta}}\left(1 \pm \sqrt{\tfrac{1}{6}\eta m^2}\right)$$



Initial conditions
$\phi_0 = \dot{\phi}_0$



Initial conditions
$\phi_0 = -\dot{\phi}_0$

De Sitter asymptotics: $H_{t\to-\infty} \approx 1/\sqrt{9\eta}(1+\frac{1}{2}\eta m^2)$,

$$H_{t\to\infty} \approx 1/\sqrt{3\eta}\left(1 \pm \sqrt{\frac{1}{6}\eta m^2}\right).$$

Screening properties of Horndeski model:

Starobinsky, Sushkov, Volkov, JCAP, 2015

The FLRW ansatz for the metric:

$$ds^2 = -dt^2 + \mathrm{a}^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right],$$

$\mathrm{a}(t)$ *cosmological factor*, $H = \dot{\mathrm{a}}/\mathrm{a}$ *Hubble parameter*

Gravitational equations:

$$-3M_{\mathrm{Pl}}^2\left(H^2 + \frac{K}{\mathrm{a}^2}\right) + \frac{1}{2}\varepsilon\,\psi^2 - \frac{3}{2}\eta\,\psi^2\left(3H^2 + \frac{K}{\mathrm{a}^2}\right) + \Lambda + \rho = 0,$$

$$-M_{\mathrm{Pl}}^2\left(2\dot{H} + 3H^2 + \frac{K}{\mathrm{a}^2}\right) - \frac{1}{2}\varepsilon\,\psi^2 - \eta\,\psi^2\left(\dot{H} + \frac{3}{2}H^2 - \frac{K}{\mathrm{a}^2} + 2H\frac{\dot{\psi}}{\psi}\right) + \Lambda - p = 0.$$

The scalar field equation:

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left(\mathrm{a}^3\left(3\eta\left(H^2 + \frac{K}{\mathrm{a}^2}\right) - \varepsilon\right)\psi\right) = 0,$$

where $\psi = \dot{\phi}$, and $\phi = \phi(t)$ is a homogeneous scalar field

Screening properties of Horndeski model

The first integral of the scalar field equation:

$$\mathrm{a}^3\left(3\eta\left(H^2+\frac{K}{\mathrm{a}^2}\right)-\varepsilon\right)\psi=Q,$$

where Q is the Noether charge associated with the shift symmetry $\phi\to\phi+\phi_0$.

Let $Q=0$. One finds in this case two different solutions:

GR branch: $\psi=0 \quad \Longrightarrow \quad H^2+\frac{K}{\mathrm{a}^2}=\frac{\Lambda+\rho}{3M_{\mathrm{Pl}}^2}$

Screening branch: $H^2+\frac{K}{\mathrm{a}^2}=\frac{\varepsilon}{3\eta} \quad \Longrightarrow \quad \psi^2=\frac{\eta\,(\Lambda+\rho)-\varepsilon\,M_{\mathrm{Pl}}^2}{\eta\,(\varepsilon-3\eta\,K/\mathrm{a}^2)}$

NOTICE: The role of the cosmological constant in the screening solution is played by $\varepsilon/3\eta$ while the Λ-term is screened and makes no contribution to the universe acceleration.

Note also that the matter density ρ is screened in the same sense.

Screening properties of Horndeski model

Let $Q \neq 0$, then

$$\psi = \frac{Q}{\mathrm{a}^3 \left[3\eta \left(H^2 + \frac{K}{\mathrm{a}^2}\right) - \varepsilon\right]},$$

and the modified Friedmann equation reads

$$3M_{\mathrm{Pl}}^2 \left(H^2 + \frac{K}{\mathrm{a}^2}\right) = \frac{Q^2 \left[\varepsilon - 3\eta \left(3H^2 + \frac{K}{\mathrm{a}^2}\right)\right]}{2a^6 \left[\varepsilon - 3\eta \left(H^2 + \frac{K}{\mathrm{a}^2}\right)\right]^2} + \Lambda + \rho.$$

Introducing dimensionless values and density parameters

$$H^2 = H_0^2\, y, \quad \mathrm{a} = \mathrm{a}_0\, a\,, \quad \rho_{\mathrm{cr}} = 3M_{\mathrm{Pl}}^2 H_0^2\,, \quad \eta = \frac{\varepsilon}{3\eta\, H_0^2}\,,$$

$$\Omega_0 = \frac{\Lambda}{\rho_{\mathrm{cr}}},\ \Omega_2 = -\frac{K}{H_0^2 \mathrm{a}_0^2},\ \Omega_6 = \frac{Q^2}{6\eta\, \mathrm{a}_0^6\, H_0^2\, \rho_{\mathrm{cr}}},\ \rho = \rho_{\mathrm{cr}} \left(\frac{\Omega_4}{a^4} + \frac{\Omega_3}{a^3}\right)$$

gives

the master equation:

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6 \left[\eta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6 \left[\eta - y + \frac{\Omega_2}{a^2}\right]^2}$$

Asymptotical behavior: Late time limit $a \to \infty$

GR branch:

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{(\eta - 3\,\Omega_0)\,\Omega_6}{(\,\Omega_0 - \eta)^2\,a^6} + \mathcal{O}\left(\frac{1}{a^7}\right) \Longrightarrow \quad H^2 \to \Lambda/3$$

Notice: The GR solution is stable (no ghost) if and only if $\eta > \Omega_0$.

Screening branches:

$$y_\pm = \eta + \frac{\Omega_2}{a^2} \pm \frac{\chi}{(\,\Omega_0 - \eta)\,a^3} \pm \frac{\Omega_2\Omega_6}{\chi\,a^5} - \frac{\Omega_6(\eta - 3\Omega_0) \pm \Omega_3\chi}{2(\Omega_0 - \eta)^2\,a^6} + \mathcal{O}\left(\frac{1}{a^7}\right)$$

$$\Longrightarrow \quad H^2 \to \varepsilon/3\alpha$$

Notice: The screening solutions are stable (no ghost) if and only if $0 < \eta < \Omega_0$.

Asymptotical behavior: The limit $a \to 0$

GR branch:

$$y = \frac{\Omega_4}{a^4} + \frac{\Omega_3}{a^3} + \frac{\Omega_2\Omega_4 - 3\Omega_6}{\Omega_4 a^2} + \frac{3\Omega_3\Omega_6}{\Omega_4 a} + \mathcal{O}(1)$$

Notice: The GR solution is unstable

Screening branch:

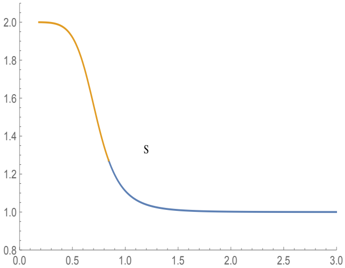
$$y_+ = \frac{3\Omega_6}{\Omega_4\, a^2} - \frac{3\Omega_3\Omega_6}{\Omega_4^2\, a} + \frac{5}{3}\eta + \frac{3\Omega_6\Omega_3^2 + 9\Omega_6^2}{\Omega_4^3} + \mathcal{O}(a),$$

$$y_- = \frac{1}{\sqrt{9\eta}} + \frac{4\,\eta^2}{27\ \Omega_6}\left(\Omega_4\, a^2 + \Omega_3\, a^3\right) + \mathcal{O}(a^4)$$

Notice: Both screening solutions are stable

Global behavior

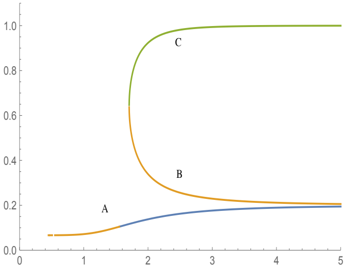
$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6 \left[\eta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6 \left[\eta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1$, $\Omega_2 = 0$, $\Omega_3 = \Omega_4 = 0$ and for $\eta = 6$

Global behavior

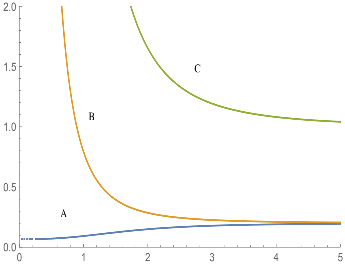
$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\eta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\eta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1,\ \Omega_2 = 0,\ \Omega_3 = \Omega_4 = 0,\ \eta = 0.2$

Global behavior

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\eta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\eta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1$, $\Omega_3 = 5$, $\Omega_4 = 0$, $\eta = 0.2$. One has $\Omega_2 = 0$.

Intermediate Summary

- The nonminimal kinetic coupling provides an *essentially new* inflationary mechanism which does not need any fine-tuned potential.
- At early cosmological times the coupling η-terms in the field equations are dominating and provide the quasi-De Sitter behavior of the scale factor: $a(t) \propto e^{H_\eta t}$ with $H_\eta = 1/\sqrt{9\eta}$.
- The model provides a natural mechanism of epoch change without any fine-tuned potential.
- The nonminimal kinetic coupling crucially changes a role of the scalar potential. Power-law and Higgs-like potentials with kinetic coupling provide accelerated regimes of the Universe evolution.

Perturbations

Scalar perturbations (Newtonian gauge):

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j,$$

$$\phi = \phi_0 + \delta\phi = \phi_0(1+\varphi),$$

$$\Psi(t,\mathbf{x}) \ll 1,\ \ \Phi(t,\mathbf{x}) \ll 1,\ \ \varphi(t,\mathbf{x}) \ll 1$$

Fourier trasformations: $\Psi(t,\mathbf{x}) = \int d\mathbf{k} e^{i\mathbf{k}\mathbf{x}}\Psi(t,\mathbf{k})$ and so on

Scalar modes:

$$-3H(\dot\Psi - H\Phi) - \frac{k^2}{a^2}\Psi = 4\pi\Big[\dot\phi^2\Phi - \dot\phi\delta\dot\phi$$

$$+\eta\left(9H\dot\phi^2\dot\Psi - 18H^2\dot\phi^2\Phi + \frac{k^2}{a^2}\dot\phi^2\Psi + 9H^2\dot\phi\delta\dot\phi + 2\frac{k^2}{a^2}H\dot\phi\delta\phi\right)\Big],$$

$$\dot\Psi - H\Phi = 4\pi\left[-\dot\phi\delta\phi + \eta\left(3H\dot\phi^2\Phi - \dot\phi^2\dot\Psi - 2H\dot\phi\delta\dot\phi + 3H^2\dot\phi\delta\phi\right)\right],$$

$$\Phi + \Psi = -4\pi\eta\left[\dot\phi^2(\Phi-\Psi) + 2(\ddot\phi + H\dot\phi)\delta\phi\right]$$

Perturbations

Scalar perturbations (Newtonian gauge):

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j,$$

$$\phi = \phi_0 + \delta\phi = \phi_0(1+\varphi),$$

$$\Psi(t,\mathbf{x}) \ll 1,\ \ \Phi(t,\mathbf{x}) \ll 1,\ \ \varphi(t,\mathbf{x}) \ll 1$$

Fourier trasformations: $\Psi(t,\mathbf{x}) = \int d\mathbf{k} e^{i\mathbf{k}\mathbf{x}}\Psi(t,\mathbf{k})$ and so on

Scalar modes:

$$-3H(\dot\Psi - H\Phi) - \frac{k^2}{a^2}\Psi = 4\pi\Big[\dot\phi^2\Phi - \dot\phi\delta\dot\phi$$
$$+\eta\left(9H\dot\phi^2\dot\Psi - 18H^2\dot\phi^2\Phi + \frac{k^2}{a^2}\dot\phi^2\Psi + 9H^2\dot\phi\delta\dot\phi + 2\frac{k^2}{a^2}H\dot\phi\delta\phi\right)\Big],$$

$$\dot\Psi - H\Phi = 4\pi\left[-\dot\phi\delta\phi + \eta\left(3H\dot\phi^2\Phi - \dot\phi^2\dot\Psi - 2H\dot\phi\delta\dot\phi + 3H^2\dot\phi\delta\phi\right)\right],$$

$$\Phi + \Psi = -4\pi\eta\left[\dot\phi^2(\Phi - \Psi) + 2(\ddot\phi + H\dot\phi)\delta\phi\right]$$

Notice: $\Psi = -\Phi$ if $\eta = 0$, but generally $\Psi \neq -\Phi$!

Perturbations in the inflationary epoch

On the inflationary stage at $t \to -\infty$ the unperturbed solutions are

$$a(t) = a_i e^{H_\eta (t-t_i)}, \quad \phi(t) = \phi_i e^{-3H_\eta (t-t_i)}, \qquad H_\eta = \frac{1}{\sqrt{9\eta}}$$

Scalar perturbations on the inflationary stage

$$\dot{\Psi} = H_\eta \Phi - \frac{1}{12 H_\eta} \frac{k^2}{a^2} (7\Psi + 3\Phi),$$

$$\dot{\Phi} = -H_\eta (6\Psi + 7\Phi) + \frac{1}{4 H_\eta} \frac{k^2}{a^2} (7\Psi + 3\Phi). \qquad a = a_i e^{H_\eta (t-t_i)}$$

Perturbations in the inflationary epoch

Limiting cases:

A. $k/a \ll H_\eta$ (modes outside the Hubble horizon)

Scalar perturbations of metric:

$$\dot{\Psi} = H_\eta \Phi - \frac{1}{12 H_\eta} \frac{k^2}{a^2} (7\Psi + 3\Phi),$$

$$\dot{\Phi} = -H_\eta (6\Psi + 7\Phi) + \frac{1}{4 H_\eta} \frac{k^2}{a^2} (7\Psi + 3\Phi). \qquad a = a_i e^{H_\eta (t - t_i)}$$

$$\Psi = \tfrac{1}{5}(6\Psi_i + \Phi_i) e^{-H_\eta (t - t_i)} - \tfrac{1}{5}(\Psi_i + \Phi_i) e^{-6 H_\eta (t - t_i)},$$

$$\Phi = -\tfrac{1}{5}(6\Psi_i + \Phi_i) e^{-H_\eta (t - t_i)} + \tfrac{6}{5}(\Psi_i + \Phi_i) e^{-6 H_\eta (t - t_i)},$$

$\Psi_i = \Psi(t_i) \ll 1, \ \Phi_i = \Phi(t_i) \ll 1, \quad t = t_i$ – beginning of inflation

Perturbs in course of inflation $t > t_i$: $\Psi = -\Phi \sim e^{-H_\eta t} \sim a^{-1}$

NOTICE: Scalar modes $k/a \ll H_\eta$ are exponentially decaying!

Perturbations in the inflationary epoch

B. $k/a \gg H_\eta$ (modes inside the Hubble horizon)

Scalar perturbations of metric:

$$\dot{\Psi} = H_\eta \Phi - \frac{1}{12 H_\eta} \frac{k^2}{a^2} (7\Psi + 3\Phi),$$

$$\dot{\Phi} = -H_\eta (6\Psi + 7\Phi) + \frac{1}{4 H_\eta} \frac{k^2}{a^2} (7\Psi + 3\Phi). \qquad a = a_i e^{H_\eta (t - t_i)}$$

$$\Psi = \tfrac{3}{2}(3\Psi_i + \Phi_i) - \tfrac{3}{2}\left(\tfrac{7}{3}\Psi_i + \Phi_i\right) \exp\left[\frac{1}{12}\left(\frac{k}{H_\eta}\right)^2 \left(\frac{1}{a_i^2} - \frac{1}{a^2}\right)\right],$$

$$\Phi = -\tfrac{7}{2}(3\Psi_i + \Phi_i) + \tfrac{9}{2}\left(\tfrac{7}{3}\Psi_i + \Phi_i\right) \exp\left[\frac{1}{12}\left(\frac{k}{H_\eta}\right)^2 \left(\frac{1}{a_i^2} - \frac{1}{a^2}\right)\right],$$

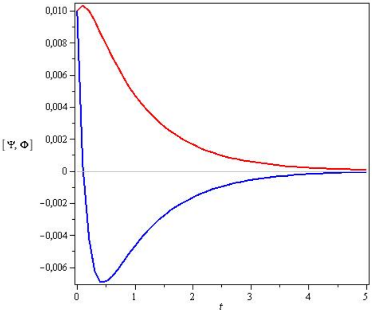
Perturbs in course of inflation $t > t_i$ $(1/a_i^2 \gg 1/a^2)$:

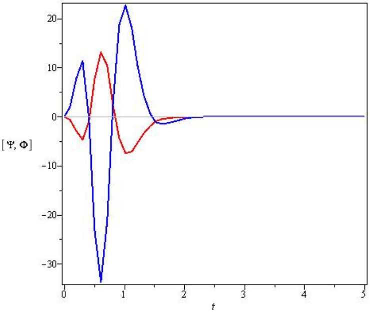
$$\Psi,\ \Phi \to \exp\left[\frac{1}{12}\left(\frac{k}{a_i H_\eta}\right)^2\right] \gg 1$$

NOTICE: Scalar modes $k/a \gg H_\eta$ are growing!

TENDENCY: During the inflation, modes with short wavelength are stretching and come beyond the Hubble horizon. After they have gone outside the Hubble horizon, they are exponentially decaying.

Examples of numerical analysis for scalar mode evolution:





Tensor perturbations

Tensor perturbations:

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j,$$

$$\partial_i h_{ij} = 0, \;\; h_{ii} = 0.$$

Two polarizations: $h_{ij} \longrightarrow h^+, \;\; h^\times$

Equation for tensor modes

$$(1 + 4\pi\eta\dot{\phi}^2)\ddot{h} + \left(3H + 4\pi\eta(2\dot{\phi}\ddot{\phi} + 3H\dot{\phi}^2)\right)\dot{h} + \frac{k^2}{a^2}(1 - 4\pi\eta\dot{\phi}^2)h = 0$$

Tensor perturbations during the kinetic inflation

Tensor perturbation on the inflationary stage:

$$\left(1+4\pi\phi_i^2 e^{-6H_\eta(t-t_i)}\right)\ddot{h}+3H_\eta\left(1-4\pi\phi_i^2 e^{-6H_\eta(t-t_i)}\right)\dot{h}$$
$$+\frac{k^2}{a^2}\left(1-4\pi\phi_i^2 e^{-6H_\eta(t-t_i)}\right)h=0$$

$$a(t)=a_i e^{H_\eta(t-t_i)},\quad \phi(t)=\phi_i e^{-3H_\eta(t-t_i)}$$

The case $4\pi\phi_i^2 \ll 1$:

$$\ddot{h}+3H_\eta\dot{h}+\frac{k^2}{a^2}h=0$$

A. $k/a \ll H_\eta$ (outside the Hubble horizon) $\Rightarrow$ *constant modes*

B. $k/a \gg H_\eta$ (inside the Hubble horizon) $\Rightarrow$ *damping oscillating modes*

Tensor perturbations during the kinetic inflation

The case $4\pi\phi_i^2 \gg 1$: $\quad \ddot{h} - 3H_\eta\dot{h} - \frac{k^2}{a^2}h = 0$

A. $k/a \ll H_\eta$ *modes outside the Hubble horizon*

$$\ddot{h} - 3H_\eta\dot{h} = 0 \quad \Longrightarrow \quad h \propto e^{3H_\eta t} \quad \Longrightarrow \quad \text{exponentially growing!}$$

B. $k/a \gg H_\eta$ *modes inside the Hubble horizon*

$$\ddot{h} - \frac{k^2}{a^2}h = 0 \quad \Longrightarrow \quad h \propto e^{\pm k e^{-H_\eta t}/H_\eta} \quad \Longrightarrow \quad \text{constant modes}$$

Final conclusions

- Long-wave scalar modes $k/a \ll H_\eta$ are exponentially decaying during the kinetic inflation. Therefore, the large-scale structure of the Universe keeps to be homogeneous and isotropic.
- Short-wave scalar modes $k/a \gg H_\eta$ are growing during the narrow time interval when $k/a \approx H_\eta$. At this moment seeds for the Universe structure (clasters, galaxies, etc) could be formed. However, this is a regime of nonlinear perturbations, and hence one needs a nonperturbative analysis.

THANKS FOR YOUR ATTENTION!