A viable compactification scenario in Gauss-Bonnet cosmology

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The action of the model reads

\[ S = \int_{M} d^{D}z \sqrt{|g|} \{ \alpha_{1}(R[g] - 2\Lambda) + \alpha_{2}L_{2}[g] \}, \]

where \( g = g_{MN}dz^{M} \otimes dz^{N} \) is the metric defined on the manifold \( M \), \( \dim M = D \), \( |g| = |\det(g_{MN})| \), \( \Lambda \) is the cosmological term,

\[ L_{2} = R_{MNPQ}R^{MNPQ} - 4R_{MN}R^{MN} + R^{2} \]

is the standard Gauss-Bonnet term and \( \alpha_{1}, \alpha_{2} \) are nonzero constants.
For a successful compactification we need:

- To start with rather general anisotropic initial conditions.
- To get isotropic 3 dimensional expanding space.
- To get extra dimensions stabilized.
A general solution for a flat anisotropic Universe in GR is the Kasner solution:

\[ a_i \sim t^{p_i} \]

where

\[ \sum p_i = 1 \]

and

\[ \sum p_i^2 = 1. \]
In the pure GB gravity there is an analog of the Kasner solution:

$$a_i \sim t^{p_i}$$

where

$$\sum p_i = 3$$

and

$$\sum p_i(p_i - 1) \sum p_j p_k = 0.$$ 

However, in GB gravity there is a solution with no analog in GR:

$$a_i \sim \exp(H_i t)$$
which means that Hubble parameters $H_i$ are constant (V.Ivashchuk, arXiv:0910.3426). Since Hubble parameters are constant, the exponential solutions are exact (not asymptotic!) solution of the theory with GB, Einstein and $\Lambda$ terms. They are stable if overall volume is increasing, apart from special families of solutions (S.Pavluchenko, arXiv:1507.01871, V.Ivashchuk, arXiv:1607.01244, D.Chirkov and AT, arXiv:1706.08889).
The parameters $H_i$ are subject to rather severe restrictions. For example, they can not be all different. Moreover, in GB gravity there are maximum 3 different Hubble parameters (V.Ivashchuk, arXiv:1607.01244). Moreover, in 4 spatial dimensions the only possible combinations are $3+1$ (with 3D isotropic subspace) and $2D+2D$ (with two different isotropic subspaces).

In 5 spatial dimensions there are the following possibilities: $(H, H, H, H, H, h) - 4+1$ splitting
\((H, H, H - H, -H, h)\) and
\((H, H, H, h, h)\) – 3+2 splitting (D.Chirkov, S.Pavluchenko and AT, arXiv:1401.2962). So that, if the exponential solution is an attractor, the isotropisation in expanding dimensions occurs automatically during dynamical evolution.
This means that if we start near anositropic GB Kasner solution with 3 expanding and 3 contracting dimensions, then from rather wide zone in the initial condition space the space-time becomes a product of isotropic subspaces.
The second question still remains: how can GB terms help to stabilize extra dimensions? An answer have been found in (F.Canfora, A.Giacomini and S.Pavluchenko, arXiv:1308.1896) where it was shown that negative spatial curvature in extra dimensions leads to their stabilization. This result is established for spaces which are already products of two isotropic subspaces. Combining with the previous result we get the following scenario.
We start from initial conditions with very small spatial curvature. Then:

- Initially anisotropic Universe with 3 expanding and 3 contracting dimensions evolves into a product of 3 exponentially expanding and 3 exponentially contracting isotropic subspaces.

- After negative spatial curvature in contracting space becomes dynamically important the extra dimensions stabilize.
The present work considers a particular example of 3+3 splitting. It is necessary to check this scenario for other exponential solutions. In particular, it would be good to set the effective $\Lambda$ of expanding dimensions to be near zero. It was shown that for successful compactification in this case we need minimum 7 spatial dimensions (F. Canfora, A. Giacomini, S. Pavluchenko and AT, arXiv:1605.00041).
For large number of extra dimensions it will be necessary to study possible influence of higher-order Lovelok terms.