THE FOUR-DIMENSIONAL SELF-CONSISTENT MODEL OF THE BUNCH OF CHARGED PARTICLES

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Abstract. The four-dimensional non-stationary model of a bunch of the particles interacting with own field is studied. For the description of behavior of a bunch the "Meshchersky's integral" allowing to give completely self-consistent kinetic description in 8-dimensional phase space is used. In the considered task the self-coordinated potential determines interaction forces in 3-dimensional space.

Introduction

The description of dynamics of non-stationary unicomponent ensemble of charged particles is rather difficult task, cm, for example, the work [1] studying behavior of a ring electron beam in magnetic field. In this work function of distribution of collisionless system undertakes as function of integrals of the movement - the moment of rather longitudinal axis and the bilinear invariant received at the solution of the linearizied equation of fluctuations concerning a bunch axis - on the substance of Kapchinsky-Vladimirsky's integral. A certain choice of function of distribution allows to receive the constant density of particles in a bunch and to solve the self-coordinated equations for a potential component.

In work [2] the similar procedure is applied to studying of dynamics of a spherical symmetric bunch, function of distribution is defined as function of two integrals of the movement - Kapchinsky-Vladimirsky's integral and a square of the full moment of number of the movement.

In the real work dynamics spherically of the symmetric clot having however the additional, 4th size is studied. The convenience of introduction of the fourth coordinate is explained by feature of the used "Meshchersky integral" only in 4- space leading to the system having the automodel decision. Also similar quantum mechanical system is considered.

1. Meshchersky's Integral

Let the system of particles be described by a look Hamiltonian:

$$H = \frac{m}{2}(\dot{r})^2 + \frac{L}{2mr^2} + \frac{1}{\xi(t)^2}U\left(\frac{r}{\xi(t)}\right) + \frac{m}{2}(\dot{s})^2, \qquad (1)$$

where - m - weight, $\frac{1}{\xi^2}U$ - the potential, s - the fourth coordinate, r - radius vector in the spherical system of coordinates, L - a square of the full moment of number of the movement. The first 4 composed in (1) describe the movement in symmetric system technical measured spherically, and the last - the movement on the fourth coordinate. According to it from (1) existence of two integrals of the movement follows:

$$I = \frac{m}{2}\left(\dot{r}\xi - r\dot{\xi}\right)^2 + \frac{L\xi^2}{2mr^2} + U\left(\frac{r}{\xi}\right) + \lambda m\frac{r^2}{2\xi^2}, \qquad (2)$$

and the integral describing the movement on s coordinate:

$$I_s = \frac{m}{2}\left(\dot{s}\xi - s\dot{\xi}\right)^2 + \lambda\frac{s^2}{\xi^2}. \quad (3)$$

In expressions (2), (3 $\xi(t)$ satisfies equation with $\ddot{\xi} = \frac{\lambda}{\xi(t)^3}$. From condition $\frac{dI}{dt} = 0$ follows $m\ddot{r} = \frac{L}{mr^3} + \frac{1}{\xi^2}U'$, and from condition $\frac{dI}{dt} = 0$ follows $\ddot{s} = 0$.

Function of distribution can be taken as function of three integrals of the movement - L, I, I_s.

2.Classical monochromatic model

We will consider a situation when all particles are characterized by one value of integral of I and inversely proportional dependence on a root from L.

we will put for function of distribution of f :

$$f = \kappa\frac{\sigma(I_1 - I_s)}{\sqrt{I_1 - I_s}}\frac{\delta(I - I_0)}{\sqrt{L}} \quad (4)$$

Here $\sigma(x)$ - the Heaviside function.

For determination of density it is necessary to calculate integral in phase space:

$$n = \frac{\pi}{2r^2}\int\frac{fdIdI_s dL}{\xi\sqrt{\frac{2}{m}(I - U) - \lambda\frac{r^2}{\xi^2} - \frac{L\xi^2}{m^2r^2}}\xi\sqrt{\frac{2I_s}{m} - \lambda\frac{s^2}{\xi^2}}}.$$

The integral for I_s is just calculated:

$$\int\frac{dI_s}{\sqrt{I_1 - I_s}\sqrt{\frac{2I_s}{m} - \lambda\frac{s^2}{\xi^2}}} = \sqrt{\frac{m}{2}}\sigma\left(I_1 - \frac{\lambda m}{2}\frac{s^2}{\xi^2}\right). \quad (5)$$

From (5) follows that density is constant for s if $|s| < \xi\sqrt{\frac{2I_1}{\lambda m}}$. Calculating integral for L we will receive

$$n = \frac{\pi\kappa m}{r\xi^3}\sqrt{\frac{m}{2}}\sigma\left(I_0 - U - \frac{\lambda m}{2}\frac{r^2}{\xi^2}\right)\sigma\left(I_1 - \frac{\lambda m}{2}\frac{s^2}{\xi^2}\right). \quad (6)$$

We will pass, further, to the solution of the three-dimensional equation of Poisson for potential Φ the bunch defining own field. Potential Φ is connected with U as follows: $q\Phi = \frac{U}{\xi^2}$, where q - a particle charge. Further we will solve Poisson's equation in three-dimensional space, considering that $s < \xi\sqrt{\frac{2I_1}{\lambda m}}$ and that potential doesn't depend on the 4th coordinate of s The Area of phase space connected with the 4th coordinate can be any small at $I_1 \to 0$. The Equation for U can be brought to a look:

$$\frac{1}{\xi^2 r}\frac{d^2}{dr^2}rU\left(\frac{r}{\xi}\right) = -4\pi q^2 n. \quad (7)$$

We will enter variable $\rho = \frac{r}{\xi}$ and we will designate $4\pi^2 q^2 \kappa m \sqrt{\frac{m}{2}} = \kappa_*$ From (7) follows:

$$\frac{d^2}{d\rho^2}\rho U(\rho) = -\kappa_* \sigma \left(I_0 - U(\rho) - \frac{\lambda m}{2}\rho^2 \right). \qquad (8)$$

We will put $U = a_0 - a_1\rho$. Then a particle are localized in the field of $I_0 - a_0 + a_1\rho - \frac{\lambda m}{2}\rho^2 > 0.$

This condition is <u>sutisfired</u> if $\rho < \rho_1 = \frac{a_1}{\lambda m} + \sqrt{\left(\frac{a_1}{\lambda m}\right)^2 + 2\frac{I_0 - a_0}{2\lambda m}}$. From (8) follows $a_1 = \frac{\kappa_*}{2}$. At $\rho > \rho_1$ $U = C_0 / \rho$, where $C_0 = \frac{\kappa_*}{2}\rho_1^2$. The full number of particles in the four-dimensional cylinder has an appearance $N = 4\pi^2 \kappa m \rho_1^2$, whereas the number of particles in volume of the three-dimensional sphere depends on time: $N_3 = \frac{2}{\xi}\pi^2 \kappa m \rho_1^2$.

As, according to (6) $n = 0$ at $r > \rho_1 \xi$ that (since $\xi^2 = a(t + b/a)^2 + \lambda/a$), condition $r < \rho_1 \sqrt{\frac{a(t+b/a)^2 + \lambda/a}{a}}$ defines area in plane r, t, in winch density of n is other than zero.

It is possible that the given model can be applied to the description of gravitating systems. If to consider particles attracted that can be considered, having replaced with $\kappa_* \to \kappa_*$, that the studied configuration keeps the look, however the characteristic size ρ_1 will be less, than in case of pushing away particles.

3 Quantum mechanical system

For the system described by a Hamiltonian (1) the Schrödinger equation has view:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 \Psi}{\partial r^2} + \frac{2}{r}\frac{\partial \Psi}{\partial r} + \frac{\partial^2 \Psi}{\partial s^2} \right) + \left(\frac{L}{2mr^2} + \frac{1}{\xi^2} V\left(\frac{r}{\xi}\right) \right)\Psi. \qquad (9)$$

We will enter new variables: $\rho = \frac{r}{\xi}, s_* = \frac{s}{\xi}, \tau = \int \frac{dt'}{\xi(t')^2}$. Then (9) will take a form::

$$i\hbar\left(\frac{\partial \Psi}{\partial \tau} - \frac{\dot{\xi}}{\xi}\rho \frac{\partial \Psi}{\partial \rho} - \frac{\dot{\xi}}{\xi} s_* \frac{\partial \Psi}{\partial s_*} \right) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 \Psi}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial \Psi}{\partial \rho} + \frac{\partial^2 \Psi}{\partial s_*^2} \right) + \left(V(\rho) + \frac{L}{2m\rho^2} \right)\Psi \qquad (10)$$

We will put, further: $\Psi = \frac{1}{\xi^2}\Psi_1(\rho, s_*, \tau)\exp\left\{ \frac{im}{\hbar}\frac{\dot{\xi}}{\xi}\left(\frac{s_*^2}{2} + \frac{\rho^2}{2} \right) \right\}$.

We will receive the equation:

$$i\hbar \frac{\partial \Psi_1}{\partial \tau} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 \Psi_1}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial \Psi_1}{\partial \rho} + \frac{\partial^2 \Psi_1}{\partial s_*^2} \right) + \left(V(\rho) + \frac{L}{2m\rho^2} + \frac{\lambda m \rho^2}{2} + \frac{\lambda m s_*^2}{2} \right)\Psi_1. \qquad (11)$$

Further, we will look for the decision in a look: $\Psi_1 = \exp\left\{-i\frac{E}{\hbar}t\right\}\psi\left(\rho, s_*\right) = \exp\left\{-i\frac{E}{\hbar}t\right\}R\left(\rho\right)X\left(s_*\right).$

For X, R we will receive the equations:

$$-\frac{\hbar^2}{2m}\frac{X''}{X} + \frac{\lambda m s_*^2}{2} = F, \tag{12}$$

$$-\frac{\hbar^2}{2mR}\left(R'' + \frac{2}{\rho}R'\right) + V + \frac{L}{2m\rho^2} + \frac{\lambda m \rho^2}{2} = G, \tag{13}$$

where $F + G = E.$ The private decision (22) has an appearance: $X\left(s_*\right) = \exp\left\{-\kappa_0 s_*^2\right\},$ where $\kappa_0 = \frac{m}{\hbar}\frac{\sqrt{\lambda}}{2}, F = \frac{\hbar\sqrt{\lambda}}{4}.$

The equation (13) has to be complemented with Poisson's equation for potential function of $V\left(\rho\right)$:

$$\frac{1}{\rho}\frac{d^2}{d\rho^2}V\left(\rho\right)\rho = -4\pi q^2 R\left(\rho\right)^2 X\left(s_*\right)^2. \tag{14}$$

Further the three-dimensional equation of Poisson is solved at $s_* << \frac{1}{\sqrt{\kappa_0}}.$

We will designate $\lambda = \frac{1}{\tau_0^2}.$Tthe Self-coordinated system takes a form:

$$-\frac{\hbar^2}{2mR}\left(R'' + \frac{2}{\rho}R'\right) + V + \frac{L}{2m\rho^2} = E - \frac{\hbar}{4\tau_0}, \tag{15}$$

$$\frac{1}{\rho}\frac{d}{d\rho^2}^2 V\left(\rho\right)\rho = -4\pi q^2 R\left(\rho\right)^2. \tag{16}$$

We will put, further, $\rho = x l_0$ where $l_0 = \sqrt{\frac{2\hbar\tau_0}{m}}.$ At $L = 0, E = 0$ we will receive the system of the equations:

$$\begin{cases} R'' + \frac{2}{x}R' - V_1 - 4x^2 R = R, \\ V_1'' + \frac{2}{x}V_1' = -QR^2 \end{cases} \tag{17}$$

In (17) $V_1 = V\frac{4\tau_0}{\hbar},$ and parameter of $Q = 4\pi q^2\sqrt{\frac{8\tau_0^3}{m\hbar}}$ is defined by an elementary charge.

Results of the solution of system (17) for the potential function of V_1 and density of a charge proportional to R^2 at $Q = 3$ and at $V_1(0) = 100, R(0) = 10, V_1'\left(0\right) = R'(0) = 0.$ are given on figures 1 and 2.

Density of a charge has a maximum near the beginning of coordinates falls with growth of distance from a center. Falling of potential at rather big distances from the center shows that the charge isn't localized in any limited area, and is distributed on all space. This circumstance essentially distinguishes quantum mechanical system from the classical system considered in the previous section in which the charge is localized in the area with a radius growing on time.

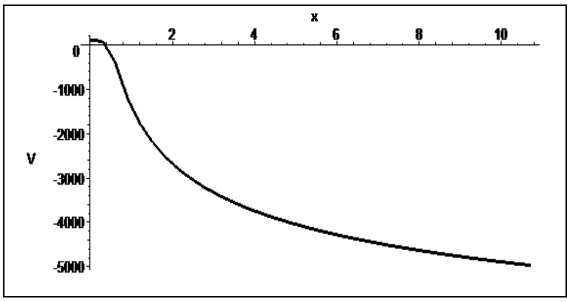

Fig1. Dependence V(x).

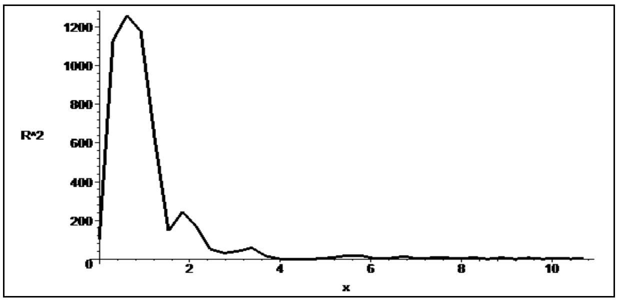

Fig2. Dependence $R(x)^2$.

Conclusion

In work model representation of the charge interacting with own field in 4 space in classical and quantum mechanical systems is considered. At this field it is defined in 3-d space and forces along the 4th coordinate are absent.

Refrences

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