IV international conference on particle physics and astrophysics

Hyperfine structure of S-states in muonic ions of lithium, beryllium and boron

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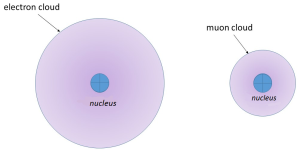
24.10.2018

- μLi, μBe, μB are two-particle bound states of muon and nucleus;

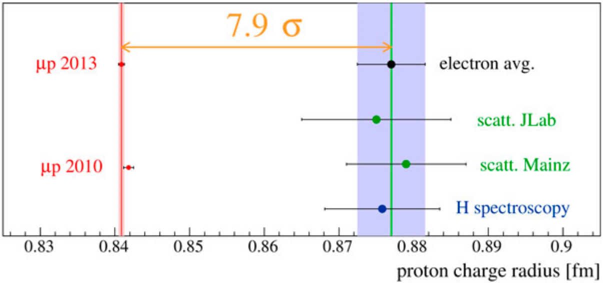


- The muon is two hundred times heavier than the electron. It leads to a lower Bohr radius of the muon. Thus, an influence of vacuum polarization and nuclear structure effects in hyperfine splitting increases;
- Muonic atoms play an important role in check of QED, theory of bound states and in precise measurement of fundamental constants;
- Measurement of the LS in light muonic atoms allows us to obtain more precise values of charge radii of corresponding atoms.

Proton radius puzzle

In the experiment carried out at PSI (Paul Scherrer Institute) transition frequency $2P^{F=2}_{3/2} - 2S^{F=1}_{1/2}$ in muonic hydrogen were measured with the following unexpected results:

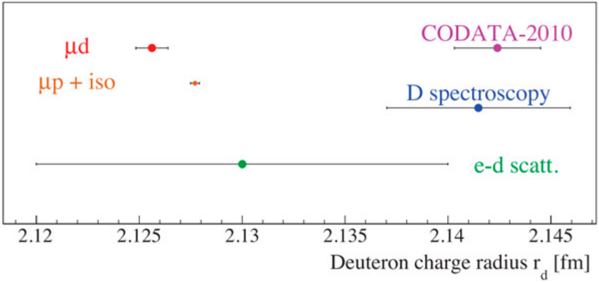
- New value of proton charge radius appeared to be 7.9 standard deviations smaller than the CODATA value $r_p = 0.8768(69) fm$.



A. Antognini et al., Science **339**, 417 (2013).

The experiment with muonic deuterium

Similar measurements for muonic deuterium also show noticeable discrepancy of 7.5σ with CODATA-2010 value. The discrepancy with CODATA-2014 value is slightly smaller - "only" 6σ.



R.Pohl et al. (the CREMA Collaboration). Laser spectroscopy of muonic deuterium // Science. 2016. V. 353. P. 669-637.

Purpose

The aim of this work is to calculate analytically and numerically corrections of order α^5 and α^6 in hyperfine structure of S-states in muonic ions Li, Be and B.

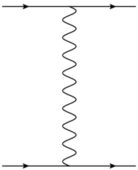
- We use quasipotential approach in QED;
- We include vacuum polarization, relativistic and nuclear structure corrections to achieve high accuracy;

The Fermi energy

A part of the Breit Hamiltonian, responsible for hyperfine splitting, has a well-known form in the coordinate representation:

$$\Delta V_B^{hfs}(r) = \frac{2\pi\alpha}{3m_1 m_p} g_d g_\mu (\mathbf{s_1 s_2})\delta(\mathbf{r}).$$

The main part of hyperfine splitting of order α^4 is called the Fermi energy. It can be obtained after averaging $\Delta V_B^{hfs}(r)$ over the Coulomb wave functions:



$$\psi_{100}(r) = \frac{W^{3/2}}{\sqrt{\pi}} e^{-Wr}, \quad W = \mu Z\alpha,$$

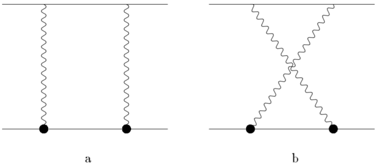
$$\psi_{200}(r) = \frac{W^{3/2}}{2\sqrt{2\pi}} e^{-Wr/2}\left(1 - \frac{Wr}{2}\right).$$

$$\Delta E_F^{hfs}(nS) = E(S=3/2) - E(S=1/2) = \frac{4\mu^3 Z^3 \alpha^4 g_N}{3m_1 m_p n^3} =$$

$$= \begin{cases} {}^6_3Li(1S) : 1416.07 \text{ meV}, & {}^6_3Li(2S) : 177.01 \text{ meV}, \\ {}^7_3Li(1S) : 5026.00 \text{ meV}, & {}^7_3Li(2S) : 628.25 \text{ meV}, \\ {}^9_4Be(1S) : -4353.49 \text{ meV}, & {}^9_4Be(2S) : -544.19 \text{ meV}, \\ {}^{10}_5B(1S) : 11420.56 \text{ meV}, & {}^{10}_5B(2S) : 19548.21 \text{ meV}, \\ {}^{11}_5B(1S) : 19548.10 \text{ meV}, & {}^{11}_5B(2S) : 2443.53 \text{ meV}. \end{cases}$$

Two-photon exchange diagrams

Basic contribution of the nuclear structure effects of order α^5 to the hyperfine splitting is determined by two-photon exchange diagrams.



Nuclear structure effects of order α^5. The bold point denotes the nucleus vertex function.

It is expressed in terms of electric $G_E(k^2)$ and magnetic $G_M(k^2)$ nuclear form factors in the form (the Zemach correction):

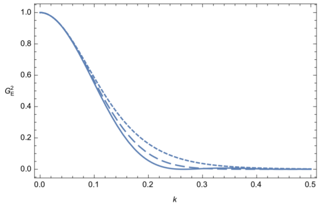
$$\Delta E_Z^{hfs}(nS) = E_F^{hfs}(nS)\frac{2\mu Z\alpha}{\pi}\int\frac{d\mathbf{k}}{k^4}\left(\frac{G_E(k^2)G_M(k^2)}{G_M(0)}-1\right).$$

We have analysed numerical values of this correction for different parameterizations of nuclear form factors: Gaussian, dipole and uniformly charged sphere:

$$G_E^G(k^2) = e^{-\frac{1}{6}r_N^2k^2}, \quad G_E^D(k^2) = \frac{1}{(1+\frac{k^2}{\Lambda^2})^2}, \quad G_E^U(k^2) = \frac{3}{(kR)^3}\left[\sin kR - kR\cos kR\right],$$

where $R = \sqrt{5}r_N/\sqrt{3}$ is the nucleus radius, $\Lambda^2 = 12/r_N^2$.

A comparison of function for different parameterizations is presented in the picture for the nucleus ^6Li. In the range $0.1 \leq k \leq 0.4$ GeV there is a difference between parameterizations which leads to different numerical values of the Zemach correction.

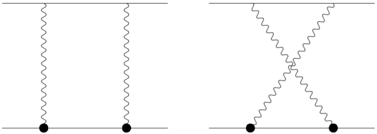


Gaussian (dashed), dipole (dotted) and uniformly charged sphere (solid) parameterizations of nuclear form factor $G_E^2(k^2)$.

The momentum integration can be done analytically:

$$\Delta E^{hfs}_{str,G} = -E_F \frac{72}{\sqrt{3\pi}} \mu Z \alpha r_N, \quad \Delta E^{hfs}_{str,D} = -E_F \frac{35}{8\sqrt{3}} \mu Z \alpha r_N, \quad \Delta E^{hfs}_{str,U} = -E_F \frac{72\sqrt{5}}{35\sqrt{3}} \mu Z \alpha r_N.$$

Muonic ion	$1S$, meV	$2S$, meV
^6_3Li	-106.54 / -109.92 / -112.02	-13.32 / -13.74 / -14.01
^7_3Li	-357.91 / -369.25 / -376.31	-44.74 / -46.16 / -47.04
^9_4Be	428.08 / 441.63 / 450.08	53.51 / 55.21 / 56.26
$^{10}_5B$	-1352.89 / -1395.72 / -1422.44	-169.11 / -174.47 / -177.81
$^{11}_5B$	-2297.31 / -2370.04 / -2415.41	-287.16 / -296.26 / -301.93



Nuclear structure effects of order α^5. The bold point denotes the nucleus vertex function.

We also can present the contribution of two-photon interactions to HFS in the form:

$$\Delta E^{hfs}(nS) = |\psi_{nS}(0)|^2 \int d^4k V_{2\gamma}(k) = \frac{64}{9}\frac{(Z\alpha)^2}{\pi^2}|\psi_{nS}(0)|^2 \int \frac{d^4k}{k^4(k^4+4m_1^2k_0^2)(k^4+4m_2^2k_0^2)}\times$$

$$\Big[F_1F_2\Big(k^6 - k^4k_0^2 + \frac{4}{15}\frac{k^4k_0^4}{m_2^2} - \frac{7}{10}\frac{k^6k_0^2}{m_2^2} + \frac{13}{30}\frac{k^8}{m_2^2}\Big) + F_2F_4\Big(-\frac{1}{30}\frac{k^2k_0^6}{m_2^2} + \frac{1}{15}\frac{k^4k_0^4}{m_2^2} - \frac{1}{30}\frac{k^6k_0^2}{m_2^2}\Big)+$$

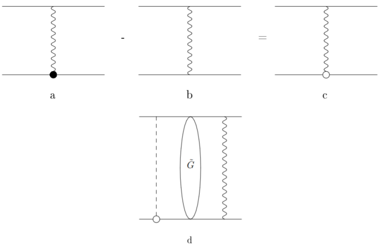
$$F_2F_3\Big(-\frac{1}{15}\frac{k^2k_0^6}{m_2^2} + \frac{11}{60}\frac{k^4k_0^4}{m_2^2} - \frac{7}{60}\frac{k^8}{m_2^2}\Big) + F_1F_4\Big(-\frac{1}{5}\frac{k^2k_0^6}{m_2^2} + \frac{3}{10}\frac{k^4k_0^4}{m_2^2} - \frac{1}{10}\frac{k^8}{m_2^2}\Big)+$$

$$F_2^2\Big(\frac{1}{15}\frac{k^2k_0^6}{m_2^2} - \frac{1}{6}k^2k_0^4 - \frac{2}{15}\frac{k^4k_0^4}{m_2^2} + \frac{1}{6}k^4k_0^2 + \frac{23}{120}\frac{k^6k_0^2}{m_2^2} - \frac{1}{4}\frac{k^8}{m_2^2}\Big)\Big].$$

Thus, we have the main contribution (the Zemach correction) and the recoil correction.

One-photon exchange diagrams

The contribution to the interaction potential in this case has the form:



Nuclear structure effects in one-photon interaction (c) and in second order perturbation theory (d). $\tilde{G}$ is the reduced Coulomb Green's function.

$$\Delta V^{hfs}_{1\gamma,str}(r) = \frac{4\pi\alpha\mu_N}{9m_1m_2} r_M^2 (\mathbf{s_1 s_2})\nabla^2\delta(\mathbf{r}),$$

$$\Delta E^{hfs}_{1\gamma,str} = \frac{2}{3}\mu^2 Z^2\alpha^2 r_M^2 \frac{3n^2+1}{n^2} E_F(nS),$$

Numerical values can be obtained assuming $r_M^2 = r_E^2$:

$$\Delta E^{hfs}_{1\gamma,str} = \begin{cases} {}^6_3Li(1S): 3.35 \text{ meV}, & {}^6_3Li(2S): 0.34 \text{ meV}, \\ {}^7_3Li(1S): 10.67 \text{ meV}, & {}^7_3Li(2S): 1.08 \text{ meV}, \\ {}^9_4Be(1S): -17.57 \text{ meV}, & {}^9_4Be(2S): -1.78 \text{ meV}, \\ {}^{10}_5B(1S): 67.06 \text{ meV}, & {}^{10}_5B(2S): 6.81 \text{ meV}, \\ {}^{11}_5B(1S): 112.97 \text{ meV}, & {}^{11}_5B(2S): 11.47 \text{ meV}. \end{cases}$$

In second order PT we should take into account a term in which the Coulomb potential is considered as a perturbation.

$$\Delta V^C_{str,1\gamma}(k) = -\frac{Z\alpha}{k^2}\left[\frac{3}{(kR)^3}(\sin kR - kR\cos kR) - 1\right] \longrightarrow \Delta V^C_{str,1\gamma}(r) = -\frac{Z\alpha}{4R^3 r}(r-R)(r+2R)(R-r+|r-R|).$$

H.F. Hameka, Jour. Chem. Phys. 47, 2728 (1967).

The second order perturbation theory corrections to the energy spectrum are determined by the reduced Coulomb Green's functions $\tilde{G}$, which have the following form:

$$\tilde{G}_{1S}(\mathbf{r},0)=\frac{Z\alpha\mu^2}{4\pi}\frac{e^{-x}}{x}g_{1S}(x),\;\; g_{1S}(x)=\left[4x(\ln 2x+C)+4x^2-10x-2\right],$$

$$\tilde{G}_{2S}(\mathbf{r},0)=-\frac{Z\alpha\mu^2}{4\pi}\frac{e^{-x/2}}{2x}g_{2S}(x), g_{2S}(x)=\left[4x(x-2)(\ln x+C)+x^3-13x^2+6x+4\right],$$

where $C=0.5772\ldots$ is the Euler constant and $x=Wr$. Contributions to HFS in second order PT have the form:

$$\Delta E^{hfs}_{SOPT}=2<\psi|\Delta V^C_{str,1\gamma}\cdot\tilde{G}\cdot\Delta V^{hfs}_B|\psi>,$$

Using the Green's functions we perform the analytical integration in second order PT:

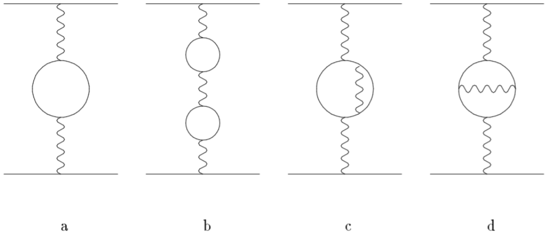
$$\Delta E^{hfs}_{str,sopt}(1S)=E_F(1S)\frac{R^2W^2}{4}\left[\frac{4}{75}(-53+15C+15\ln RW)-\frac{RW}{12}(-15+4C+4\ln RW)\right],$$

$$\Delta E^{hfs}_{str,sopt}(2S)=E_F(2S)\frac{R^2W^2}{4}\left[\frac{4}{75}(-107+60C+60\ln RW)+\frac{RW}{3}(17-8C-8\ln RW)\right],$$

where we present an expansions in (RW) up to terms of first order in square brackets ($RW(^6_3Li)=0.038$, $RW(^7_3Li)=0.036$, $RW(^9_4Be)=0.050$, $RW(^{10}_5B)=0.060$, $RW(^{11}_5B)=0.060$).

Effects of one- and two-loop vacuum polarization

The potential of one-loop vacuum polarization can be obtained in momentum representation after a standard modification of hyperfine muon-nucleus interaction due to vacuum polarization effect.



Effects of one- and two-loop vacuum polarization in one-photon interaction.

In coordinate representation it is defined by the following integral expression

$$\Delta V^{hfs}_{1\gamma,VP}(r) = \frac{4\alpha g_N(1+a_\mu)}{3m_1 m_p}(\mathbf{s_1 s_2})\frac{\alpha}{3\pi}\int_1^\infty \rho(\xi)d\xi\left[\pi\delta(\mathbf{r}) - \frac{m_e^2\xi^2}{r}e^{-2m_e\xi r}\right],$$

where $g_N = \mu_N/s_2$, $\rho(\xi) = \sqrt{\xi^2-1}(2\xi^2+1)/\xi^4$. We include in this contribution the anomalous magnetic moment of muon, which leads to the additional contribution of order α^6

Averaging the expression over wave functions, we get the contribution of order α^5 to hyperfine structure of 1S- and 2S-states ($W=\mu Z\alpha$). After integrating over particle coordinates, the results have a fairly simple form:

$$\Delta E^{hfs}_{1\gamma,VP,1s}=\frac{2\alpha^4\mu^3(1+a_\mu)\mu_N}{m_1 m_p}\frac{\alpha}{3\pi}\int_1^\infty\rho(\xi)d\xi\times\left[\ 1-\frac{{m_e}^2\xi^2}{w^2}\int_0^\infty e^{-x(1+\frac{m_e\xi}{w})}xdx\ \right]$$

$$\Delta E^{hfs}_{1\gamma,VP,2s}=\frac{\alpha^4\mu^3(1+a_\mu)\mu_N}{4m_1 m_p}\frac{\alpha}{3\pi}\int_1^\infty\rho(\xi)d\xi\times\left[\ 1-\frac{4{m_e}^2\xi^2}{w^2}\int_0^\infty e^{-x(1+\frac{2m_e\xi}{w})}(1-\frac{x}{2})^2xdx\ \right]$$

With a simple replacement m_e to muon mass m_1 one can obtain the muon vacuum polarization correction to HFS. It has higher order α^6 because the ratio $W/m_1\ll 1$.

α^6 contribution is given also by two-loop vacuum polarization diagrams:

$$\Delta V^{hfs}_{1\gamma,VP-VP}=\frac{4\pi\alpha^3 g_N(1+a_\mu)(\mathbf{s_1 s_2})}{3m_1 m_p(3\pi)^2}\int_1^\infty\rho(\xi)d\xi\int_1^\infty\rho(\eta)d\eta\times\left[\delta(\mathbf{r})-\frac{m_e^2\left(\eta^4e^{-2m_e\eta r}-\xi^4e^{-2m_e\xi r}\right)}{\pi r(\eta^2-\xi^2)}\right],$$

$$\Delta V^{hfs}_{1\gamma,2-loop\ VP}(r)=\frac{4\alpha^3 g_N(1+a_\mu)}{9\pi^2 m_1 m_p}(\mathbf{s_1 s_2})\int_0^1\frac{f(v)dv}{1-v^2}\left[\pi\delta(\mathbf{r})-\frac{m_e^2}{r(1-v^2)}e^{-\frac{2m_e r}{\sqrt{1-v^2}}}\right],$$

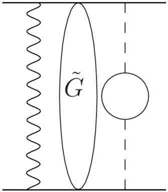
where two-loop spectral function

$$f(v)=v\Big\{(3-v^2)(1+v^2)\left[Li_2\left(-\frac{1-v}{1+v}\right)+2Li_2\left(\frac{1-v}{1+v}\right)+\frac{3}{2}\ln\frac{1+v}{1-v}\ln\frac{1+v}{2}-\ln\frac{1+v}{1-v}\ln v\right]$$
$$+\left[\frac{11}{16}(3-v^2)(1+v^2)+\frac{v^4}{4}\right]\ln\frac{1+v}{1-v}+\left[\frac{3}{2}v(3-v^2)\ln\frac{1-v^2}{4}-2v(3-v^2)\ln v\right]+\frac{3}{8}v(5-3v^2)\Big\},$$

$Li_2(z)$ the Euler dilogarithm.

Averaging two-loop vacuum polarization potentials over wave functions the integration over r can be done analytically, while two other integrations over ξ and η are calculated numerically with the use of Wolfram Mathematica.

We present summary vacuum polarization correction in first order PT in the following table:



Muonic ion	$1S$, meV	$2S$, meV
^6_3Li	5.22	0.67
^7_3Li	18.54	2.38
^9_4Be	-17.97	-2.30
$^{10}_5B$	50.99	6.51
$^{11}_5B$	87.31	11.14

To achieve the desired accuracy of calculations one-loop and two-loop contributions to HFS have to be taken into account in second order perturbation theory. The main contribution of the electron vacuum polarization to HFS in second order PT has the form:

$$\Delta E^{hfs}_{SOPT\ VP\ 1} = 2 < \psi|\Delta V^C_{VP} \cdot \tilde{G} \cdot \Delta V^{hfs}_B|\psi >,$$

where the Coulomb potential, modified by the one-loop vacuum polarization effect, has the form:

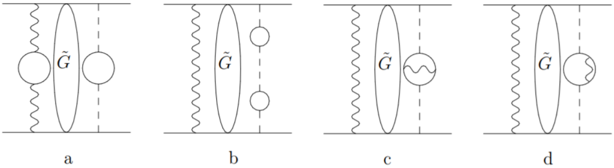
$$\Delta V^C_{VP}(r) = \frac{\alpha}{3\pi}\int_1^\infty \rho(\xi)d\xi\left(-\frac{Z\alpha}{r}\right)e^{-2m_e\xi r}.$$

As a result necessary corrections to HFS can be presented as follows:

$$\Delta E^{hfs}_{VP\ 1}(1S) = -E_F(1S)\frac{\alpha}{3\pi}(1+a_\mu)\int_1^\infty \rho(\xi)d\xi\int_0^\infty e^{-x\left(1+\frac{m_e\xi}{W}\right)}g_{1S}(x)dx,$$

$$\Delta E^{hfs}_{VP\ 1}(2S) = E_F(2S)\frac{\alpha}{3\pi}(1+a_\mu)\int_1^\infty \rho(\xi)d\xi\int_0^\infty e^{-x\left(1+\frac{2m_e\xi}{W}\right)}g_{2S}(x)(1-\frac{x}{2})dx.$$

Two-loop corrections of order α^6:



Effects of two-loop vacuum polarization in second order PT.

Let us consider first contribution (a). General structure of this contribution:

$$\Delta E^{hfs}_{SOPT\ VP\ 2} = 2 < \psi|\Delta V^C_{VP} \cdot \tilde{G} \cdot \Delta V^{hfs}_B|\psi > .$$

The convenient representation for reduced Coulomb Green's function with nonzero arguments has form:

$$\tilde{G}_{1S}(r, r') = -\frac{Z\alpha\mu^2}{\pi} e^{-(x_1+x_2)} g_{1S}(x_1, x_2), \quad \tilde{G}_{2S}(r, r') = -\frac{Z\alpha\mu^2}{16\pi x_1 x_2} e^{-(x_1+x_2)} g_{2S}(x_1, x_2),$$

$$g_{1S}(x_1, x_2) = \frac{1}{2x_<} - \ln 2x_> - \ln 2x_< + Ei(2x_<) + \frac{7}{2} - 2C - (x_1 + x_2) + \frac{1 - e^{2x_<}}{2x_<},$$

$$g_{2S}(x_1, x_2) = 8x_< - 4x_<^2 + 8x_> + 12x_< x_> - 26x_<^2 x_> + 2x_<^3 x_> - 4x_>^2 - 26x_< x_>^2 + 23x_<^2 x_>^2 - x_<^3 x_>^2 +$$
$$+2x_< x_>^3 - x_<^2 x_>^3 + 4e^x(1 - x_<)(x_> - 2)x_> + 4(x_< - 2)x_<(x_> - 2)x_>[-2C + Ei(x_<) - \ln(x_<) - \ln(x_>)],$$

where $x_< = min(x_1, x_2)$, $x_> = max(x_1, x_2)$, $Ei(x)$ is the integral exponential function.

The substitution provides two terms for each 1-S and 2-S level in integral form:

$$\Delta E^{hfs}_{VP\ 21}(1S)=-\frac{2\alpha^6\mu^3 g_N(1+a_\mu)}{9\pi^2 m_1 m_p}\int_1^\infty \rho(\xi)d\xi\int_1^\infty \rho(\eta)d\eta\int_0^\infty dx e^{-x\left(1+\frac{m_e\xi}{W}\right)} g_{1S}(x),$$

$$\Delta E^{hfs}_{VP\ 22}(1S)=-\frac{4\alpha^6\mu^3 g_N(1+a_\mu)m_e^2}{9\pi^2 m_1 m_p W^2}\int_1^\infty \rho(\xi)\rho(\eta)\eta^2 d\xi d\eta\int_0^\infty x_1 x_2 e^{-(x_1+x_2)\left(1+\frac{m_e\xi}{W}\right)} dx_1 dx_2 g_{1S}(x_1,x_2),$$

$$\Delta E^{hfs}_{VP\ 21}(2S)=\frac{\alpha^6\mu^3 g_N(1+a_\mu)}{36\pi^2 m_1 m_p}\int_1^\infty \rho(\xi)d\xi\int_1^\infty \rho(\eta)d\eta\int_0^\infty \left(1-\frac{x}{2}\right) dx e^{-x\left(1+\frac{2m_e\xi}{W}\right)} g_{2S}(x),$$

$$\Delta E^{hfs}_{VP\ 22}(2S)=-\frac{\alpha^6\mu^3 g_N(1+a_\mu)m_e^2}{18\pi^2 m_1 m_p W^2}\int_1^\infty \rho(\xi)d\xi\int_1^\infty \rho(\eta)\eta^2 d\eta\times$$

$$\times\int_0^\infty \left(1-\frac{x_1}{2}\right) dx_1 e^{-x_1\left(1+\frac{2m_e\xi}{W}\right)}\int_0^\infty \left(1-\frac{x_2}{2}\right) dx_2 e^{-x_2\left(1+\frac{2m_e\xi}{W}\right)} g_{2S}(x_1,x_2).$$

Separately, the contributions are divergent but their sum is finite. Corresponding numerical values are

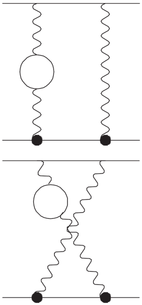
Muonic ion	$1S$, meV	$2S$, meV
^6_3Li	0.05	0.01
^7_3Li	0.20	0.02
^9_4Be	-0.21	-0.02
$^{10}_5B$	0.65	0.06
$^{11}_5B$	1.11	0.11

The contributions of other amplitudes (b, c, d) to HFS can be calculated in the same way with the replacement with the following potentials:

$$\Delta V^{C}_{VP-VP}(r)=\left(\frac{\alpha}{3\pi}\right)^2\int_1^\infty\rho(\xi)d\xi\int_1^\infty\rho(\eta)d\eta\left(-\frac{Z\alpha}{r}\right)\frac{1}{\xi^2-\eta^2}\left(\xi^2e^{-2m_e\xi r}-\eta^2e^{-2m_e\eta r}\right),$$

$$\Delta V^{C}_{2-loop\ VP}(r)=-\frac{2Z\alpha^3}{3\pi^2 r}\int_0^1\frac{f(v)dv}{(1-v^2)}e^{-\frac{2m_e r}{\sqrt{1-v^2}}}.$$

Omitting further intermediate expressions we include total numerical values of two-loop vacuum polarization corrections in second order PT to final value of HFS.



There is another correction for the polarization of the vacuum, which also includes the effect of the nuclear structure. To calculate it, it is necessary to use the potential $V_{2\gamma}(k)$, modifying it accordingly.

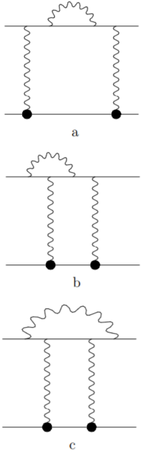
$$\frac{1}{k^2}\rightarrow\frac{\alpha}{3\pi}\int_1^\infty\rho(\xi)d\xi\frac{1}{k^2+4m_e^2\xi^2}.$$

As a result, the contribution to the HFS spectrum is determined by the following expression (the factor 2 corresponds to two exchange photons):

$$\Delta E^{hfs}_{2\gamma,VP}=-\frac{2\mu^3Z^3\alpha^4}{9\pi^2n^3}\int\frac{V_{2\gamma(k)d^4k}}{k^3}[5k^3-12m_ek^2-$$

$$-6(k^2-2m_e^2)\sqrt{k^2+4m_e^2}Arcth[\frac{k}{\sqrt{k^2+4m_e^2}}]].$$

Radiative corrections to two-photon exchange diagrams



Wave line on the diagram denotes the photon. Bold point on the diagram denotes the vertex operator of the nucleus.

The results already clearly show that the corrections to the structure of the nucleus are dominant. Thus, it seems useful to consider another correction for the structure of the nucleus of order α^6 to refine the results. Three types of contributions to hyperfine splitting are expressed in integral form:

$$\Delta E^{hfs}_{se} = E_F 6 \frac{\alpha(Z\alpha)}{\pi^2} \int_0^1 x dx \int_0^\infty \frac{F_1(k^2) F_3(k^2) dk}{x + (1-x)k^2},$$

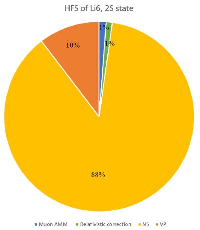
$$\Delta E^{hfs}_{vertex-1} = -E_F 24 \frac{\alpha(Z\alpha)}{\pi^2} \int_0^1 dz \int_0^1 x dx \int_0^\infty \frac{F_1(k^2) F_3(k^2) \ln[\frac{x + k^2 z(1-xz)}{x}] dk}{k^2},$$

$$\Delta E^{hfs}_{vertex-2} = E_F 8 \frac{\alpha(Z\alpha)}{\pi^2} \int_0^1 dz \int_0^1 dx \int_0^\infty \frac{dk}{k^2} \left\{ \frac{F_1(k^2) F_3(k^2)}{[x + k^2 z(1-xz)]^2} \times \right.$$
$$\left. \times \left[-2xz^2(1-xz)k^4 + zk^2(3x^3 z - x^2(9z+1) + x(4z+7) - 4) + x^2(5-x) \right] - \frac{1}{2} \right\},$$

$$\Delta E^{hfs}_{jellyfish} = E_F 4 \frac{\alpha(Z\alpha)}{\pi^2} \int_0^1 (1-z) dz \int_0^1 (1-x) dx \times$$
$$\times \int_0^\infty \frac{F_1(k^2) F_3(k^2) dk}{[x + (1-x)k^2]^3} \Big[6x + 6x^2 - 6x^2 z + 2x^3 - 12x^3 z - 12x^4 z +$$
$$+ k^2(-6z + 18xz + 4xz^2 + 7x^2 z - 30x^2 z^2 - 2x^2 z^3 - 36x^3 z^2 +$$
$$+ 12x^3 z^3 + 24x^4 z^3) + k^4(9xz^2 - 31x^2 z^3 + 34x^3 z^4 - 12x^4 z^5 \Big].$$

Conclusion

In this work we carry out a calculation of S-states hyperfine splitting in a number of muonic ions.



Muonic ion	$1S$, meV	$2S$, meV
^6_3Li	1325.02	164.65
^7_3Li	4693.40	583.38
^9_4Be	-4080.71	-505.82
$^{10}_5B$	10233.86	1265.03
$^{11}_5B$	17572.70	2172.56

Corrections to the structure of the nucleus, which are determined by two-photon exchange amplitudes play a very important role in achieving high accuracy of calculation. They are defined in our approach by the electromagnetic form factors of the nuclei, which in this case must be taken from experimental data. The results of calculating various corrections are presented with an accuracy of 0.01 meV. But this does not mean that the accuracy of our calculation is that high. Unfortunately, due to the electromagnetic form factors of the nuclei, can be about 1 percent of the correction to the structure of the nucleus of the order α^5.

Light muonic ions of the lithium, beryllium and boron can be used in experiments of the CREMA (Charge Radius Experiment with Muonic Atoms) collaboration. Intensive experimental studies of the scattering of leptons by light nuclei were carried out several years ago. The results obtained then are reflected. We use these results, although the accuracy of determining all the required form factors is not very high, as would be desirable. For this reason, we use different parameterizations (Gaussian, uniformly charged sphere) for form factors and compared the numerical results for them to understand how they can differ.

The hyperfine structure of muonic ions of lithium, beryllium and boron was investigated previously in paper:

Drake G.W.F., Byer L.L. Lamb shifts and fine-structure splittings for the muonic ions μLi, μBe, and μB: A proposed experiment //Phys. Rev. A. 1985. V. 32. P. 713-719.

The authors gave only estimates of basic contributions in hyperfine structure. In this work we improve their results accounting for different corrections.

Thanks for your attention!