One-Loop Amplitudes of Charged Fermions in Constant Homogeneous Electromagnetic Field

Ilya Karabanov, Alexandra Dobrynina,
Alexander Parkhomenko & Lubov Vassilevskaya

Department of Theoretical Physics
P. G. Demidov Yaroslavl State University

The IV International Conference
on Particle Physics and Astrophysics
National Research Nuclear University "MEPhI" , Moscow,
Russia, 22–26 October 2018

Outline

Introduction: General Case of Two-Point Correlator

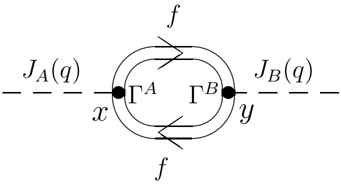
[M. Yu. Borovkov et al., Phys. At. Nucl. 62 (1999) 1601]

- Lagrangian density of local fermion interaction

$$\mathcal{L}_{\text{int}}(x) = \left[\bar{f}(x)\Gamma^A f(x)\right] J_A(x)$$

- J_A — generalized current (photon, neutrino current, etc.)
- Γ_A — any of γ-matrices from the set $\{1,\, \gamma_5,\, \gamma_\mu,\, \gamma_\mu\gamma_5, \sigma_{\mu\nu} = i\left[\gamma_\mu, \gamma_\nu\right]/2\}$
- Interaction constants are included into the current J_A

Introduction: General Case of Two-Point Correlator



- Two-point correlation function of general form

$$\Pi_{AB} = \int d^4X \, \mathrm{e}^{-i(qX)} \, \mathrm{Sp}\left\{ S_{\mathrm{F}}(-X) \, \Gamma_A \, S_{\mathrm{F}}(X) \, \Gamma_B \right\}$$

- $S_{\mathrm{F}}(X)$ — Lorentz-invariant part of exact fermion propagator
- $X^\mu = x^\mu - y^\mu$ — integration variable
- Correlations of scalar, pseudoscalar, vector and axial-vector currents were studied by Borovkov et al. [Phys. At. Nucl. 62 (1999) 1601]
- Consider correlations of a tensor current with the other ones

Basic Tensors in Presence of Magnetic Field

- Minkowski space filled with external magnetic field is divided into two subspaces:
  - Euclidean with the metric tensor $\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu}$; plane orthogonal to the field strength vector
  - Pseudo-Euclidean with the metric tensor $\tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$
  - Metric tensor of Minkowski space $g_{\mu\nu} = \tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu}$
- Dimensionless tensor of the external magnetic field and its dual

$$\varphi_{\alpha\beta} = \frac{F_{\alpha\beta}}{B}\,, \qquad \tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\,\varepsilon_{\alpha\beta\rho\sigma}\varphi^{\rho\sigma}$$

- Arbitrary four-vector $a^\mu = (a_0, a_1, a_2, a_3)$ can be decomposed into two orthogonal components

$$a_\mu = \tilde{\Lambda}_{\mu\nu}a^\nu - \Lambda_{\mu\nu}a^\nu = a_{\parallel\mu} - a_{\perp\mu}$$

- For the scalar product of two four-vectors one has

$$(ab) = (ab)_\parallel - (ab)_\perp$$

$$(ab)_\parallel = (a\tilde{\Lambda}b) = a^\mu\tilde{\Lambda}_{\mu\nu}b^\nu, \quad (ab)_\perp = (a\Lambda b) = a^\mu\Lambda_{\mu\nu}b^\nu$$

Propagator in the Fock-Schwinger Representation

- General representation of the propagator [Itzikson & Zuber]

$$G_{\mathrm{F}}(x,y) = \mathrm{e}^{i\Omega(x,y)}\, S_{\mathrm{F}}(x-y)$$

- Lorentz non-invariant phase factor

$$\Omega(x,y) = -eQ_f \int_y^x d\xi^\mu \left[A_\mu(\xi) + \frac{1}{2} F_{\mu\nu}(\xi - y)^\nu\right]$$

- In two-point correlation function phase factors canceled

$$\Omega(x,y) + \Omega(y,x) = 0$$

- Lorentz-invariant part of the fermion propagator ($\beta = eB|Q_f|$)

$$\begin{aligned} S_{\mathrm{F}}(X) &= -\frac{i\beta}{2(4\pi)^2} \int_0^\infty \frac{ds}{s^2} \left\{ (X\widetilde{\Lambda}\gamma)\cot(\beta s) - i(X\widetilde{\varphi}\gamma)\gamma_5 - \right. \\ &- \left. \frac{\beta s}{\sin^2(\beta s)} (X\Lambda\gamma) + m_f s\, [2\cot(\beta s) + (\gamma\varphi\gamma)] \right\} \times \\ &\times \exp\left(-i\left[m_f^2 s + \frac{1}{4s}(X\widetilde{\Lambda}X) - \frac{\beta\cot(\beta s)}{4}(X\Lambda X)\right]\right) \end{aligned}$$

Orthogonal Basis Motivated by Magnetic Field

- Correlators having rank non-equal to zero, should be decomposed in some orthogonal set of vectors
- In magnetic field, such a basis naturally exists

$$b_\mu^{(1)} = (q\varphi)_\mu, \qquad b_\mu^{(2)} = (q\tilde{\varphi})_\mu$$
$$b_\mu^{(3)} = q^2 (\Lambda q)_\mu - (q\Lambda q)\, q_\mu, \quad b_\mu^{(4)} = q_\mu$$

- Arbitrary vector a_μ can be presented as

$$a_\mu = \sum_{i=1}^{4} a_i \frac{b_\mu^{(i)}}{(b^{(i)} b^{(i)})}, \qquad a_i = a^\mu b_\mu^{(i)}$$

- Third-rank tensor $T_{\mu\nu\rho}$ can be decomposed similarly

$$T_{\mu\nu\rho} = \sum_{i,j,k=1}^{4} T_{ijk} \frac{b_\mu^{(i)} b_\nu^{(j)} b_\rho^{(k)}}{(b^{(i)} b^{(i)})(b^{(j)} b^{(j)})(b^{(k)} b^{(k)})},$$

$$T_{ijk} = T^{\mu\nu\rho} b_\mu^{(i)} b_\nu^{(j)} b_\rho^{(k)}.$$

Correlator of Vector and Tensor Currents

- Example: correlation function of vector and tensor currents
- Correlator of vector and tensor currents is rank-3 tensor
- Vector-current conservation and anti-symmetry of the tensor current reduce the number of independent coefficients in the basis decomposition to 18
- From them, four coefficients only are non-trivial
- Double-integral representation of coefficients is used

$$\Pi_{ijk}^{(\mathrm{VT})}(q^2, q_\perp^2, \beta) = \frac{1}{4\pi^2} \int_0^\infty \frac{dt}{t} \int_0^1 du\, Y_{ijk}^{(\mathrm{VT})}(q^2, q_\perp^2, \beta; t, u)$$
$$\times \exp\left\{ -i \left[m_f^2 t - \frac{q_\parallel^2}{4}\, t\,(1-u^2) + q_\perp^2\, \frac{\cos(\beta t u) - \cos(\beta t)}{2\beta \sin(\beta t)} \right] \right\}$$

- Integration variables and relation between momenta squared $t = s_1 + s_2,\ u = (s_1 - s_2)/(s_1 + s_2); \quad q_\parallel^2 = q^2 + q_\perp^2$

Integrands of Vector-Tensor Correlator

$$Y^{(\mathrm{VT})}_{114}(q^2, q_\perp^2, \beta; t, u) = -Y^{(\mathrm{VT})}_{141}(q^2, q_\perp^2, \beta; t, u) == -m_f\, q_\perp^2\, q^2\, \frac{\beta t \cos(\beta t u)}{\sin(\beta t)}$$

$$Y^{(\mathrm{VT})}_{223}(q^2, q_\perp^2, \beta; t, u) = -Y^{(\mathrm{VT})}_{232}(q^2, q_\perp^2, \beta; t, u) = \\ = m_f\, q_\perp^2\, (q_\parallel^2)^2\, \frac{\beta t}{\sin(\beta t)}\, [\cos(\beta t) - \cos(\beta t u)]$$

$$Y^{(\mathrm{VT})}_{224}(q^2, q_\perp^2, \beta; t, u) = -Y^{(\mathrm{VT})}_{242}(q^2, q_\perp^2, \beta; t, u) = \\ = m_f\, q_\parallel^2\, \frac{\beta t}{\sin(\beta t)}\, \left[q_\perp^2 \cos(\beta t) - q_\parallel^2 \cos(\beta t u)\right]$$

$$Y^{(\mathrm{VT})}_{334}(q^2, q_\perp^2, \beta; t, u) = -Y^{(\mathrm{VT})}_{343}(q^2, q_\perp^2, \beta; t, u) == -m_f\, q_\perp^2\, q_\parallel^2\, (q^2)^2\, \frac{\beta t \cos(\beta t u)}{\sin(\beta t)}$$

- Choice of basic vectors is dictated by the conservation of the vector current and $Y^{(\mathrm{VT})}_{4jk}$ vanish in this basis
- Anti-symmetry in last two indices are due to the tensor current

Crossed-Field Limit

- Field parameter vanishes $(\beta_f \to 0)$
- As basic vectors, it is convenient to accept the following orthonormalized set

$$b_\mu^{(1)} = \frac{e_f}{\chi_f}(qF)_\mu, \qquad b_\mu^{(2)} = \frac{e_f}{\chi_f}(q\tilde{F})_\mu$$

$$b_\mu^{(3)} = \frac{e_f^2}{\chi_f\sqrt{q^2}}\left[q^2\,(qFFq)_\mu - (qFFq)\,q_\mu\right], \quad b_\mu^{(4)} = \frac{q_\mu}{\sqrt{q^2}}$$

- Dynamical parameter: $\chi_f^2 = e_f^2\,(qFFq) = \beta_f^2 q_\perp^2$
- Coefficients of the vector-tensor correlator in this basis:

$$\Pi_{ijk}^{(VT)}(q^2, \chi_f) = \frac{1}{4\pi^2}\int_0^\infty \frac{dt}{t}\int_0^1 du\, Y_{ijk}^{(VT)}(q^2, \chi_f; t, u)$$

$$\times \exp\left\{-i\left[\left(m_f^2 - \frac{q^2}{4}(1-u^2)\right)t + \frac{1}{48}\chi_f^2(1-u^2)^2 t^3\right]\right\}$$

Vector-Tensor Correlator Integrands in Crossed Fields

- Results for integrands in external electromagnetic crossed fields

$$
\begin{aligned}
Y_{114}^{(\mathrm{VT})} &= -Y_{141}^{(\mathrm{VT})} = -m_f \sqrt{q^2} \\
Y_{223}^{(\mathrm{VT})} &= -Y_{232}^{(\mathrm{VT})} = m_f \frac{\chi_f^2 t^2}{2\sqrt{q^2}} \left(1 - u^2\right) \\
Y_{224}^{(\mathrm{VT})} &= -Y_{242}^{(\mathrm{VT})} = -m_f \sqrt{q^2} \left[1 + \frac{\chi_f^2 t^2}{2q^2} \left(1 - u^2\right)\right] \\
Y_{334}^{(\mathrm{VT})} &= -Y_{343}^{(\mathrm{VT})} = -m_f \sqrt{q^2}
\end{aligned}
$$

Applications of Correlators

- Polarization operator is related with correlator of two vector currents
- Contribution linear in the fermion AMM is related with correlator of vector and tensor currents
- Its influence on photon requires detail discussion

Conclusions

- Two-point correlators in presence of constant homogeneous external magnetic field are considered
- This analysis extended the previous one by inclusion of tensor currents into consideration
- The research of correlators of tensor fermionny current with the others allows to investigate the effects arising at the expense of the abnormal magnetic moment of fermion