One-Loop Amplitudes of Charged Fermions in Constant Homogeneous Electromagnetic Field

Ilya Karabanov, Alexandra Dobrynina, Alexander Parkhomenko & Lubov Vassilevskaya

Department of Theoretical Physics
P. G. Demidov Yaroslavl State University

The IV International Conference on Particle Physics and Astrophysics
National Research Nuclear University "MEPhI", Moscow, Russia, 22–26 October 2018
Outline

1. Introduction
2. Correlators in Constant Homogeneous Magnetic Field
3. Correlators in the Crossed-Field Limit
4. Applications of Correlators
5. Conclusions
Lagrangian density of local fermion interaction

\[ \mathcal{L}_{\text{int}}(x) = \left[ \bar{f}(x) \Gamma^A f(x) \right] J_A(x) \]

- \( J_A \) — generalized current (photon, neutrino current, etc.)
- \( \Gamma_A \) — any of \( \gamma \)-matrices from the set
  \[ \{ 1, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu} = i [\gamma_\mu, \gamma_\nu] / 2 \} \]
- Interaction constants are included into the current \( J_A \)
Two-point correlation function of general form

\[ \Pi_{AB} = \int d^4 X \ e^{-i(qX)} \ S_F \{ S_F(-X) \Gamma_A \ S_F(X) \Gamma_B \} \]

- \( S_F(X) \) — Lorentz-invariant part of exact fermion propagator
- \( X^\mu = x^\mu - y^\mu \) — integration variable
- Correlations of scalar, pseudoscalar, vector and axial-vector currents were studied by Borovkov et al. [Phys. At. Nucl. 62 (1999) 1601]
- Consider correlations of a tensor current with the other ones
Basic Tensors in Presence of Magnetic Field

- Minkowski space filled with external magnetic field is divided into two subspaces:
  - Euclidean with the metric tensor \( \Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu} \);
  - plane orthogonal to the field strength vector.
  - Pseudo-Euclidean with the metric tensor \( \tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu} \).
  - Metric tensor of Minkowski space \( g_{\mu\nu} = \tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu} \).
- Dimensionless tensor of the external magnetic field and its dual:
  \[ \varphi_{\alpha\beta} = \frac{F_{\alpha\beta}}{B}, \quad \tilde{\varphi}_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\rho\sigma} \varphi^{\rho\sigma} \]
- Arbitrary four-vector \( a^\mu = (a_0, a_1, a_2, a_3) \) can be decomposed into two orthogonal components:
  \[ a_\mu = \tilde{\Lambda}_{\mu\nu} a^\nu - \Lambda_{\mu\nu} a^\nu = a_{||\mu} - a_{\bot\mu} \]
- For the scalar product of two four-vectors one has:
  \[ (ab) = (ab)_{||} - (ab)_{\bot} \]
  \[ (ab)_{||} = (a\tilde{\Lambda}b) = a^\mu \tilde{\Lambda}_{\mu\nu} b^\nu, \quad (ab)_{\bot} = (a\Lambda b) = a^\mu \Lambda_{\mu\nu} b^\nu \]
Propagator in the Fock-Schwinger Representation

- General representation of the propagator [Itzikson & Zuber]
  \[ G_F(x, y) = e^{i\Omega(x, y)} S_F(x - y) \]
- Lorentz non-invariant phase factor
  \[ \Omega(x, y) = -eQ_f \int_y^x d\xi^\mu \left[ A_\mu(\xi) + \frac{1}{2} F_{\mu\nu}(\xi - y)^\nu \right] \]
- In two-point correlation function phase factors canceled
  \[ \Omega(x, y) + \Omega(y, x) = 0 \]
- Lorentz-invariant part of the fermion propagator (\( \beta = eB|Q_f| \))
  \[ S_F(X) = -\frac{i\beta}{2(4\pi)^2} \int_0^\infty \frac{ds}{s^2} \left\{ (X\bar{\Lambda}\gamma) \cot(\beta s) - i(X\bar{\varphi}\gamma)\gamma_5 - \right. \]
  \[ \left. - \frac{\beta s}{\sin^2(\beta s)} (X\Lambda\gamma) + m_f s [2 \cot(\beta s) + (\gamma\varphi\gamma)] \right\} \times \]
  \[ \exp \left( -i \left[ m_f^2 s + \frac{1}{4s} (X\bar{\Lambda}X) - \frac{\beta \cot(\beta s)}{4} (X\Lambda X) \right] \right) \]
Orthogonal Basis Motivated by Magnetic Field

- Correlators having rank non-equal to zero, should be decomposed in some orthogonal set of vectors.
- In magnetic field, such a basis naturally exists:
  
  \[
  b^{(1)}_{\mu} = (q\varphi)_{\mu}, \quad b^{(2)}_{\mu} = (q\bar{\varphi})_{\mu}
  \]
  
  \[
  b^{(3)}_{\mu} = q^2 (\Lambda q)_{\mu} - (q\Lambda q) q_{\mu}, \quad b^{(4)}_{\mu} = q_{\mu}
  \]

- Arbitrary vector \( a_{\mu} \) can be presented as:
  
  \[
  a_{\mu} = \sum_{i=1}^{4} a_i \frac{b^{(i)}_{\mu}}{(b(i)b(i))}, \quad a_i = a^{\mu} b^{(i)}_{\mu}
  \]

- Third-rank tensor \( T_{\mu\nu\rho} \) can be decomposed similarly:
  
  \[
  T_{\mu\nu\rho} = \sum_{i,j,k=1}^{4} T_{ijk} \frac{b^{(i)}_{\mu} b^{(j)}_{\nu} b^{(k)}_{\rho}}{(b(i)b(i))(b(j)b(j))(b(k)b(k))},
  \]
  
  \[
  T_{ijk} = T^{\mu\nu\rho} b^{(i)}_{\mu} b^{(j)}_{\nu} b^{(k)}_{\rho}.
  \]
Correlator of Vector and Tensor Currents

- Example: correlation function of vector and tensor currents
- Correlator of vector and tensor currents is rank-3 tensor
- Vector-current conservation and anti-symmetry of the tensor current reduce the number of independent coefficients in the basis decomposition to 18
- From them, four coefficients only are non-trivial
- Double-integral representation of coefficients is used

\[
\Pi^{(VT)}_{ijk}(q^2, q^2_\perp, \beta) = \frac{1}{4\pi^2} \int_0^\infty \frac{dt}{t} \int_0^1 du \ Y^{(VT)}_{ijk}(q^2, q^2_\perp, \beta; t, u) \\
\times \exp \left\{ -i \left[ m_f^2 t - \frac{q^2_\parallel}{4} t (1 - u^2) + q^2_\perp \frac{\cos(\beta tu) - \cos(\beta t)}{2\beta \sin(\beta t)} \right] \right\}
\]

- Integration variables and relation between momenta squared
  \[ t = s_1 + s_2, \ u = (s_1 - s_2)/(s_1 + s_2); \quad q^2_\parallel = q^2 + q^2_\perp \]
Integrands of Vector-Tensor Correlator

\[ Y^{(VT)}_{114}(q^2, q^2_\perp, \beta; t, u) = - Y^{(VT)}_{141}(q^2, q^2_\perp, \beta; t, u) = - m_f q^2_\perp \frac{\beta t \cos(\beta tu)}{\sin(\beta t)} \]

\[ Y^{(VT)}_{223}(q^2, q^2_\perp, \beta; t, u) = - Y^{(VT)}_{232}(q^2, q^2_\perp, \beta; t, u) = m_f q^2_\perp (q_\parallel^2)^2 \frac{\beta t}{\sin(\beta t)} [\cos(\beta t) - \cos(\beta tu)] \]

\[ Y^{(VT)}_{224}(q^2, q^2_\perp, \beta; t, u) = - Y^{(VT)}_{242}(q^2, q^2_\perp, \beta; t, u) = m_f q_\parallel^2 \frac{\beta t}{\sin(\beta t)} [q^2_\perp \cos(\beta t) - q_\parallel^2 \cos(\beta tu)] \]

\[ Y^{(VT)}_{334}(q^2, q^2_\perp, \beta; t, u) = - Y^{(VT)}_{343}(q^2, q^2_\perp, \beta; t, u) = - m_f q^2_\perp q_\parallel^2 (q^2)^2 \frac{\beta t \cos(\beta tu)}{\sin(\beta t)} \]

- Choice of basic vectors is dictated by the conservation of the vector current and \( Y^{(VT)}_{4jk} \) vanish in this basis
- Anti-symmetry in last two indices are due to the tensor current
Crossed-Field Limit

- Field parameter vanishes \((\beta_f \to 0)\)
- As basic vectors, it is convenient to accept the following orthonormalized set

\[ b_{(1)}^{\mu} = \frac{e_f}{\chi_f} (qF)_\mu, \quad b_{(2)}^{\mu} = \frac{e_f}{\chi_f} (q\bar{F})_\mu \]

\[ b_{(3)}^{\mu} = \frac{e_f^2}{\chi_f \sqrt{q^2}} \left[ q^2 (qFFq)_\mu - (qFFq) q_\mu \right], \quad b_{(4)}^{\mu} = \frac{q_\mu}{\sqrt{q^2}} \]

- Dynamical parameter: \(\chi^2_f = e_f^2 (qFFq) = \beta^2_f q^2_\perp\)

- Coefficients of the vector-tensor correlator in this basis:

\[
\Pi_{ijk}^{(VT)}(q^2, \chi_f) = \frac{1}{4\pi^2} \int_0^1 \frac{dt}{t} \int_0^1 du \ Y_{ijk}^{(VT)}(q^2, \chi_f; t, u) \\
\times \exp \left\{ -i \left[ \left( m^2_f - \frac{q^2}{4} (1 - u^2) \right) t + \frac{1}{48} \chi_f^2 (1 - u^2)^2 t^3 \right] \right\}
\]
Results for integrands in external electromagnetic crossed fields

\[ Y^{(VT)}_{114} = -Y^{(VT)}_{141} = -m_f \sqrt{q^2} \]

\[ Y^{(VT)}_{223} = -Y^{(VT)}_{232} = m_f \frac{x_f^2 t^2}{2 \sqrt{q^2}} (1 - u^2) \]

\[ Y^{(VT)}_{224} = -Y^{(VT)}_{242} = -m_f \sqrt{q^2} \left[ 1 + \frac{x_f^2 t^2}{2q^2} (1 - u^2) \right] \]

\[ Y^{(VT)}_{334} = -Y^{(VT)}_{343} = -m_f \sqrt{q^2} \]
Applications of Correlators

- Polarization operator is related with correlator of two vector currents
- Contribution linear in the fermion AMM is related with correlator of vector and tensor currents
- Its influence on photon requires detail discussion
Conclusions

- Two-point correlators in presence of constant homogeneous external magnetic field are considered.
- This analysis extended the previous one by inclusion of tensor currents into consideration.
- The research of correlators of tensor fermionny current with the others allows to investigate the effects arising at the expense of the abnormal magnetic moment of fermion.