Reconstruction of particle's energy spectrum in experiment with Unfolding technique

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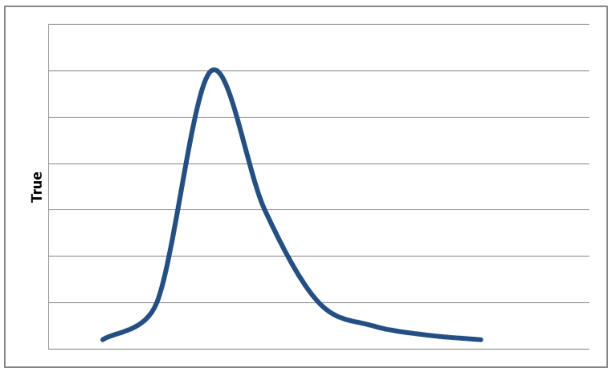
Plan

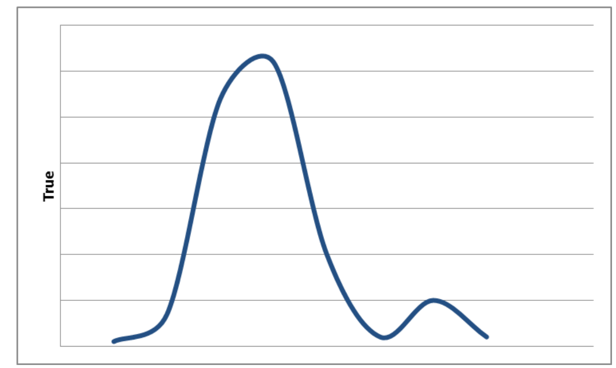
1. Main problem and direct method of solving
2. Unfolding methods overview
3. Software implementation: ROOT, RooUnfold
4. Computational experiments results: comparison

Main problem

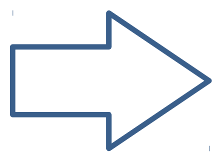
True value

Measured value

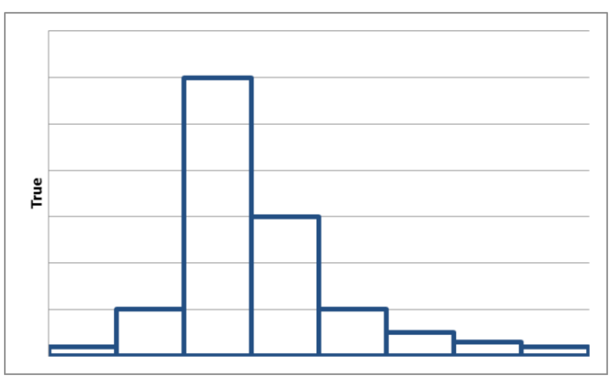


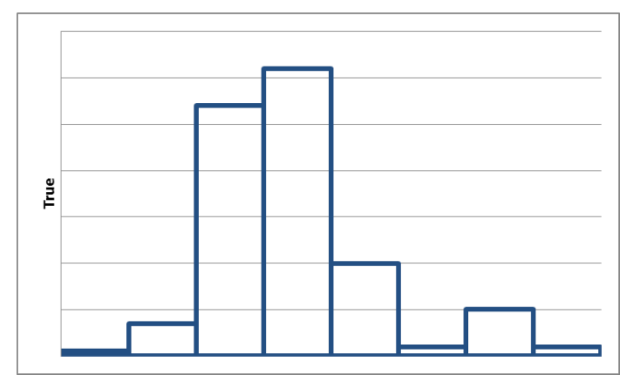


True distribution



Measured distribution





Main problem

Measurement
(folding)

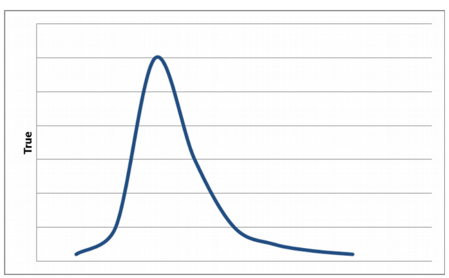
True distribution

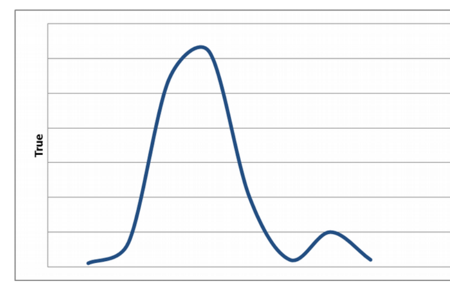
$f_{true}(x)$

Measured distribution

$f_{meas}(y)$

$$f_{meas}(y) = \int_{\Omega} K(x,y)\, f_{true}(x)\, dx$$





?

Spectrum unfolding
(deconvolution)

Main problem
(sampling)

$$\nu_i = \sum_j R_{ij} \tau_j$$

$\tau = (\tau_1, \tau_2, ..., \tau_k)$ **— true distribution**

$\mu = (\mu_1, \mu_2, ..., \mu_n)$ **— measured distribution**

$$M\mu = \nu = (\nu_1, \nu_2, ..., \nu_n)$$

Unfolding

“Naive” method

$$\nu = R\tau$$

τ — true distribution

μ — measured distribution

$$M\mu = \nu$$

$$\nu = \mu$$

$$R\tau = \mu$$

$$\Downarrow$$

$$\tau = R^{-1}\mu$$

Unfolding

Maximum likelihood estimation

$$L(\tau) = \prod_i P(\mu_i, \nu_i(\tau)) = \prod_i e^{-\nu_i} \frac{\nu_i^{\mu_i}}{\mu_i!} \rightarrow \max_\tau$$

True distribution (spectrum) estimation: $\hat{\tau} = R^{-1}\mu$

(the same as in “naive” method)

Properties: consistent and unbiased estimator, minimal error for unbiased estimators class.

Problems: unstable for μ fluctuations, too large statistical errors.

Conclusion: biased estimators (add systematic bias).

Unfolding methods ideas

1. Correction by multiplicative coefficients
   (for measured spectrum)

2. Bayesian methods
   (D Agostini methods + modifications)

3. Metods based on regularization

1. Tikhonov’s regularization + **SVD** (Kartvelishvili, Hoecker)
2. Tikhonov’s regularization , bias limitation + **L-curve** (Schmitt)
3. regularization based on entropy

Bayesian method

(D Agostini)

$(C_1, C_2, ..., C_k)$ — causes — particle is in the given energy interval (bin)

$(E_1, E_2, ..., E_n)$ — effects — particle is detected in the given energy interval (bin)

$R_{ij} = P(E_i \mid C_j)$ — migration matrix

Basic idea

$$M_{ji} = P(C_j \mid E_i) = \frac{P(E_i \mid C_j) P(C_j)}{\sum_k P(E_i \mid C_k) P(C_k)}$$

$$\hat{\tau} = M\mu$$

Iterative bayesian method (D Agostini)

$$R_{ij} = P(E_i \mid C_j)$$ — migration matrix

$$\varepsilon_i = \sum_k P(E_i \mid C_k)$$ — efficiency

1. Initialization: $$P(C_j) = P_0(C_j)$$

2. Iteration: $$M_{ji} = \frac{R_{ij} P(C_j)}{\varepsilon_i \sum_k R_{ik} P(C_k)}$$

$$\hat{\tau} = M\mu \qquad P(C_j) = \hat{\tau}_j / \sum_k \hat{\tau}_k$$

3. Evaluate errors, repeat step 2 if necessary.

G. D'Agostini. A multidimensional unfolding method based on Bayes' theorem Nuclear Instruments and Methods in Physics Research A, 362:487498, Febr. 1995.

Methods based on regularization

$$R\tau = \mu \quad \Longleftrightarrow \quad L(\tau) = (R\tau - \mu)^T (R\tau - \mu) \rightarrow \min_{\tau}$$

$$\boxed{\Phi(\tau) = L(\tau) + \alpha \cdot S(\tau) \rightarrow \min_{\tau}}$$

$S(\tau)$ — regularization function

α — regularization coefficient

Tasks: * choose S,

* choose optimal α.

SVD Unfolding

$$\Phi(\tau) = L(\tau) + \alpha \cdot S(\tau) \to \min_{\tau}$$

$$S(\tau) = \sum_i (\tau_{i-1} - 2\tau_i + \tau_{i+1})^2 = (C\tau)^T (C\tau)$$

Solution $R\tau = \mu$ **system by SVD**

$$R = USV^T$$

U, V — orthogonal matrices

S — diagonal matrix

$$USV^T \tau = \mu$$

$$z = V^T \tau, \quad d = U^T \mu$$

$$Sz = d \quad \Rightarrow \quad z_i = \frac{d_i}{s_{ii}}$$

$$\tau = Vz$$

Solution of new system SVD

$$z_i = \frac{d_i}{s_{ii}} \cdot \frac{s_{ii}^2}{s_{ii}^2 + \alpha}$$

SVD Unfolding

$$\Phi(\tau) = L(\tau) + \alpha \cdot S(\tau) \to \min_{\tau}$$

$$S(\tau) = \sum_i (\tau_{i-1} - 2\tau_i + \tau_{i+1})^2 = (C\tau)^T (C\tau)$$

Extras:

1. Renormalization of the original system

2. Choose regularization coefficient as $\alpha = s_{kk}^2$

   where s_{kk} — last «large» singular value s_{ii}

A. Hoecker and V. Kartvelishvili SVD Approach to Data Unfolding
(Nucl. Instrum. Meth. A 372 (1996) 469 [arXiv:hep-ph/9509307])

TUnfold

$$\Phi(\tau) = L_1(\tau) + \alpha \cdot L_2(\tau) + \lambda \cdot L_3(\tau) \to \min_{\tau}$$

$$L_1(\tau) = L(\tau)$$

$L_2(\tau)$ — regularization function (smoothness)

$L_3(\tau)$ — bias limitation

Regularization coefficients choosing

1. L-curve
2. Correlation coefficient minimization

Testing on proton's energy spectrum

Software implementation

ROOT (5.28, 5.34)

- TUnfold

RooUnfold 1.1.1

- Tunfold
- SVD unfolding
- Bayes unfolding

Data

Exponential spectrums (sp1 and sp2) are generated

1. Exponential parameters -1 and -2
2. Generative and measure energy values
3. Migration matrix constructed by sp2
4. Migration matrix applied for sp1

Problems

1. Migration matrix construction

   Condition $R_{ii} > 0,5$ (Blobel)

   Possibility of using for another distribution (spectrum)

2. Binning

   The effect of extreme (additional) bin

   Irregular binninig

   Different binning for true/measured spectrum

   The effect of binning on the quality of the unfolding

3. Methods particular features

   Bayesian methods: number of iterations

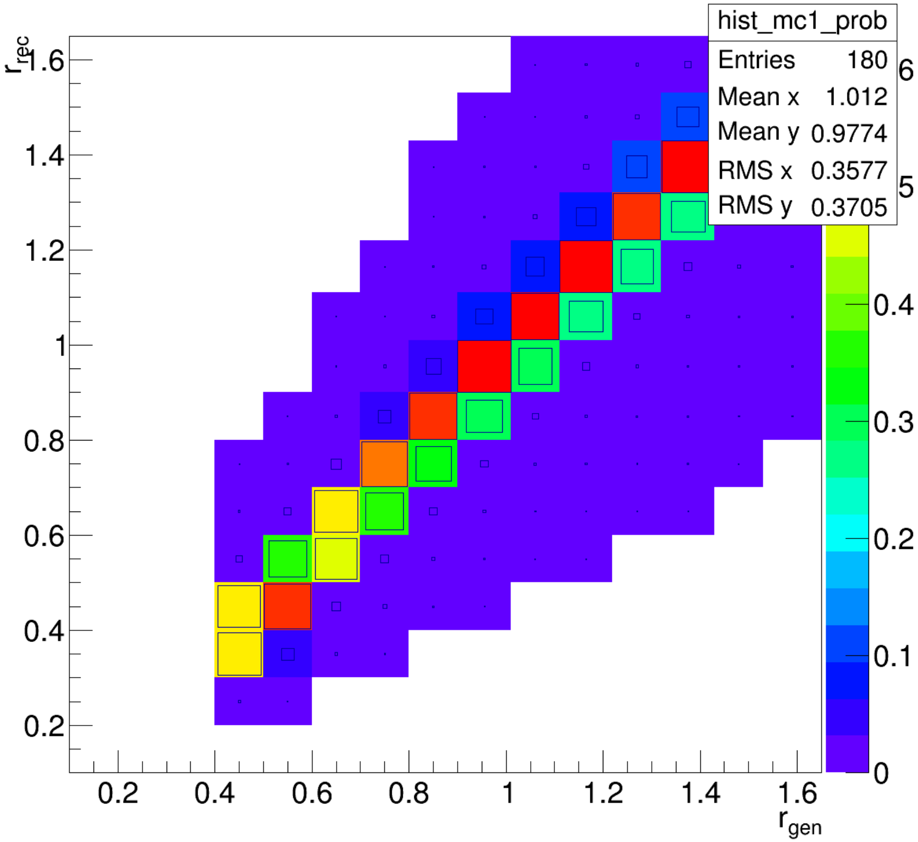
   Regularization methods: coefficient

4. General

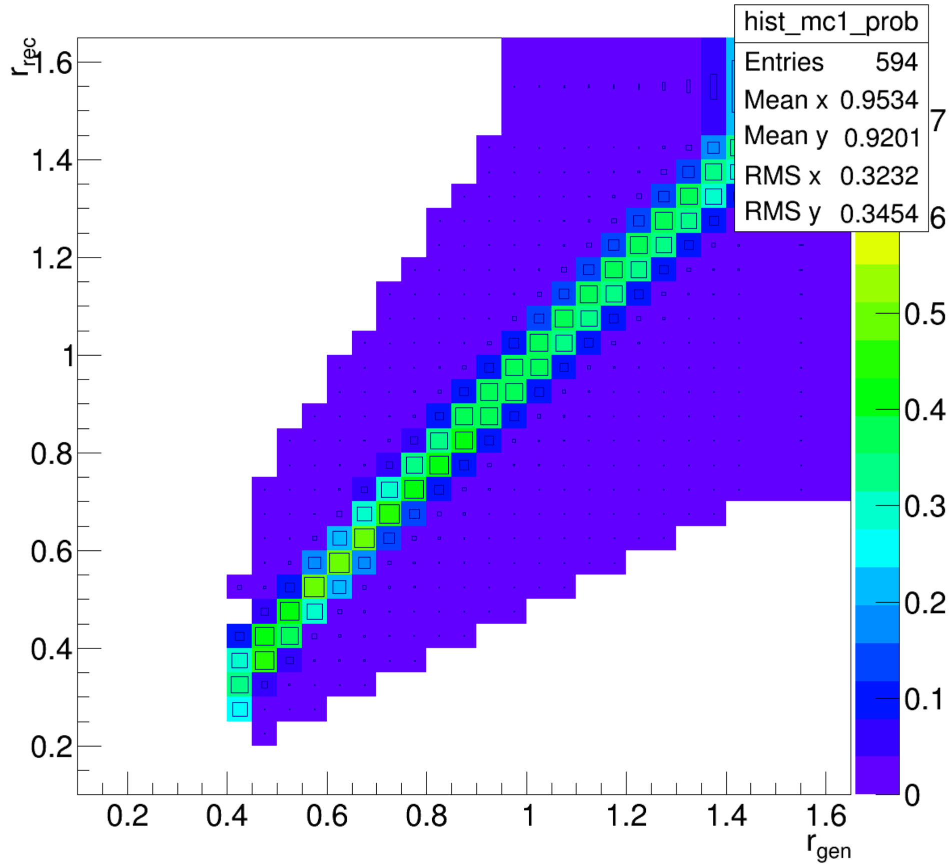
   Unfolding quality estimation (comparision)

   Using methods for several measured values

Migration matrix
Different binning



Wide bins, max Rig = 1.6Gv

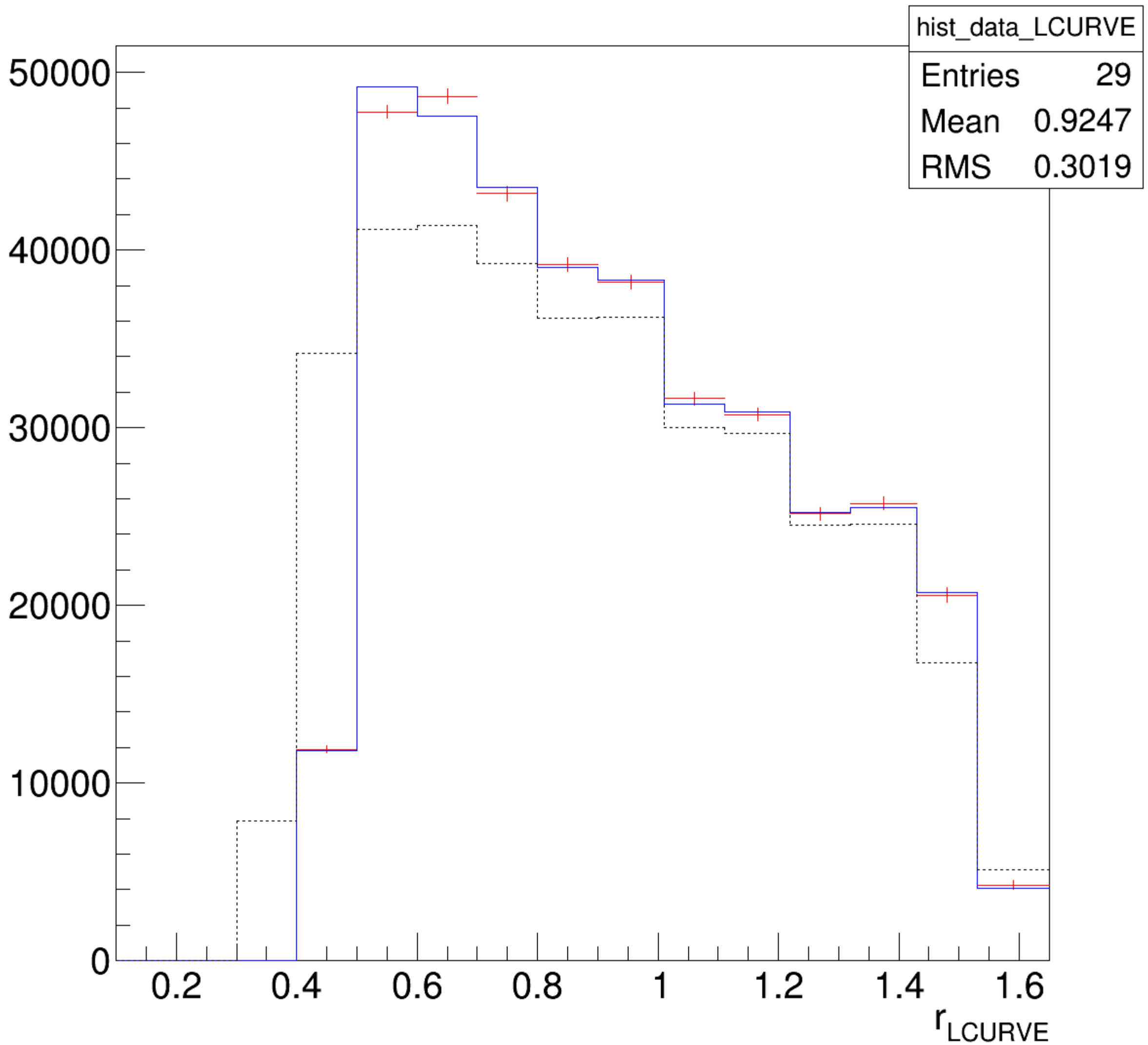


Short bins, max Rig = 1.6Gv

Results

TUnfold (ROOT)

Wide bins



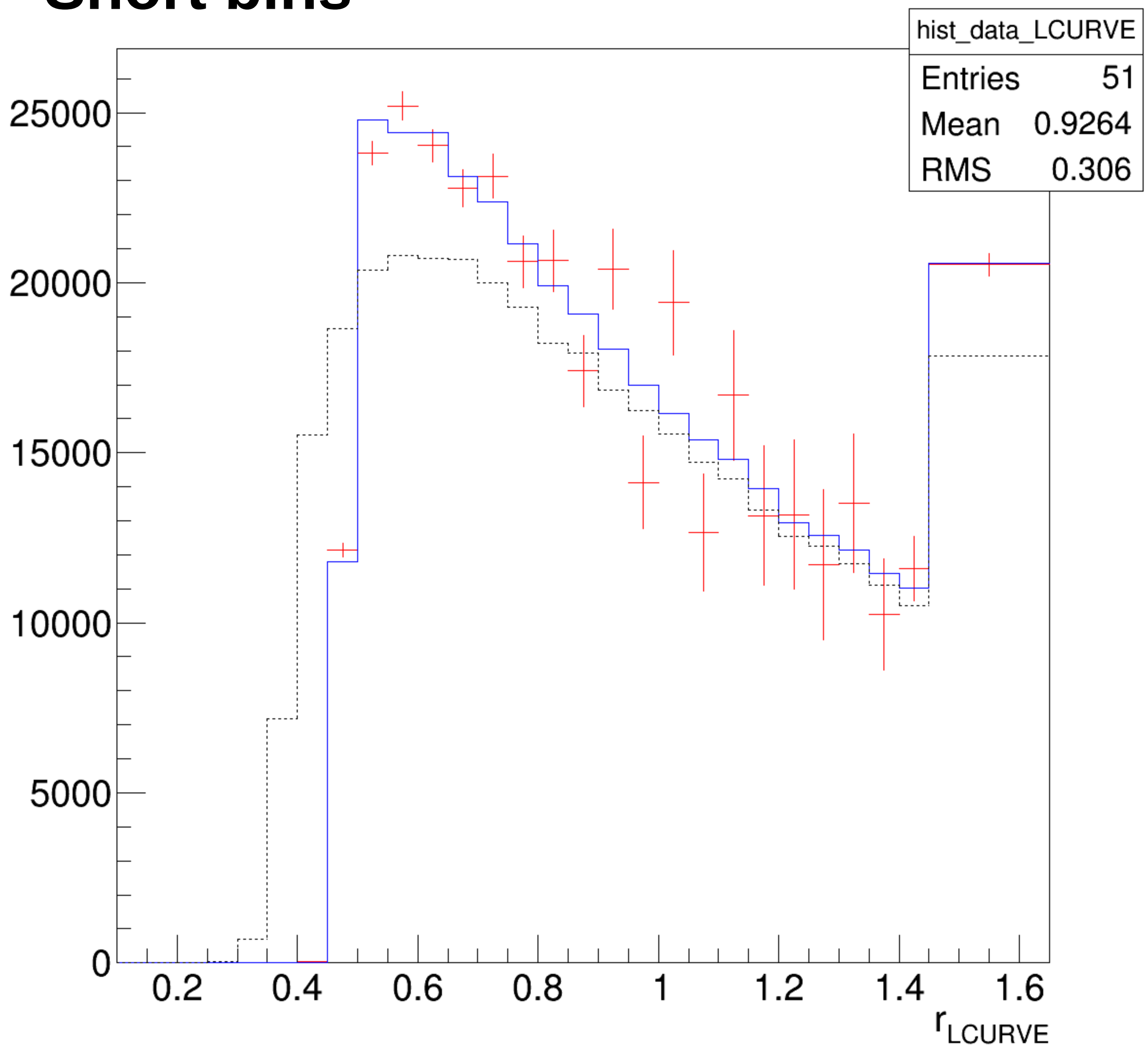
True

Unfolded

Measured

Results
TUnfold (ROOT)

Short bins

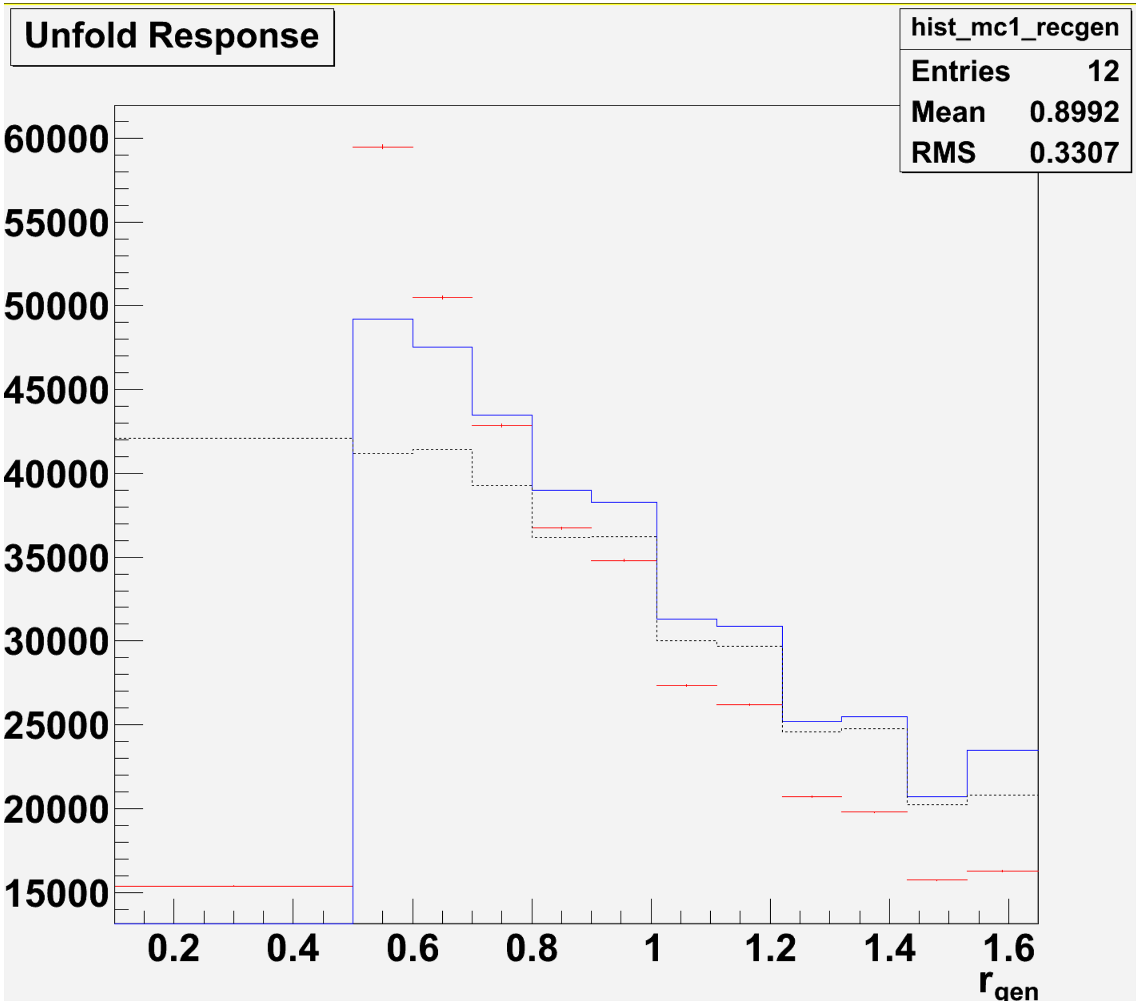


Method is sensitive to binning

Results

SVD unfolding (RooUnfold)

Wide bins



K_reg = 2

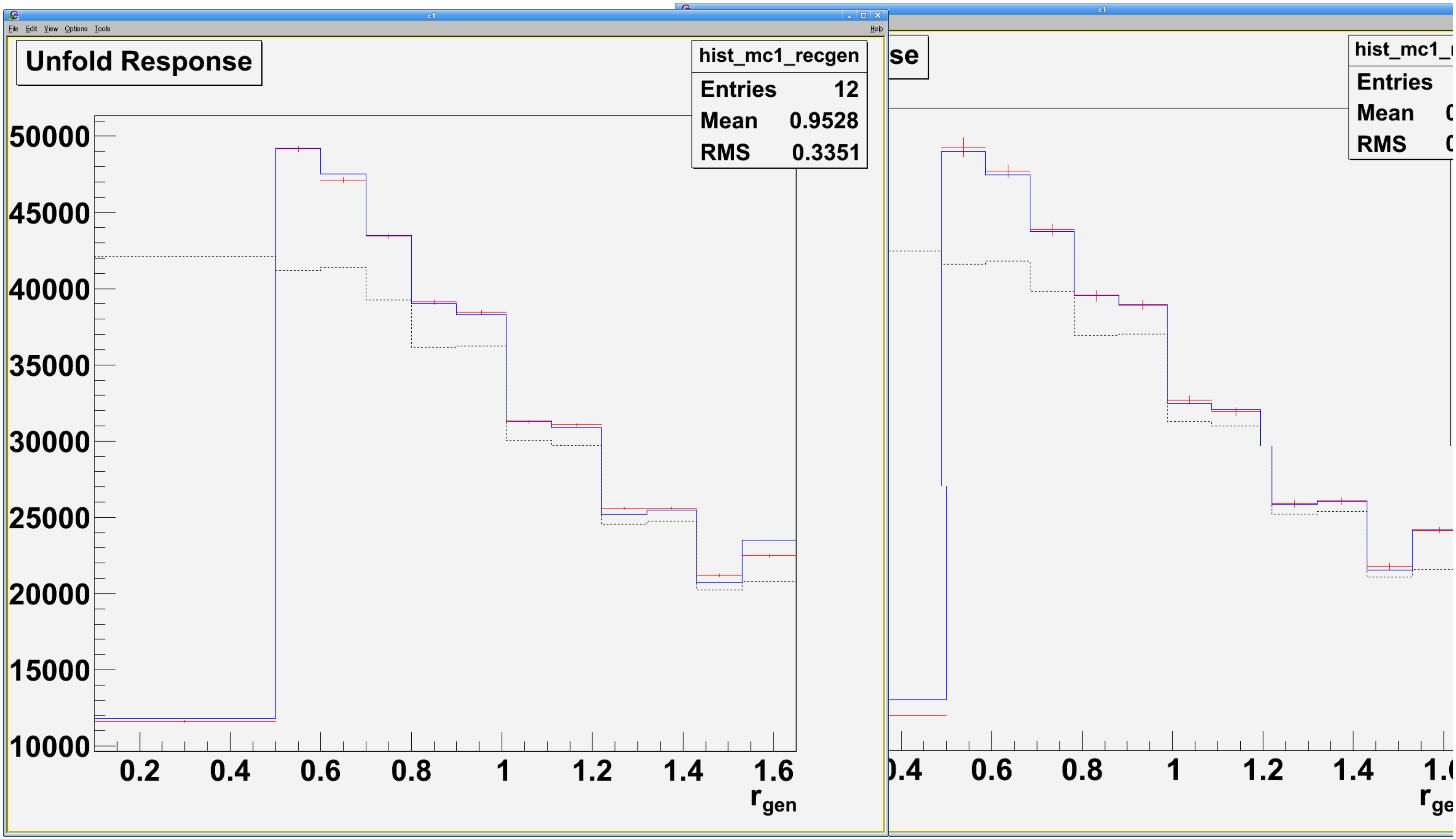
small regularization

→ Right choose regularization

Results

Wide bins

SVD unfolding (RooUnfold)



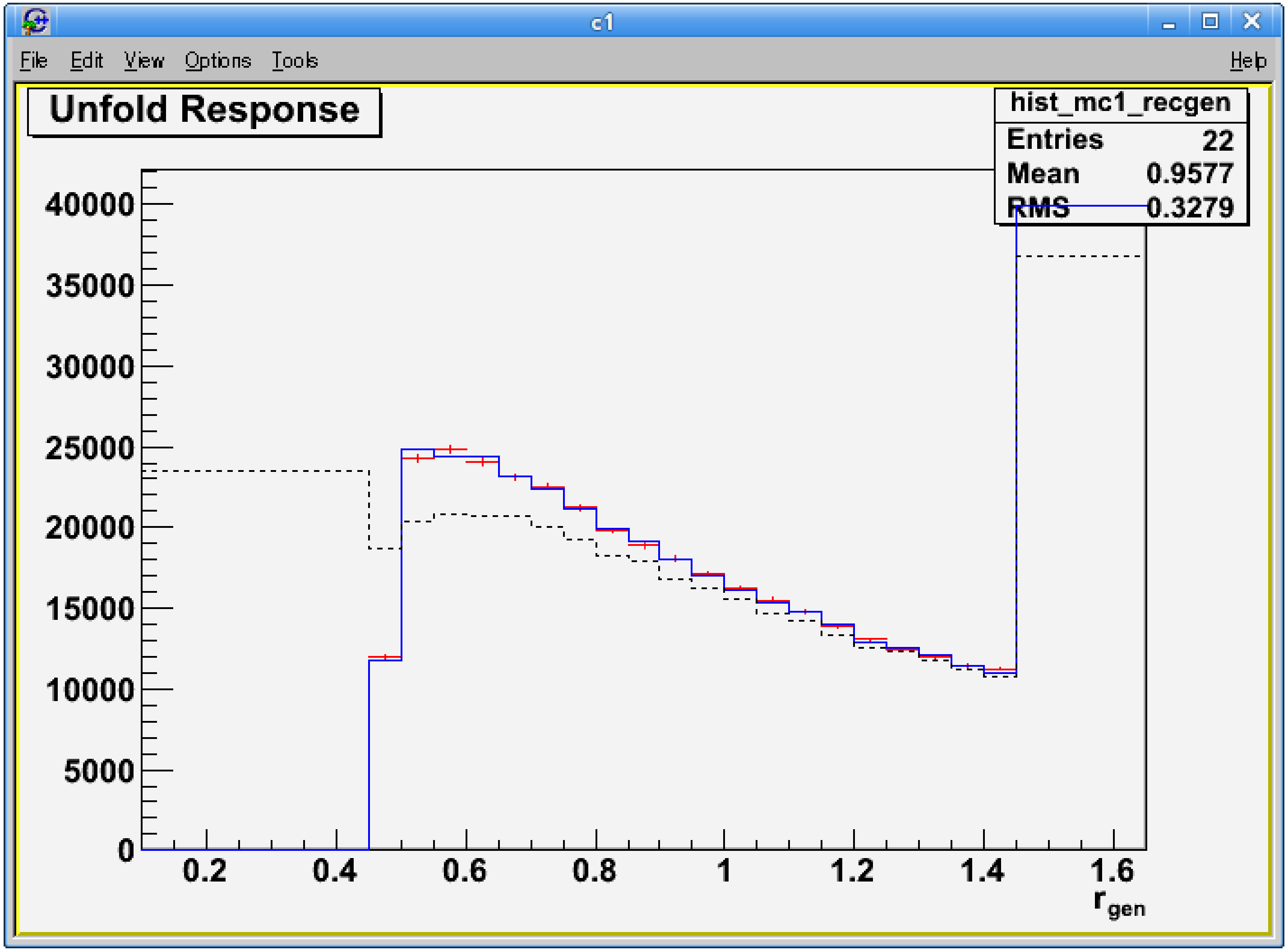
K_reg = 5

K_reg = 12

Results

SVD unfolding (RooUnfold)

Short bins

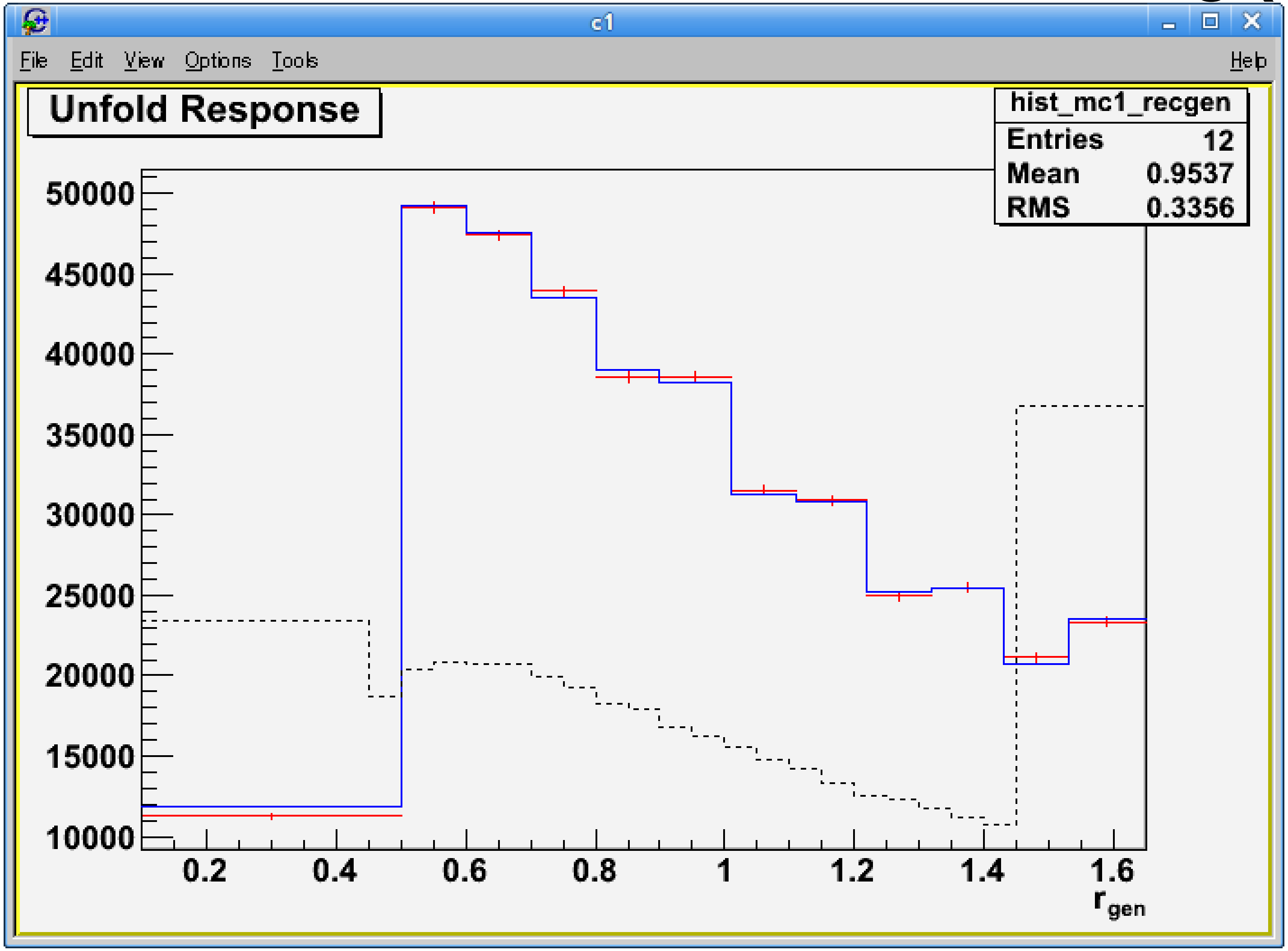


Same behavior

Results

SVD unfolding (RooUnfold)

Different bins



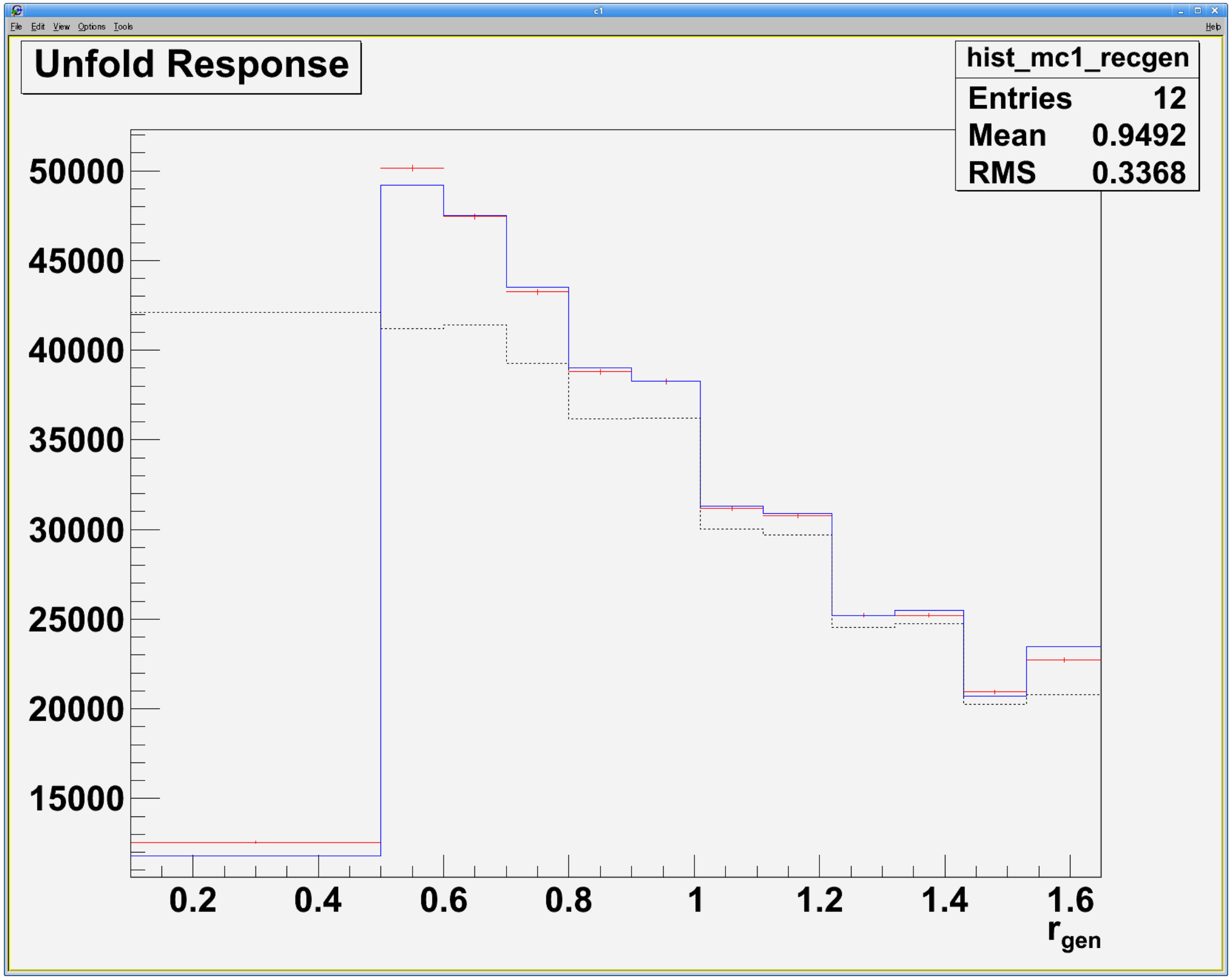
K_reg = 12

Stable for binning

Results

Wide bins

Bayes unfolding (RooUnfold)

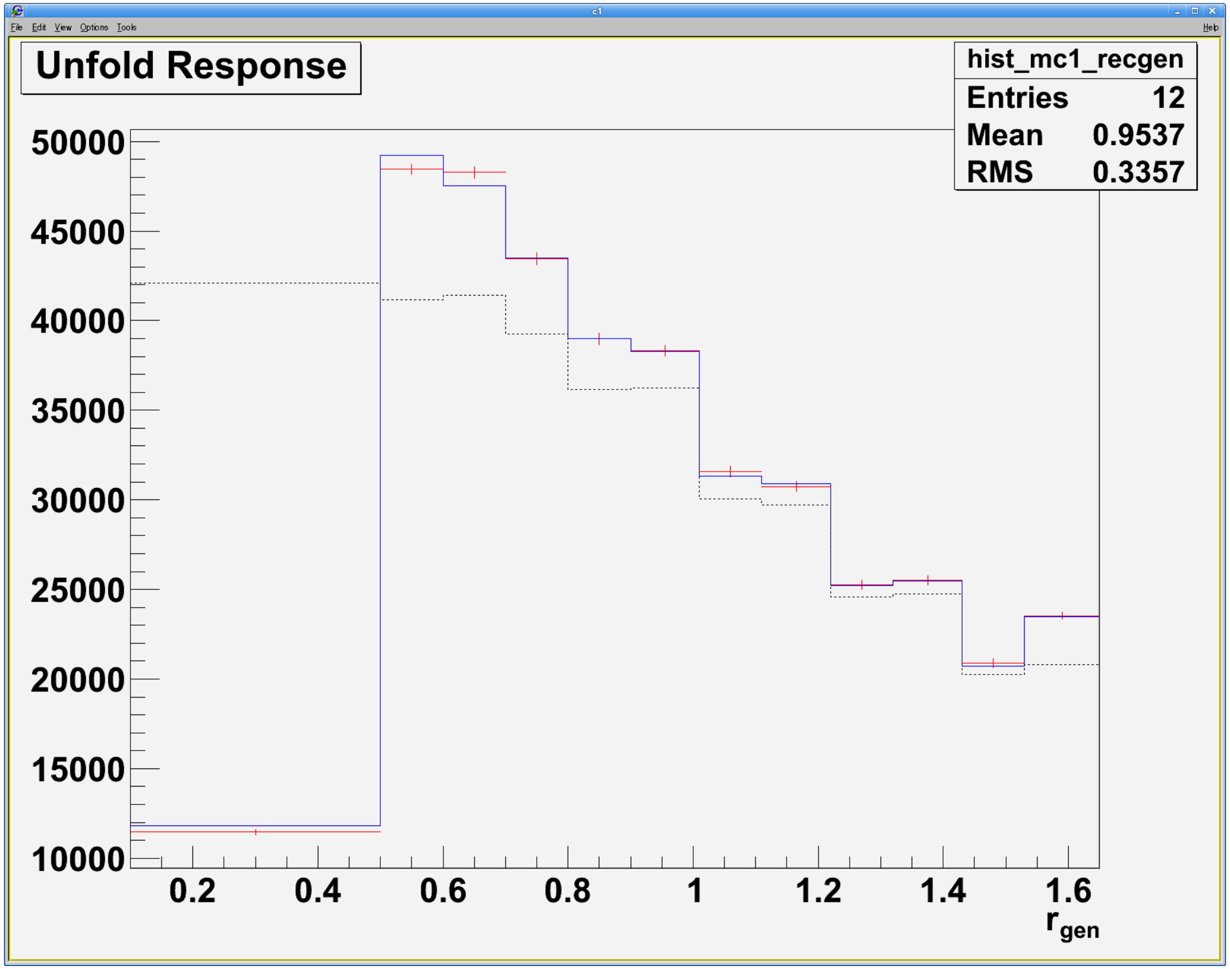


iterations = 1

Results

Bayes unfolding (RooUnfold)

Wide bins

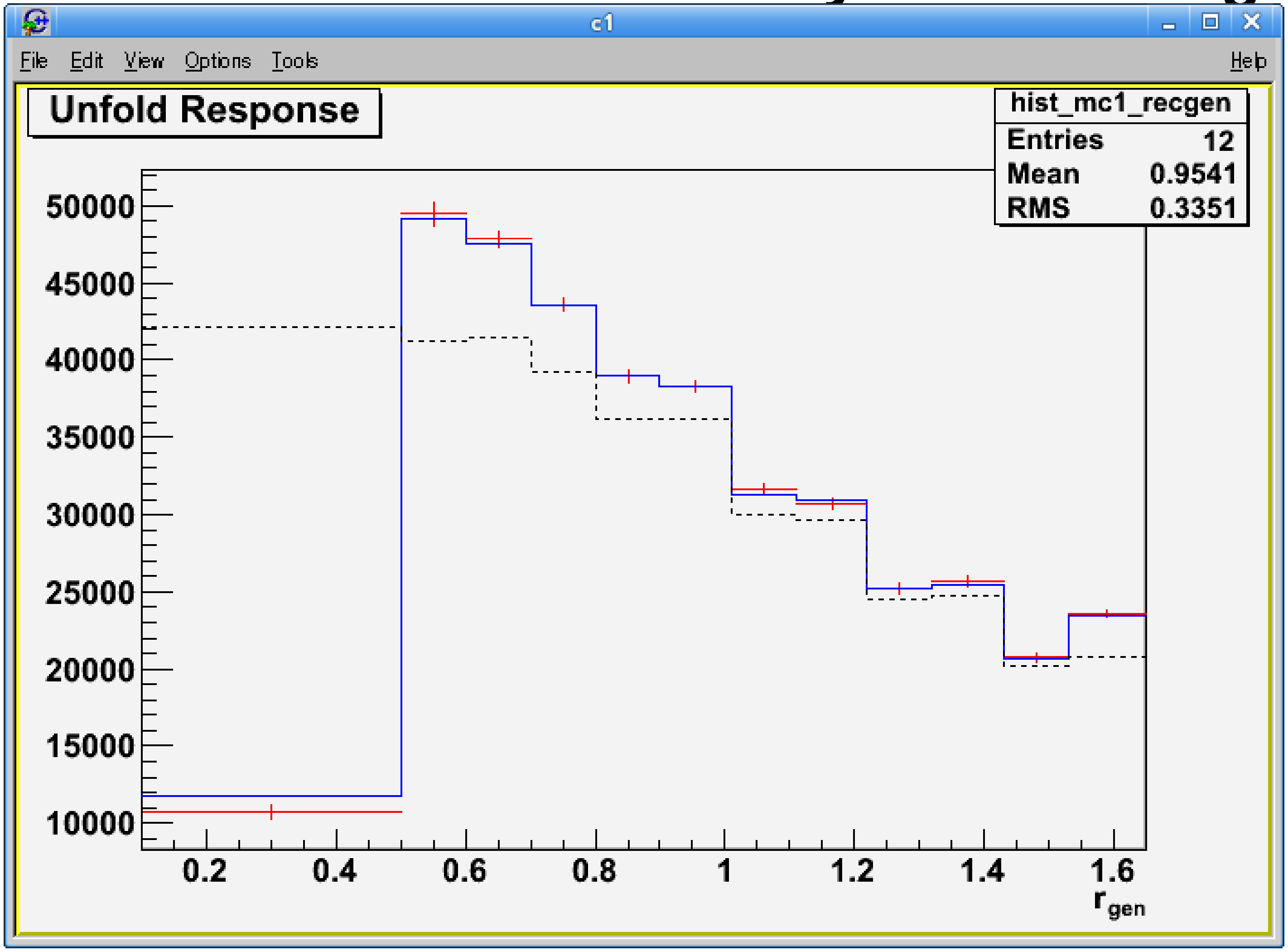


iterations = 10

Results

Wide bins

Bayes unfolding (RooUnfold)



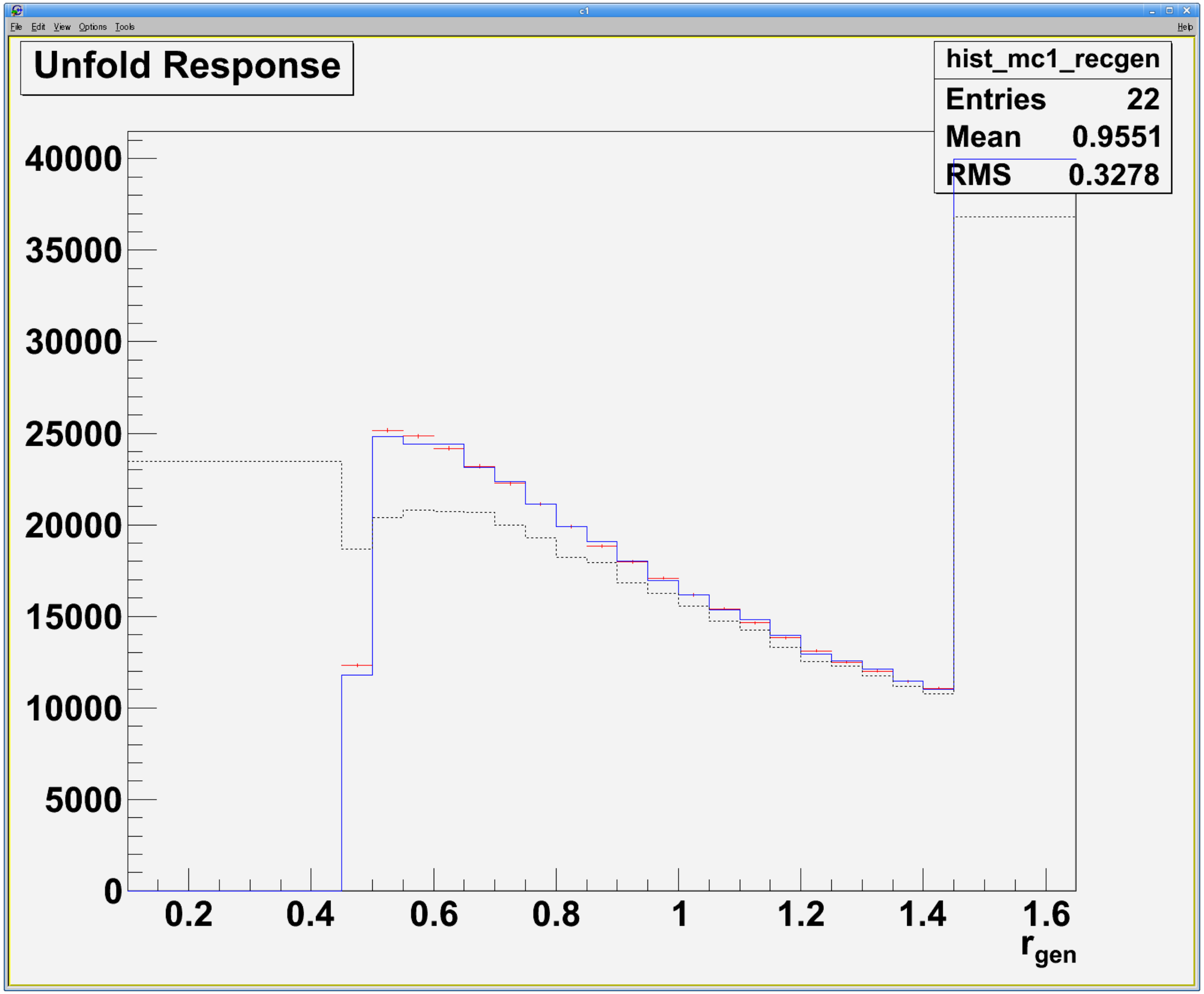
iterations = 100

Need to measure the quality

Results

Short bins

Bayes unfolding (RooUnfold)

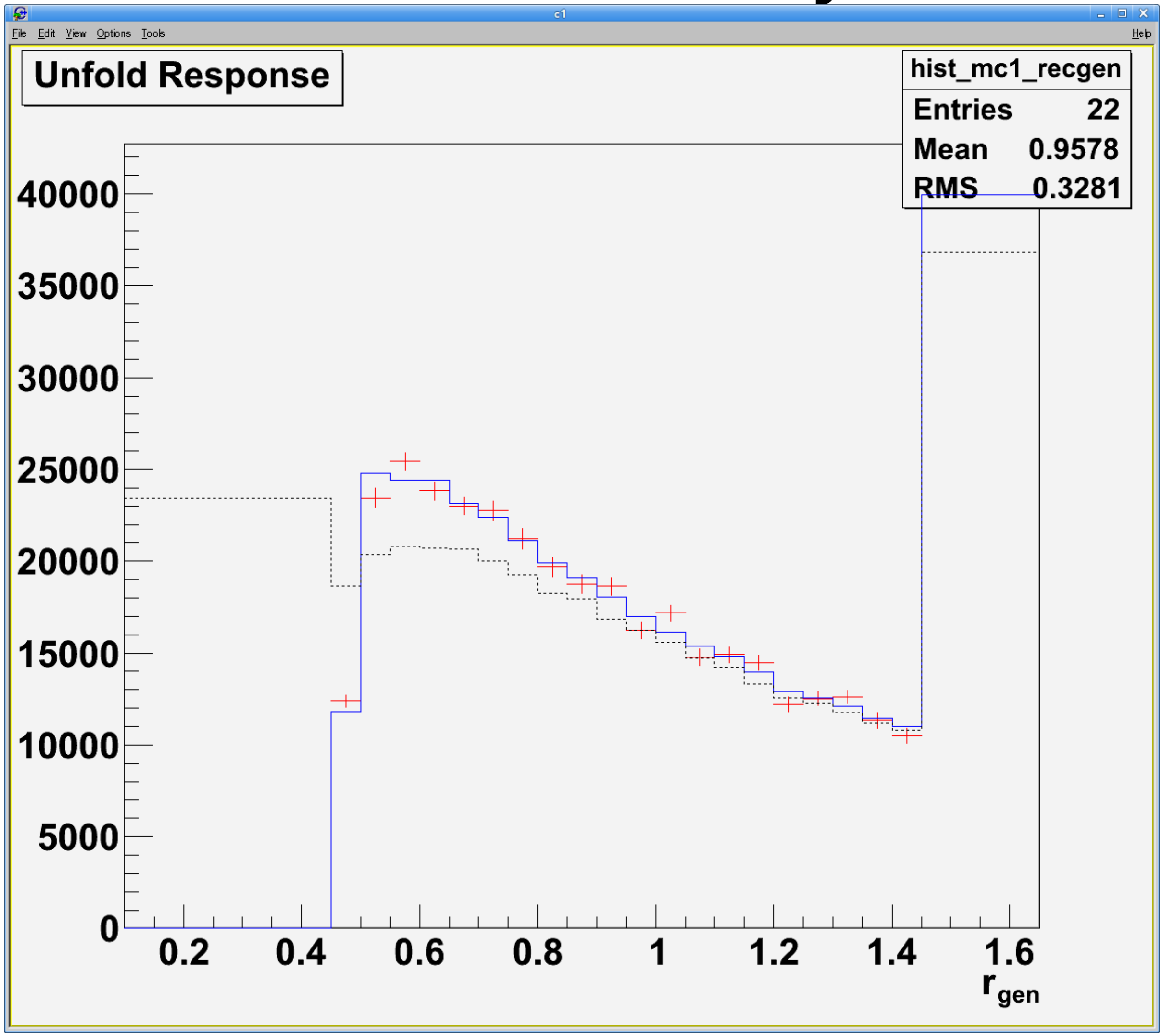


iterations = 1

Results

Short bins

Bayes unfolding (RooUnfold)



iterations = 100

Iterations convergence depends on binning

Conclusions

Methods features

*** Bayesian method**

perceptible dependence on iterations and
some dependence on binning
+ can be used for several measured parameters (2D)

*** SVD unfolding**

regularization coefficient choosing

*** TUnfold**

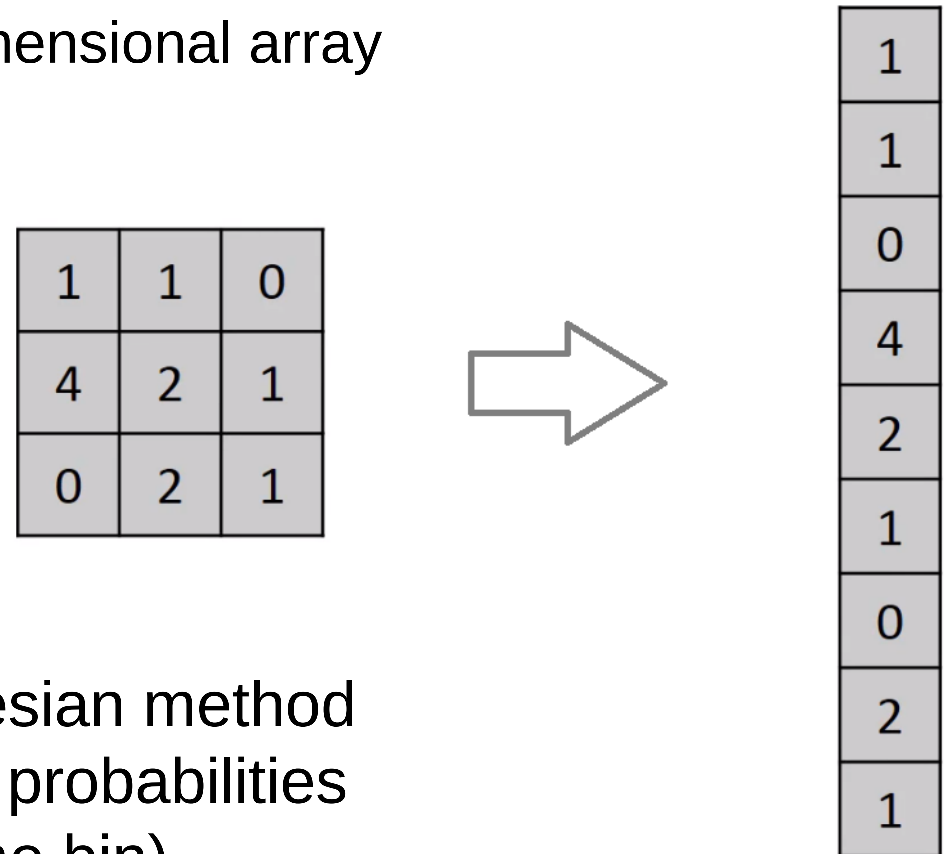
perceptible sensitivity to binning

Thank you for your attention!

Olga Dunaeva
Yuri Bogomolov
Andrey Mayorov

Idea of extending for several parameters

Unpack the matrix in a one-dimensional array

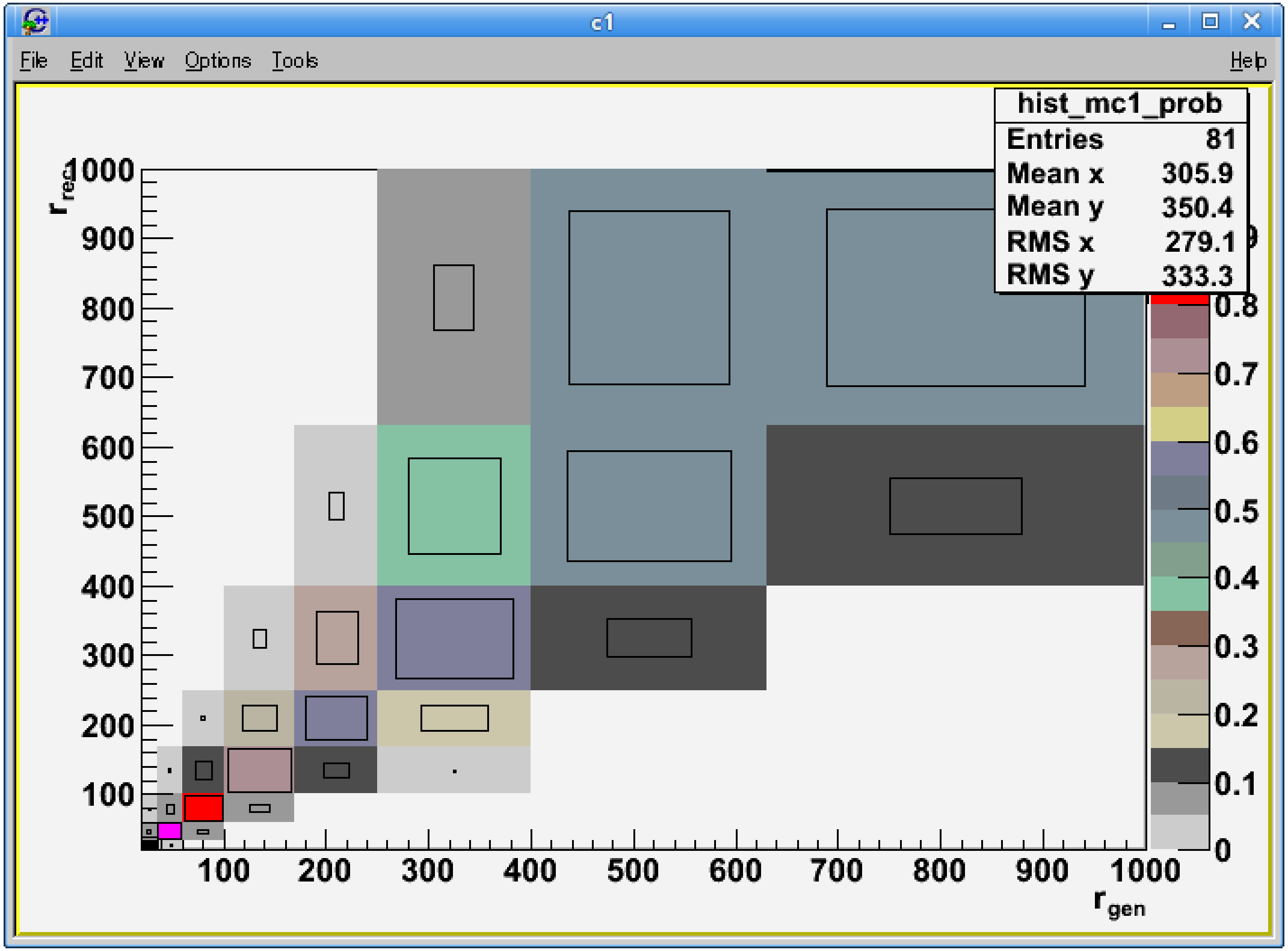


Then possible to use Bayesian method because it’s based on just probabilities (to detect the event in some bin)

Other methods use the information about nearest bins

High energy

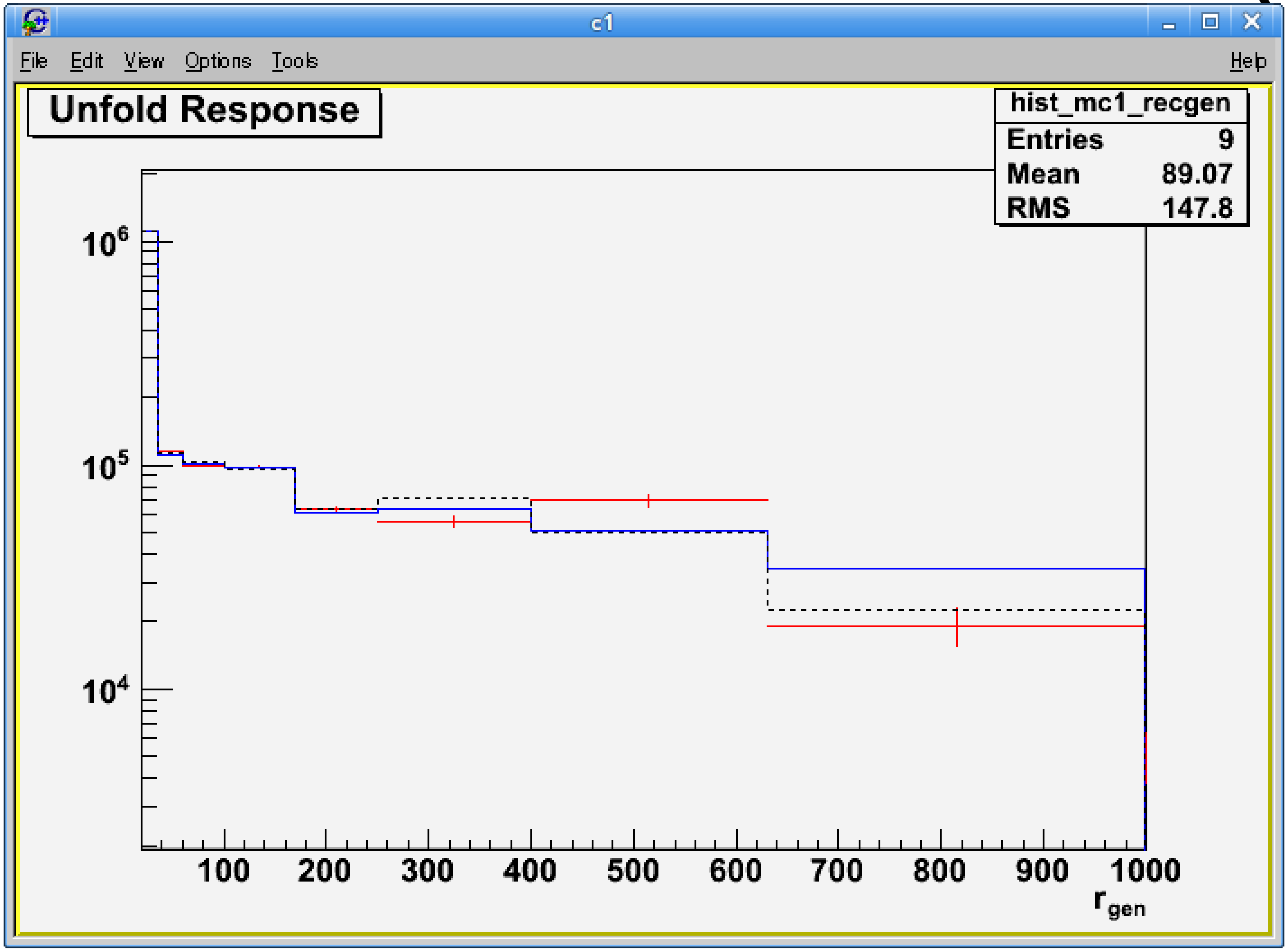
Migration matrix



High energy (10-1000 GeV)

Results

TUnfold (RooUnfold)

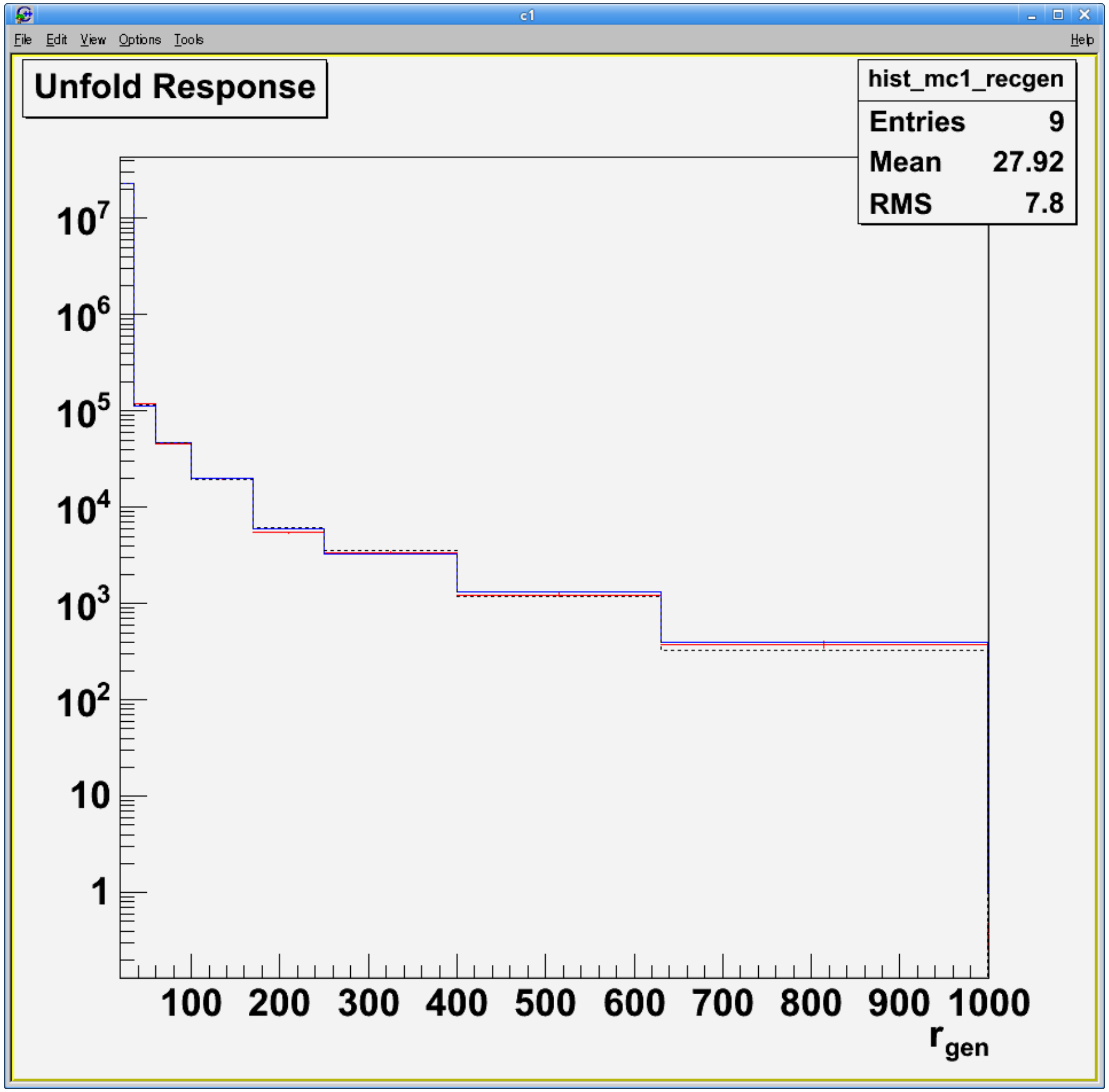


log-
scale

Aplied to cosmic rays spectrum (-2.7)

Results

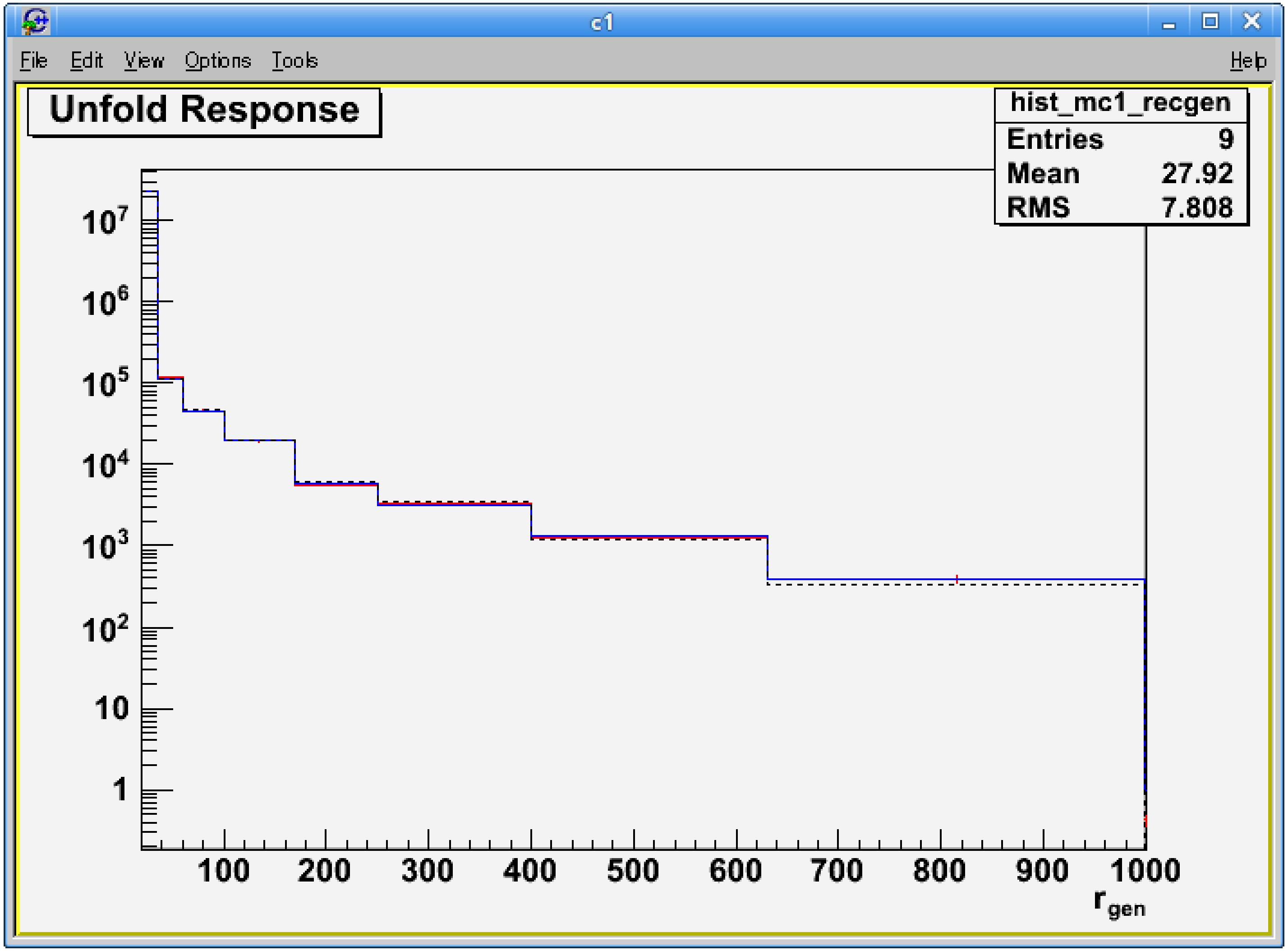
Bayes unfolding (RooUnfold)



iteration = 10

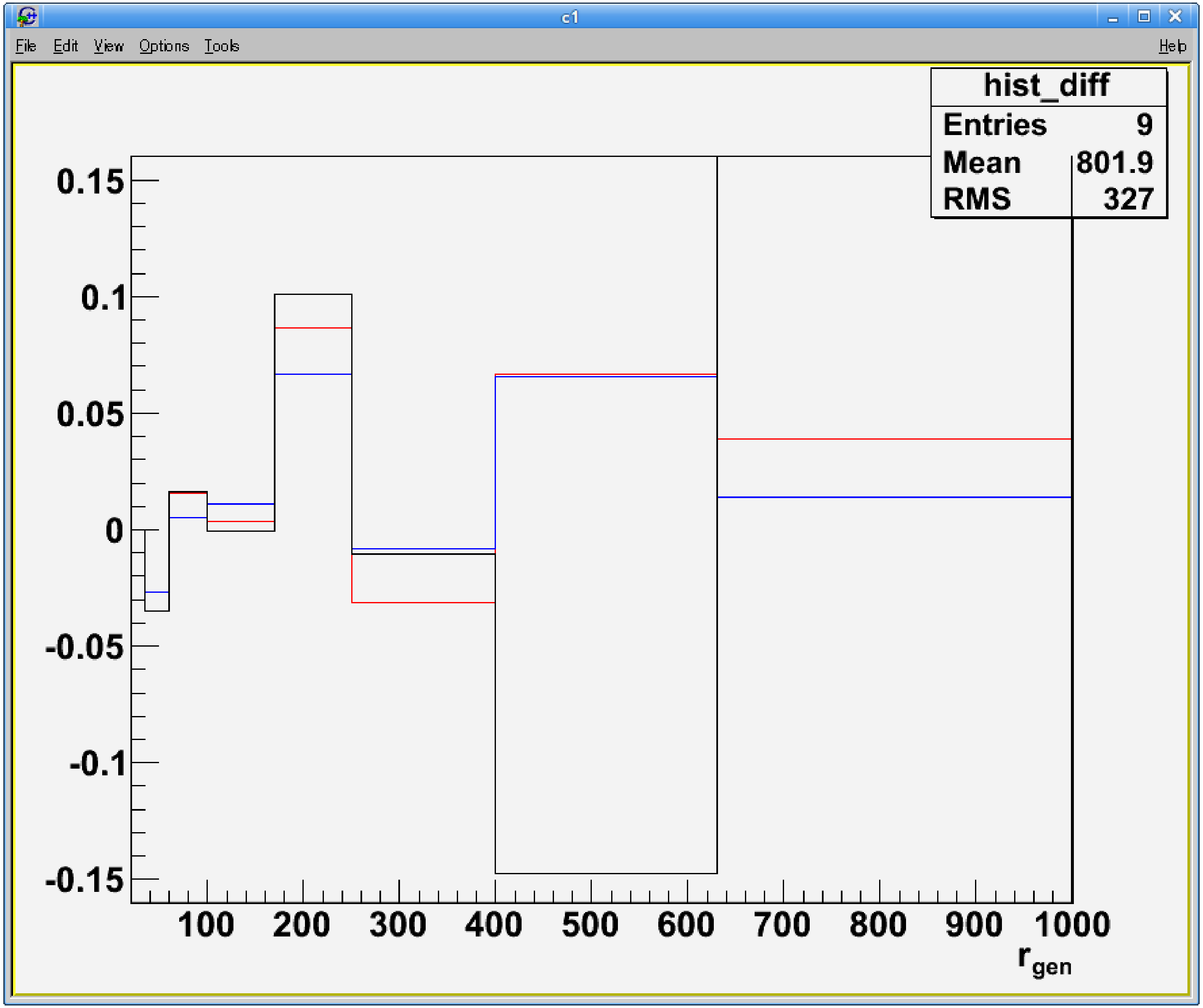
Results

SVD unfolding (RooUnfold)



k_reg = 7

Results



TUnfold

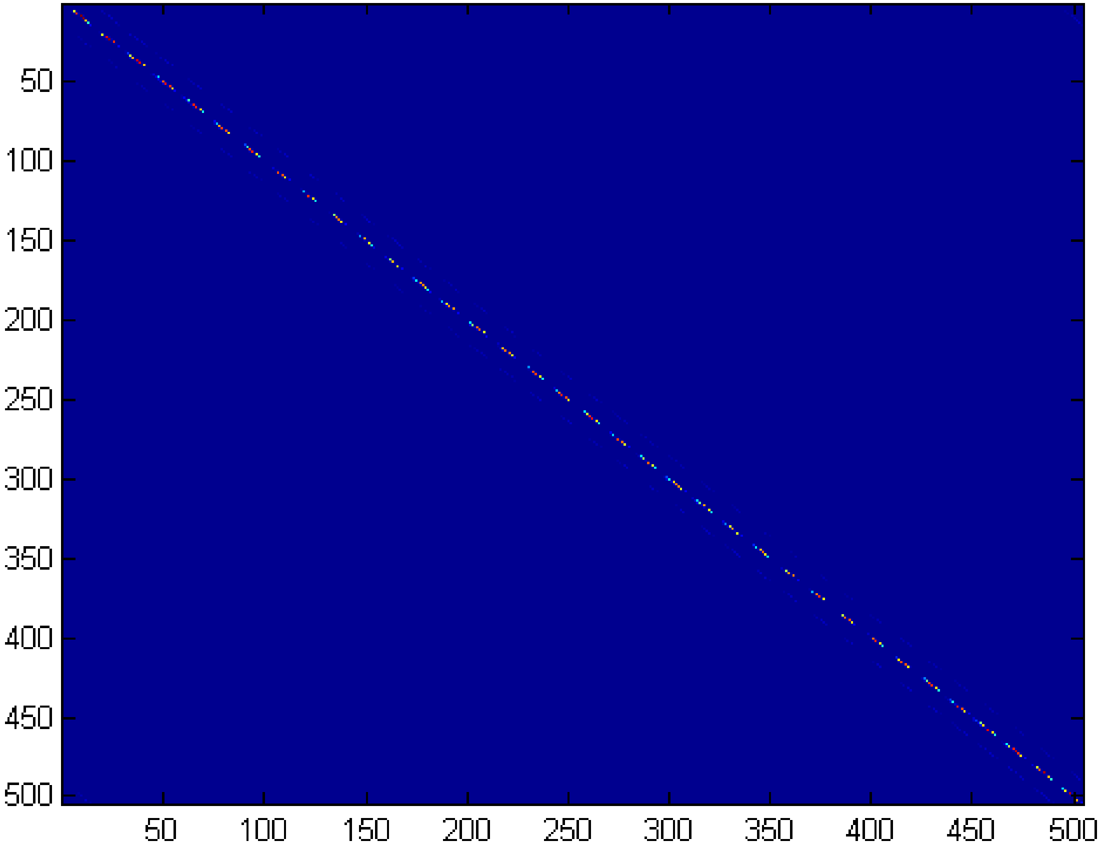
SVD

Bayes

High energy (10-1000 GeV), cosmic rays

Unpacked migration matrix

Migration matrix for Azimuthal and Zenith angle



Use 36 bins for azimut and 14 bins for zenit angle
gives migration matrix 504x504