WHY THE HYDRODYNAMICS IS VALID IN HEAVY-ION COLLISIONS ?

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Dynamic Regimes

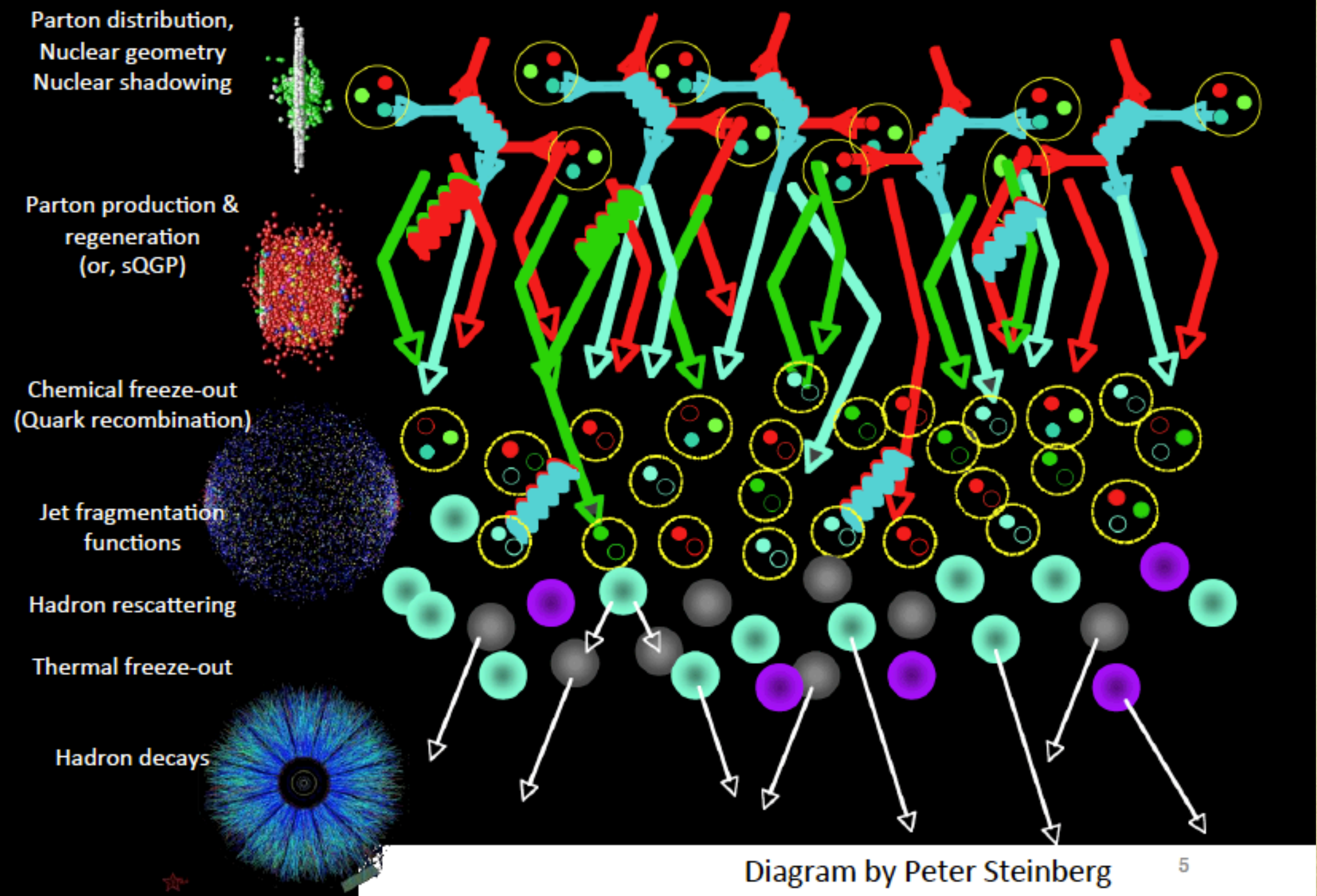


Diagram by Peter Steinberg

Models at our disposal

Relativistic Hydrodynamics

Relativistic Hydrodynamics

Basic Equations

Energy-momentum tensor

$$\mathbf{T}^{\mu\nu} = \underbrace{(\varepsilon + \mathbf{P})\mathbf{u}^{\mu}\mathbf{u}^{\nu} - \mathbf{P}\mathbf{g}^{\mu\nu}}_{\text{inertial}} + \underbrace{\eta^{\mu\nu}}_{\text{dissipative}}$$

The space-time evolution of relativistic fluid is described by the set of differential equations

$$\boxed{\begin{gathered} \partial_{\mu}\mathbf{N}^{\mu}(\mathbf{x}) = \mathbf{0} \\ \partial_{\mu}\mathbf{T}^{\mu\nu} = \mathbf{0};\ \ \mu,\nu = \mathbf{0},\mathbf{1},\mathbf{2},\mathbf{3} \end{gathered}}$$

For perfect fluid (i.e. $\eta^{\mu\nu} = 0$) these equations take the familiar form

$$\boxed{\begin{aligned} (\partial_t + \vec{\mathbf{v}}\cdot grad)\mathcal{N} &= -\mathcal{N}\,div\vec{\mathbf{v}} & \mathcal{N} &\equiv \gamma\,\mathbf{N}^{\mu}\mathbf{u}_{\mu} \\ (\partial_t + \vec{\mathbf{v}}\cdot grad)\vec{\mathcal{M}} &= -\vec{\mathcal{M}}\cdot div\vec{\mathbf{v}} - grad\,\mathbf{P} & \vec{\mathcal{M}} &\equiv \mathbf{T}^{\mathbf{0i}} = (\varepsilon + \mathbf{P})\gamma^2\vec{\mathbf{v}} \\ (\partial_t + \vec{\mathbf{v}}\cdot grad)\mathcal{E} &= -\mathcal{E}div\vec{\mathbf{v}} - div(\mathbf{P}\vec{\mathbf{v}}) & \mathcal{E} &\equiv \mathbf{T}^{\mathbf{00}} = (\varepsilon + \mathbf{P}\vec{\mathbf{v}}^{\mathbf{2}})\gamma^2 \end{aligned}}$$

$$\boxed{\begin{gathered}\partial_\mu \mathbf{N}^\mu(\mathbf{x}) = 0 \\ \partial_\mu \mathbf{T}^{\mu\nu} = 0; \ \ \mu,\nu = 0,1,2,3\end{gathered}}$$

Number of variables – 6

Number of equations – 4

Missing equations:

$$\mathbf{T}^{\mu\nu} = \underbrace{(\varepsilon + \mathbf{P})\mathbf{u}^\mu\mathbf{u}^\nu - \mathbf{P}\mathbf{g}^{\mu\nu}}$$

(1) EOS, that links energy density and pressure

- **Four-velocity**

$$\mathbf{u}^\mu = (\gamma, \gamma\vec{\mathbf{v}}); \ \ \vec{\mathbf{v}} \equiv \frac{\vec{\mathbf{p}}}{\mathbf{p}^0}; \ \ \gamma = \frac{1}{\sqrt{1-(\vec{\mathbf{v}})^2}}$$

thus

(2) $$\boxed{\mathbf{u}^\mu \mathbf{u}_\mu = 1}$$

Bjorken Model

Using the thermodynamic relations

$$\varepsilon + \mathbf{P} = \mathbf{Ts}\,, \quad \mathbf{d}\varepsilon = \mathbf{Tds}\,,$$

where $\mathbf{s}$ is the *entropy density* in the local rest frame, we find

$$\boxed{\frac{\mathbf{ds}}{\mathbf{d}\tau} = -\frac{\mathbf{s}}{\tau}\,, \quad \longrightarrow \mathbf{s}(\tau) = \mathbf{s}(\tau_0)\frac{\tau_0}{\tau}}$$

Consequently, expansion proceeds adiabatically: $\mathbf{S} = \mathbf{sV} = \mathbf{const}$

EOS: $\mathbf{P} = \varepsilon/\mathbf{3}$ - ultrarelativistic gas. The basic equation reduces to

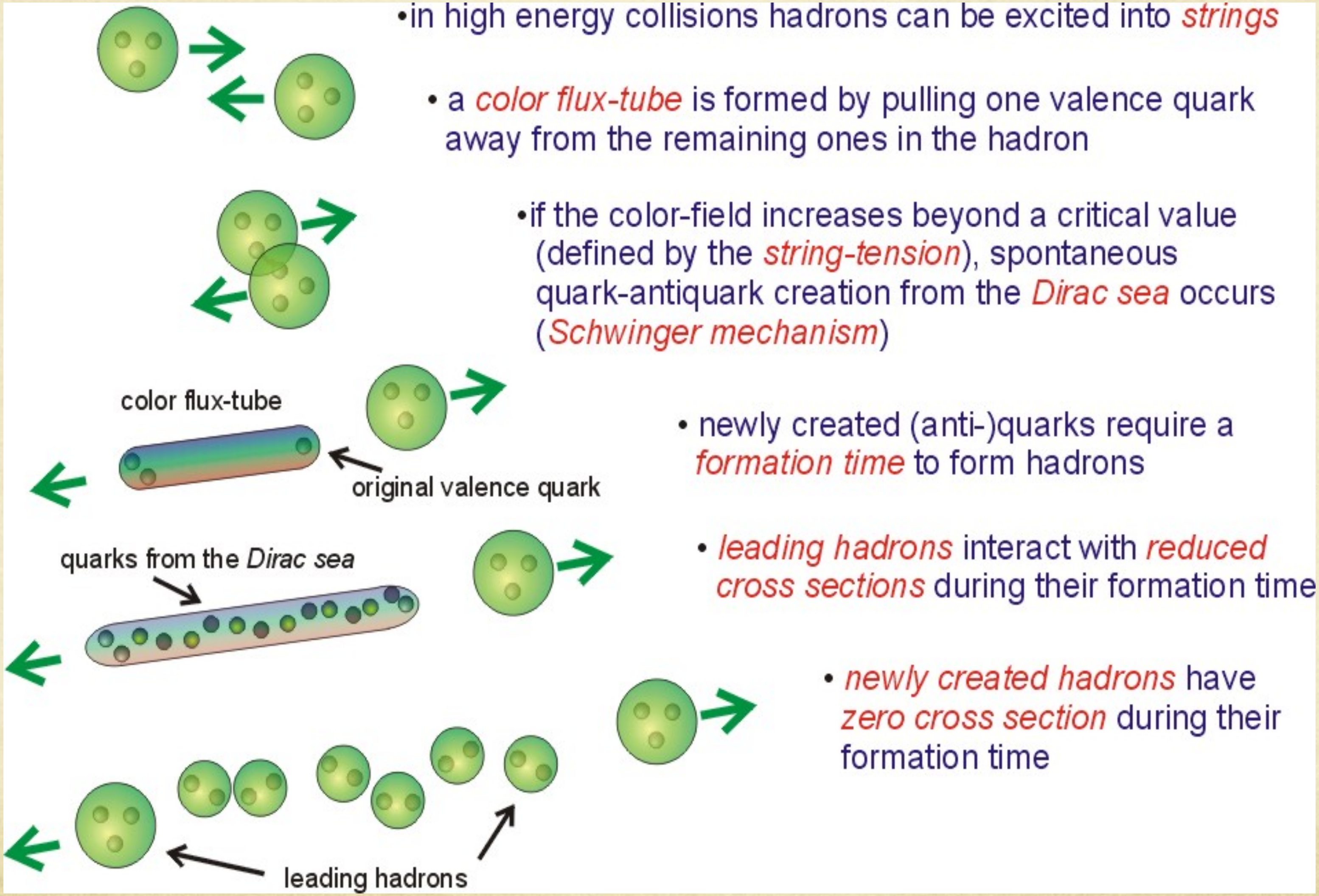
$$\boxed{\frac{\partial\varepsilon}{\partial\tau} = -\frac{\mathbf{4}}{\mathbf{3}}\frac{\varepsilon}{\tau}\,, \quad \longrightarrow \varepsilon(\tau) = \varepsilon(\tau_0)\left(\frac{\tau_0}{\tau}\right)^{\mathbf{4/3}}}$$

Note: volume increases $\propto \tau$, while energy density drops $\propto \tau^{-\mathbf{4/3}}$!

Models at our disposal

Microscopic Transport Models

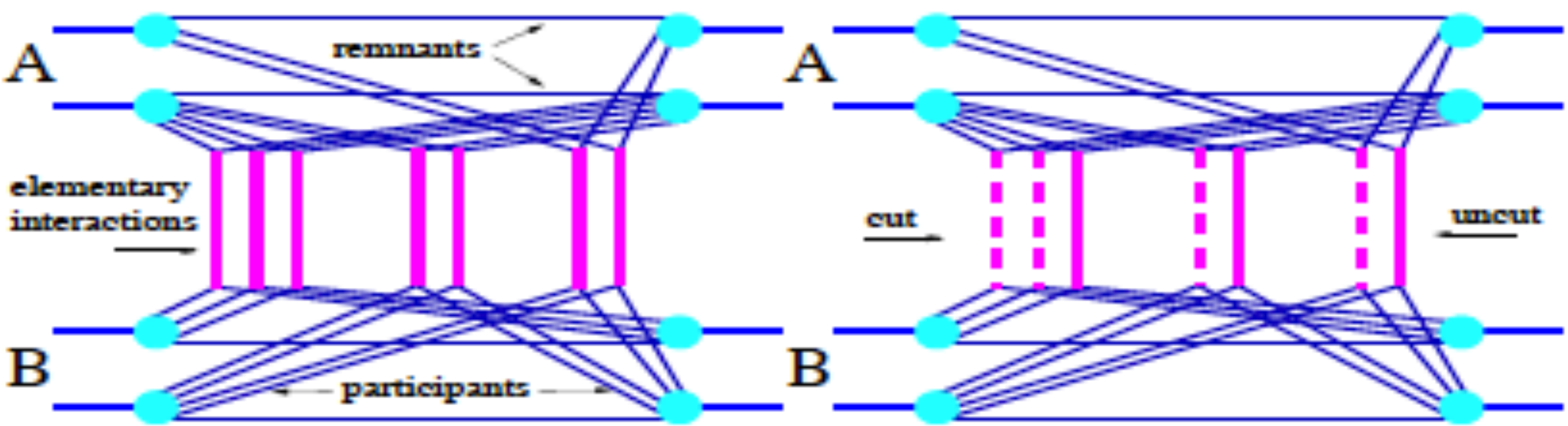
Initial Particle Production in UrQMD



Steffen A. Bass

The Quark-Gluon String Model (QGSM)

- microscopic model, based on Gribov-Regge theory (GRT)
- interaction is modelled via the exchange of reggeons and pomenrons

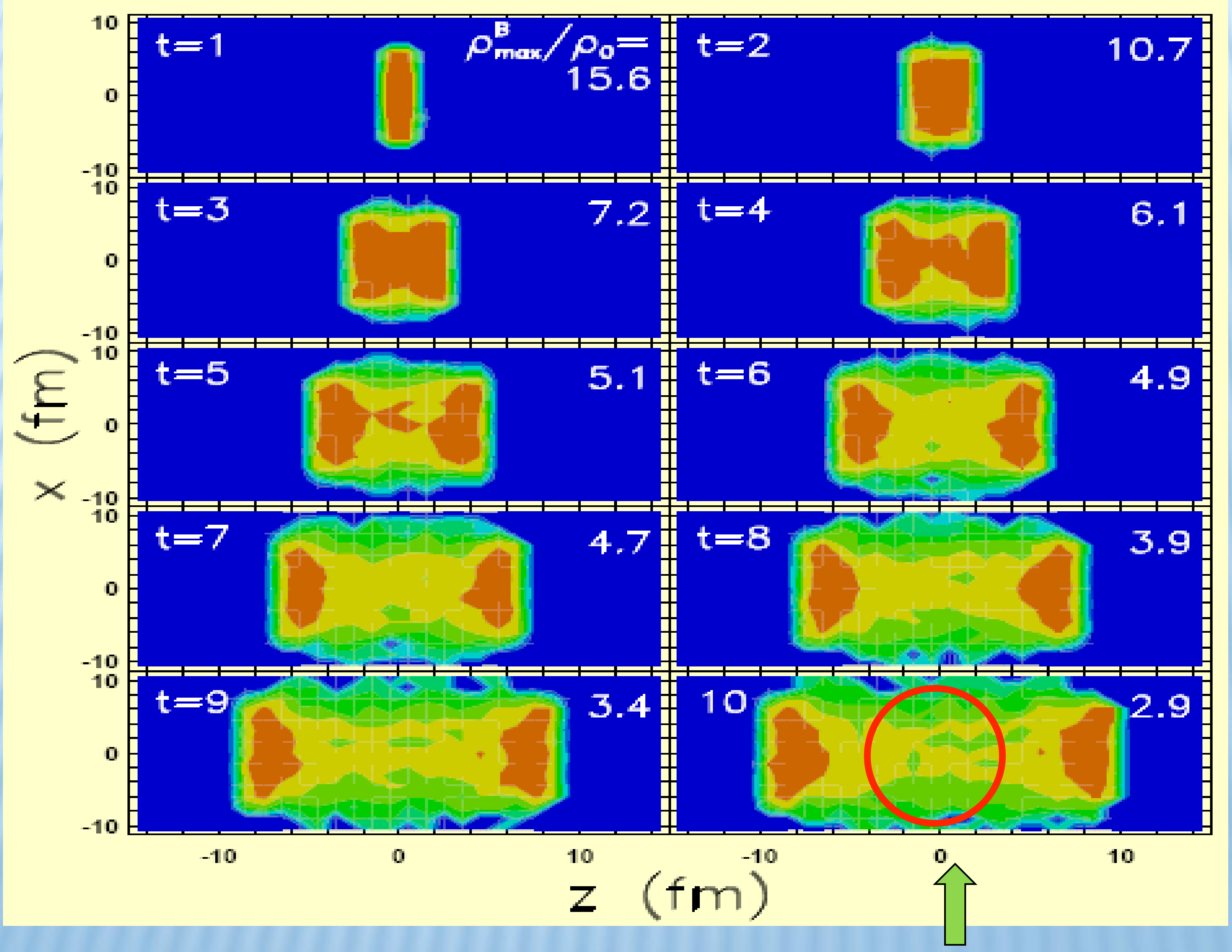


- the production of particles is described with the excitation and decay of color neutral strings

Central cell:
Relaxation to (local) equilibrium

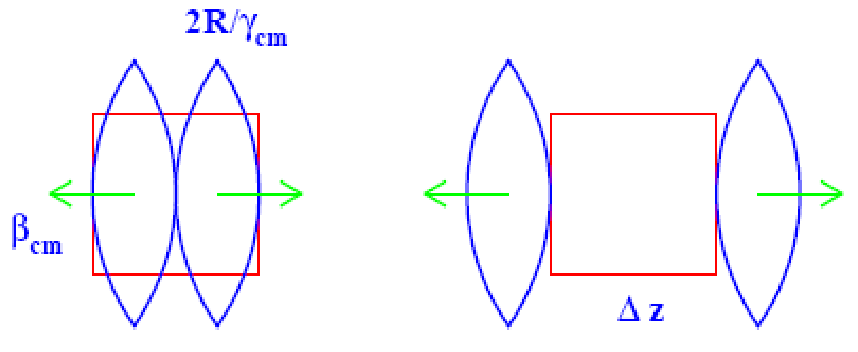
Pre-equilibrium: Homogeneity of baryon matter

L.Bravina et al., PRC 60 (1999) 024904



The local equilibrium in the central zone is quite possible

Equilibration in the Central Cell



$t^{cross} = 2R/(\gamma_{cm}\,\beta_{cm})$ $\qquad$ $t^{eq} \geq t^{cross} + \Delta z/(2\beta_{cm})$

Kinetic equilibrium:

Isotropy of velocity distributions

Isotropy of pressure

Thermal equilibrium:

Energy spectra of particles are described by Boltzmann distribution

L.Bravina et al., PLB 434 (1998) 379; JPG 25 (1999) 351

$$\frac{dN_i}{4\pi pEdE} = \frac{Vg_i}{(2\pi\hbar)^3}\exp\left(\frac{\mu_i}{T}\right)\exp\left(-\frac{E_i}{T}\right)$$

Chemical equlibrium:

Particle yields are reproduced by SM with the same values of $(T,\ \mu_B,\ \mu_S)$:

$$N_i = \frac{Vg_i}{2\pi^2\hbar^3}\int_0^\infty p^2dp\exp\left(\frac{\mu_i}{T}\right)\exp\left(-\frac{E_i}{T}\right)$$

Statistical model of ideal hadron gas

input values

output values

$$\varepsilon^{\rm mic} = \frac{1}{V}\sum_i E_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm B}^{\rm mic} = \frac{1}{V}\sum_i B_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm S}^{\rm mic} = \frac{1}{V}\sum_i S_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}).$$

Multiplicity

Energy

Pressure

Entropy density

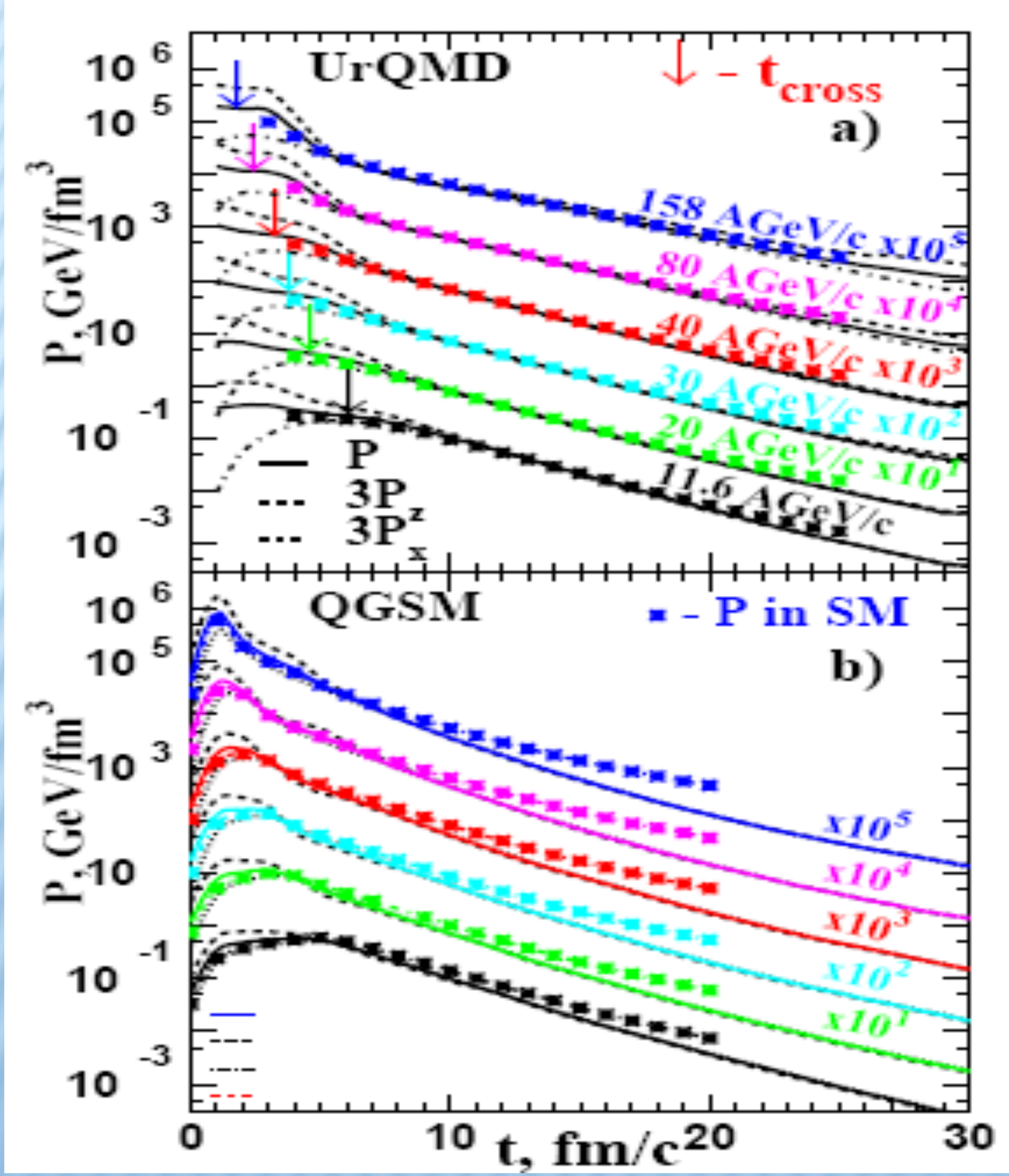
$$N_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 f(p,m_i)dp,$$

$$E_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 \sqrt{p^2+m_i^2}\, f(p,m_i)dp$$

$$P^{\rm SM} = \sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 \frac{p^2}{3(p^2+m_i^2)^{1/2}}\, f(p,m_i)dp$$

$$s^{\rm SM} = -\sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty f(p,m_i)\,\left[\ln f(p,m_i)-1\right]\, p^2 dp$$

Kinetic Equilibrium



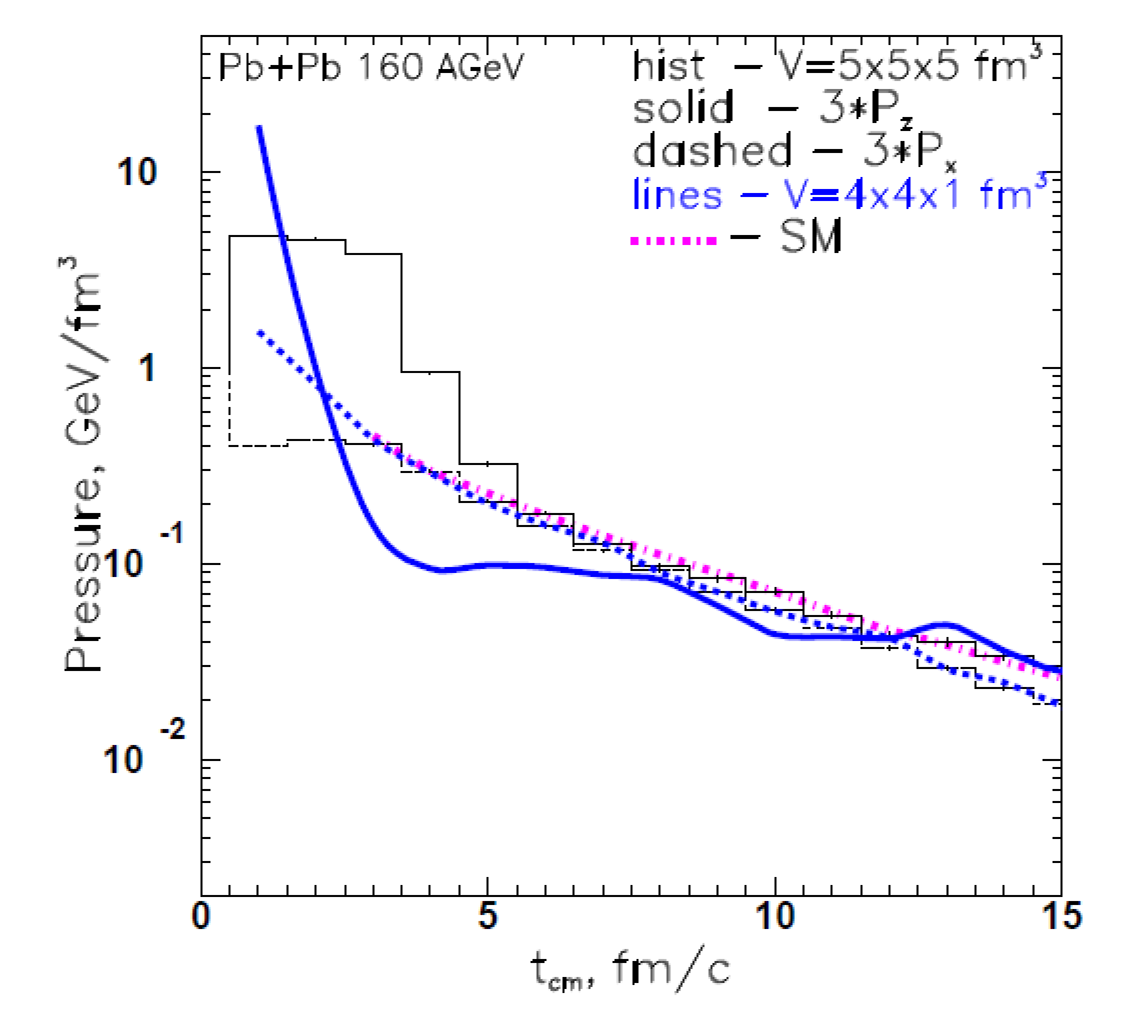
Isotropy of pressure

L.Bravina et al., PRC 78 (2008) 014907

Pressure becomes isotropic for all energies from 11.6 AGeV to 158 AGeV

Kinetic Equilibrium in a Small Cell

Isotropy of pressure

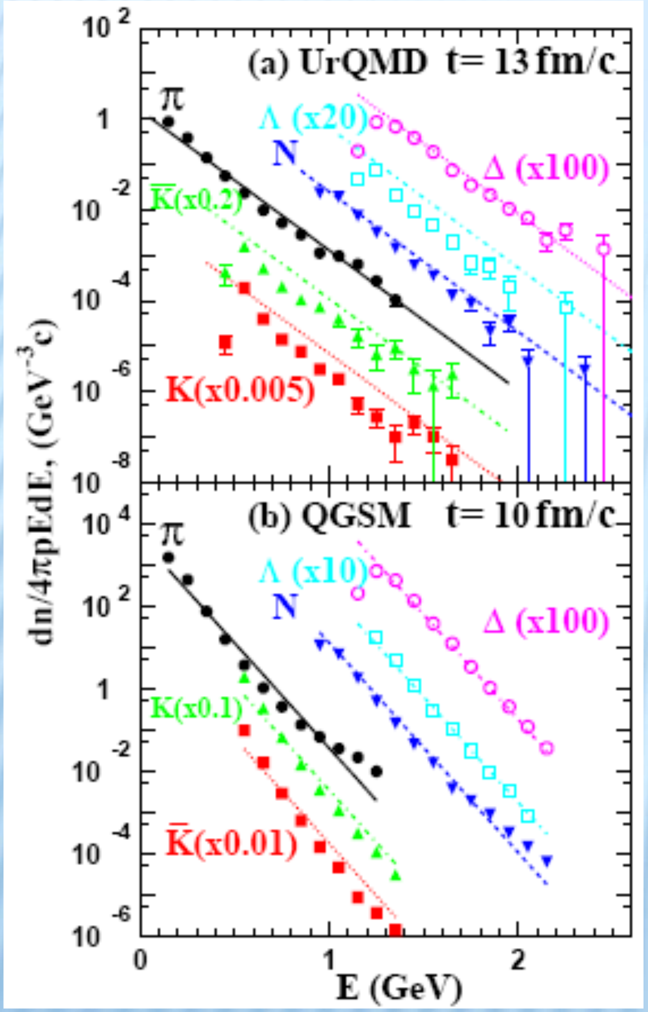


L. Bravina et al., JPG 25 (1999) 351

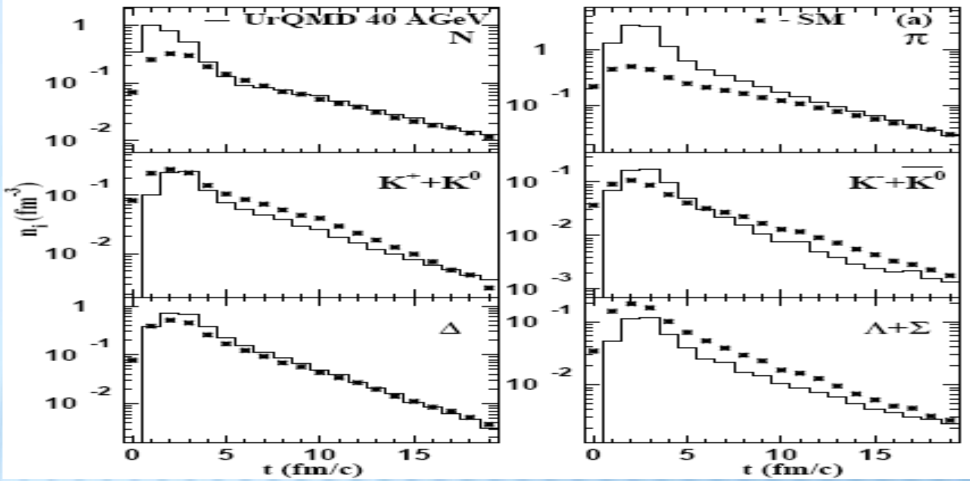
Longitudinal and transverse pressures in a small cell (4x4x1)fm^3 converge at the same rate as those in a larger cell (5x5x5)fm^3

Thermal and Chemical Equilibrium

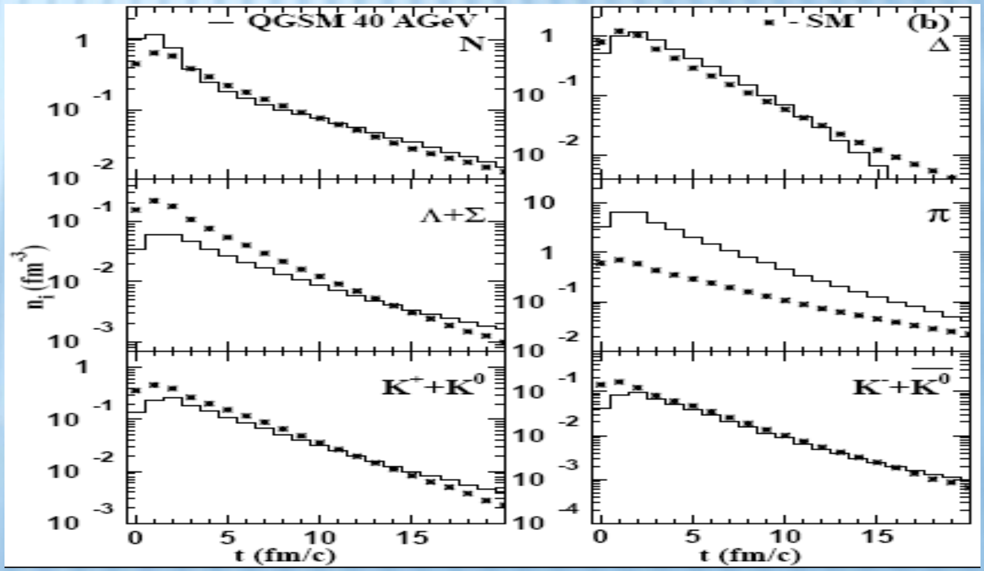
Boltzmann fit to the energy spectra



Particle yields



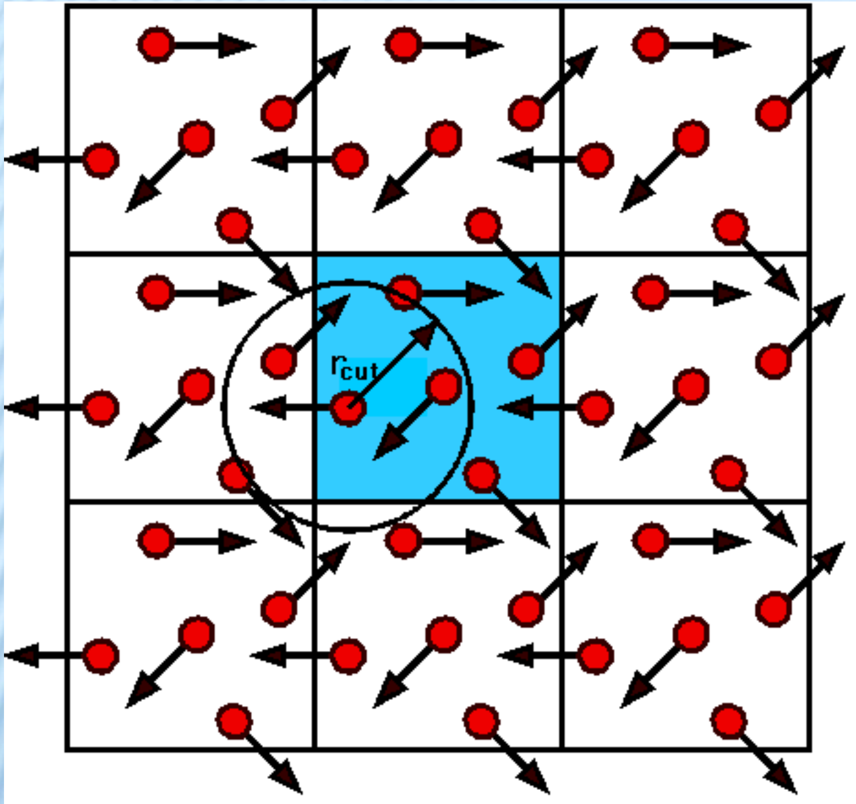
PRC 78 (2008) 014907



Thermal and chemical equilibrium seems to be reached

Infinite hadron gas: a box with periodic boundary conditions

BOX WITH PERIODIC BOUNDARY CONDITIONS



M.Belkacem et al., PRC 58, 1727 (1998)

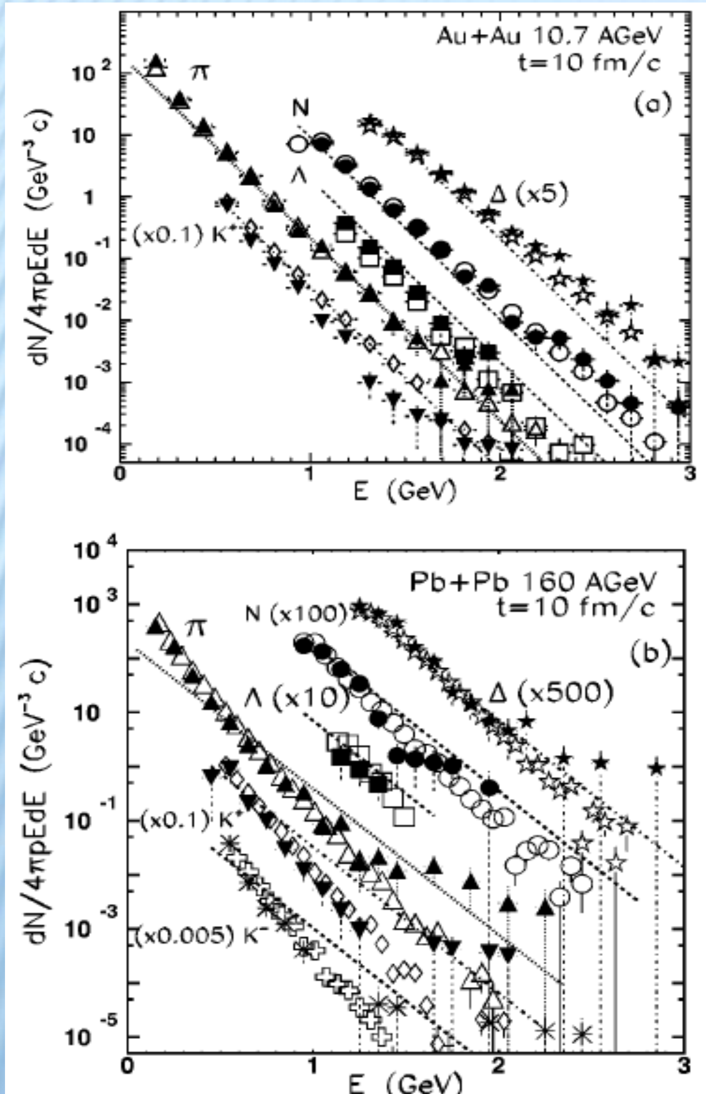
Model employed: UrQMD
55 different baryon species
(N, Δ, hyperons and their
resonances with
$m \leq 2.25$ GeV/c^2),
32 different meson species
(including resonances with
$m \leq 2$ GeV/c^2) and their
respective antistates.
For higher mass excitations
a string mechanism is invoked.

Initialization: (i) nucleons are uniformly distributed in a configuration space;
(ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

Test for equilibrium: particle yields and energy spectra

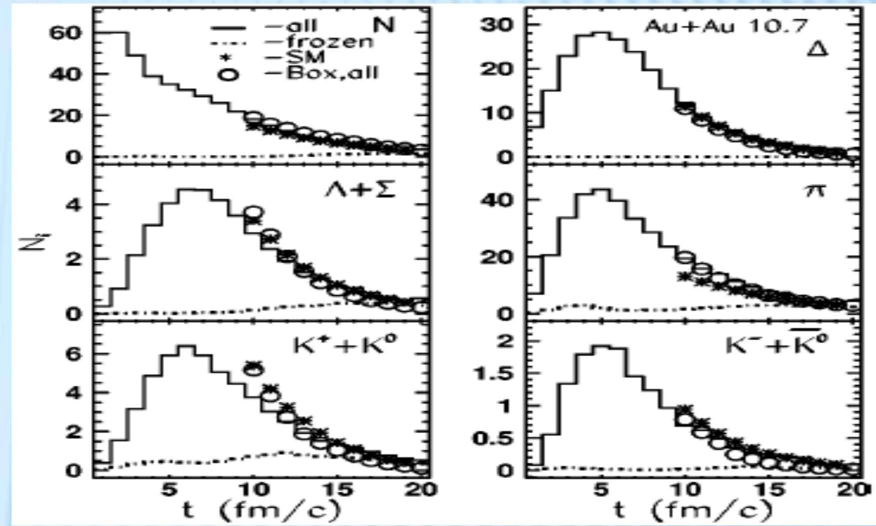
THERMAL AND CHEMICAL EQUILIBRIUM

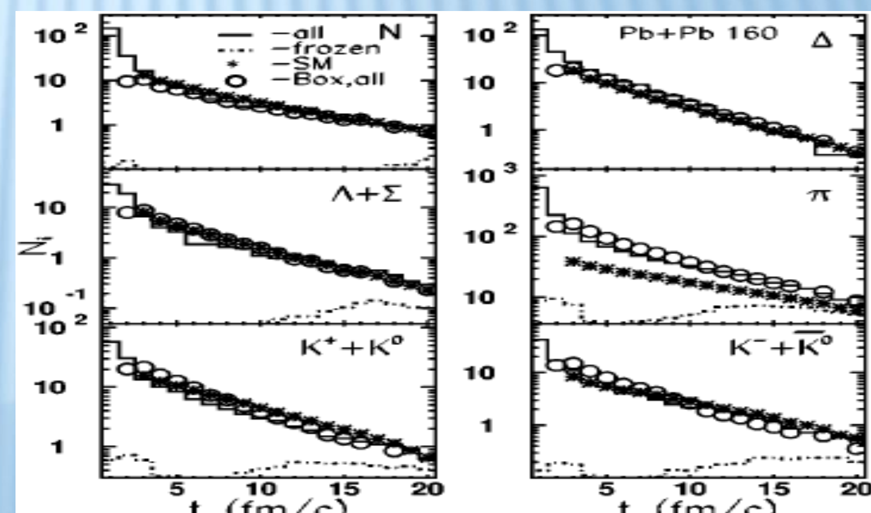
Boltzmann fit to the energy spectra



L.Bravina et al., PRC 62, 064906 (2000)

Hadron yields



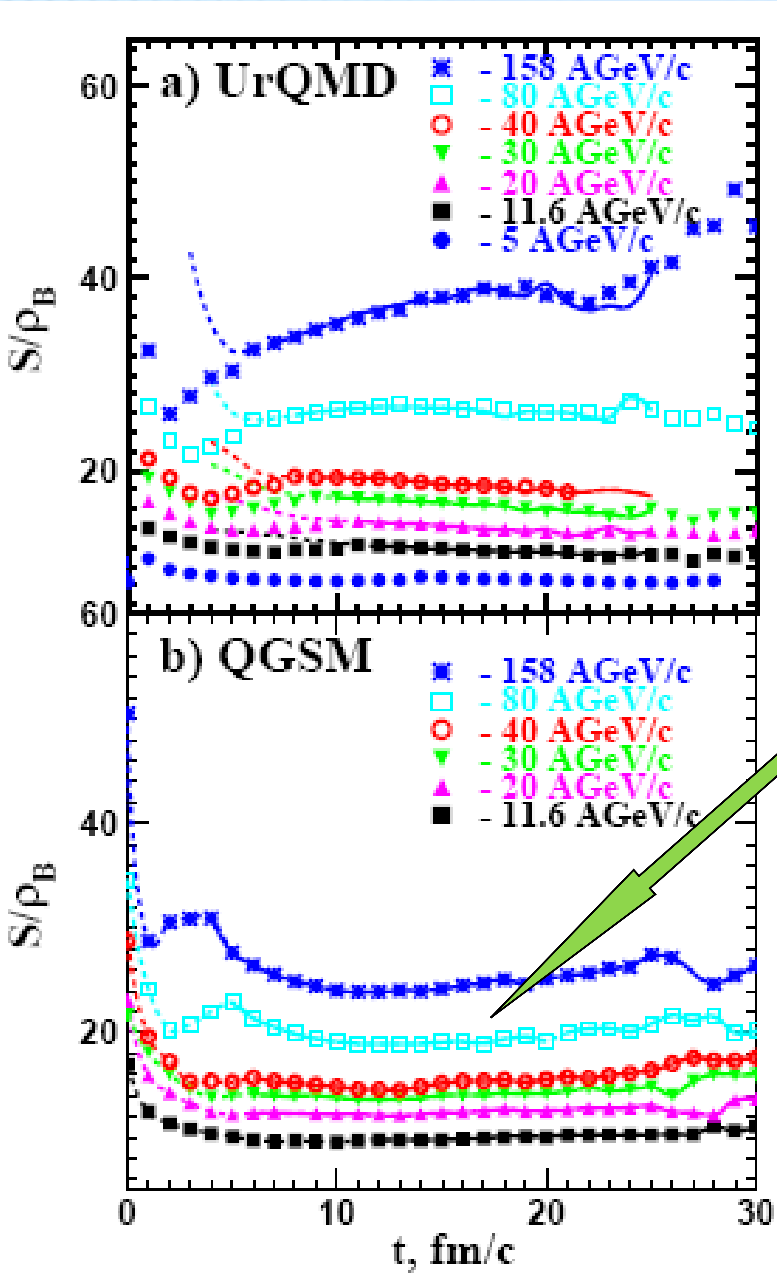


Box calculations are on the top of the cell results

Equation of State

T vs. energy, etc

Isentropic expansion

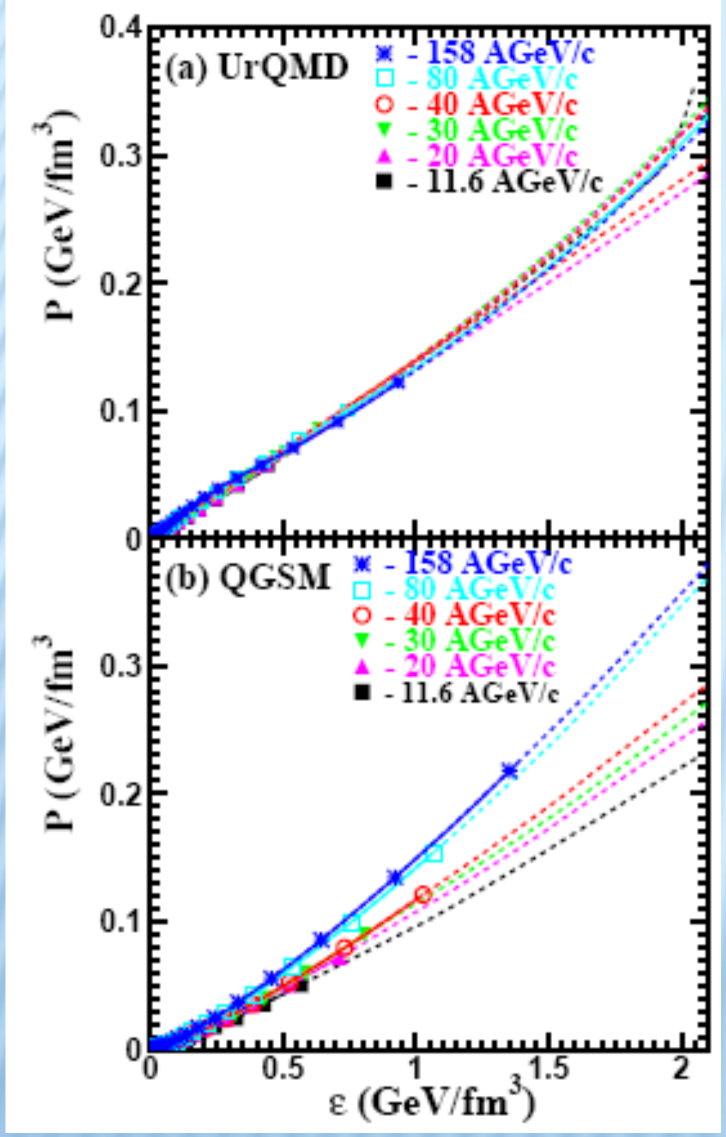


Expansion proceeds isentropically (with constant entropy per baryon). This result supports application of hydrodynamics

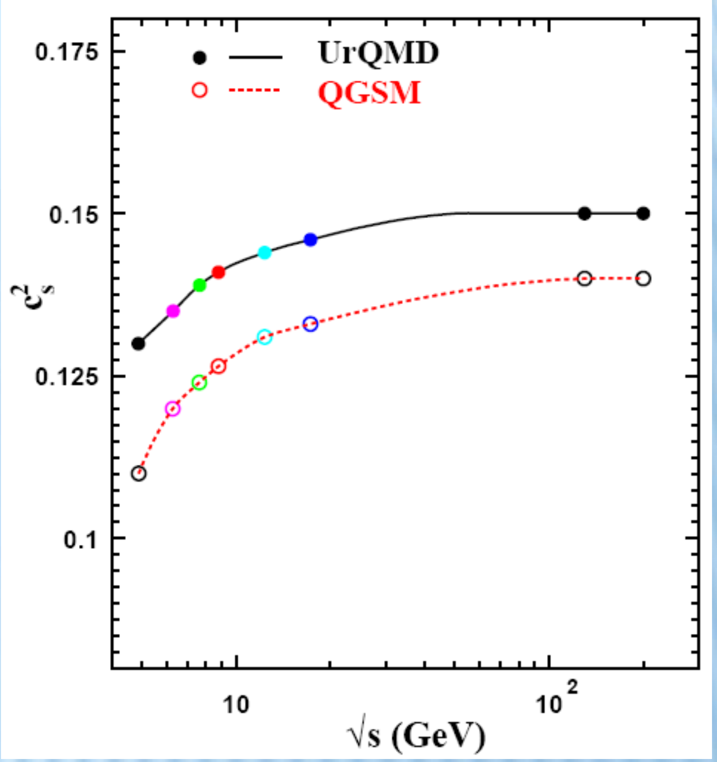
$$s/\rho_{\mathrm{B}} = \mathrm{const} = 12(\mathrm{AGS}),\ 20(40),\ 38(\mathrm{SPS})$$

Equation of State in the cell

pressure vs. energy



sound velocity



P/ε = const very early (!)

$P/\varepsilon = 0.13(\mathrm{AGS}),\ 0.14(40),\ 0.146(\mathrm{SPS}),\ 0.15(\mathrm{RHIC})$

Conclusions

- *MC models favor early pre-equilibration of hot and dense nuclear matter already at $t \approx 2$ fm/c*
- *After that the expansion of matter in the central cell proceeds isentropically with constant s/ρ_B (hydro!)*
- *The EOS has a simple form: $P/\varepsilon = const$ (hydro!) even at far-from-equilibrium stage*
- *The speed of sound C^2_s varies from 0.12 (AGS) to 0.14 (40 AGeV), and to 0.15 (SPS & RHIC) => saturation*
- *Good agreement between the cell and box results*

Back-up Slides

Bjorken Model

Lorentz-invariant solution

[1] J.D. Bjorken, Phys. Rev. D **27** (1983) 140
[2] E.P.T. Liang, Astrophys. J. **211** (1977) 361

Initial conditions, assumptions:

- the system expands into vacuum (i.e. no walls, pistons, etc)
- the system occupies initial volume V_0, which possesses certain symmetry (e.g., rectangular box, cylinder, or sphere) - otherwise the hydrodynamic equations are too cumbersome to be solved analytically
  But: such initial conditions are relativistically non-invariant.

Let us introduce *proper time* $\tau = \mathrm{t}/\gamma = \sqrt{\mathrm{t^2 - x^2}}$ and assume that the initial conditions depend only on τ_0 and that the macroscopic variables $(\mathrm{T}, \varepsilon, \mathrm{P}, \ldots)$ are functions of τ . For simplicity, consider the one-dimensional case $\mathrm{x}^\mu = (\mathrm{t}, 0, 0, \mathrm{z})$, $\mathrm{u}^\mu = \frac{1}{\tau}(\mathrm{t}, 0, 0, \mathrm{z})$. From the equations of perfect fluid

$$\partial_\mu \mathrm{T}^{\mu\nu} = 0$$

we get

$$\frac{\partial \varepsilon}{\partial \tau} = -\frac{\varepsilon + \mathrm{P}}{\tau}, \qquad \varepsilon(\tau_0) = \varepsilon_0$$

This is the basic differential equation of Bjorken hydrodynamic model.

Bibliography:

Microscopic models and effective equation of state in nuclear collisions at FAIR energies

L.V. Bravina et al.. Phys. Rev. C78 (2008) 014907 ; DOI: 10.1103/PhysRevC.78.014907

Equation of state at FAIR energies and the role of resonances

E E Zabrodin et al.. J. Phys. G36 (2009) 064065; DOI: 10.1088/0954-3899/36/6/064065

Effective equation of state of hot and dense matter in nuclear collisions around FAIR energy

L. Bravina , E. Zabrodin , EPJ Web Conf. 95 (2015) 01003; DOI: 10.1051/epjconf/20159501003