RADIATIVE TRANSITIONS OF ELECTRONS BETWEEN LANDAU LEVELS IN A MODERATELY STRONG MAGNETIC FIELD

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Abstract

We investigate the processes of radiative transitions of electrons between the Landau levels, $e_l \to e_n + \gamma$, in a moderately strong magnetic field. Under such conditions, it is necessary to take into account transitions in which both the initial and final electrons can be in states corresponding to arbitrary Landau levels. The results obtained can be used as building blocks in calculating the efficiency of the electron-positron plasma generation in accretion disks of the Kerr black holes, considered by specialists as the most probable sources of a short cosmological gamma-ray bursts.

Introduction

When a magnetic field $B \sim B_e = m^2/e \approx 4.4 * 10^{13} G$, we need to use the exact solutions of the Dirac equation, which are the eigenfunctions of the covariant magnetic polarization operator [1,2] $\hat{\mu}_z = m\Sigma_z - i\gamma_0\gamma_5[\boldsymbol{\Sigma} \times \boldsymbol{P}]_z$, where $\boldsymbol{\Sigma} = \gamma_0\boldsymbol{\gamma}\gamma_5, \boldsymbol{P} = -i\boldsymbol{\nabla} + e\boldsymbol{A}$. We take the frame where the field is directed along the z axis, and the Landau gauge with the four-potential $A^\lambda = (0,0,xB,0)$. In this approach, the electron wave functions have the form

$$\Psi^s_{p,n} = \frac{e^{-i(E_n t - p_y y - p_z z)}\; U^s_n(\xi)}{\sqrt{4E_n M_n (E_n + M_n)(M_n + m) L_y L_z}}, \text{ where } E_n = \sqrt{M_n^2 + p_z^2},\; M_n = \sqrt{m^2 + 2n\beta},\; \beta = eB,\; \xi = \sqrt{\beta}\left(x + \frac{p_y}{\beta}\right).$$

The functions $\Psi^s_{p,n}$ satisfy the equation $\hat{\mu}_z \Psi^s_{p,n} = s M_n \Psi^s_{p,n}$. The bispinors take the form:

$$U_n^-(\xi) = \begin{pmatrix} -i\sqrt{2n\beta}\, p_z V_{n-1}(\xi) \\ (E_n + M_n)(M_n + m)V_n(\xi) \\ -i\sqrt{2n\beta}(E_n + M_n)V_{n-1}(\xi) \\ -p_z(M_n + m)V_n(\xi) \end{pmatrix},\; U_n^+(\xi) = \begin{pmatrix} (E_n + M_n)(M_n + m)V_{n-1}(\xi) \\ -i\sqrt{2n\beta}\, p_z V_n(\xi) \\ p_z(M_n + m)V_{n-1}(\xi) \\ i\sqrt{2n\beta}(E_n + M_n)V_n(\xi) \end{pmatrix}.$$

Here, $V_n(\xi)$ $(n = 0, 1, 2, \dots)$ are the well-known normalized harmonic oscillator functions, which are expressed in terms of the Hermite polynomials $H_n(\xi)$: $V_n(\xi) = \sqrt[4]{\beta/\pi}\, \frac{e^{-\xi^2/2}}{\sqrt{2^n n!}} H_n(\xi)$.

Results

Using the standard computational technique, see e.g. Refs. [3-5], we found a set of formulas for the probabilities of different polarization channels $e^s_l \to e^{s'}_n + \gamma^\lambda$. Here, the following notations are used: l and n – initial and final Landau levels; $s, s' = \pm 1$ and $\lambda = 1, 2$ are the electron and photon polarization states respectively.

$$W^{1ss'}_{ln} = \frac{\alpha m^2 b}{4E} \int_0^{\rho_1} d\rho\, e^{-\rho} \rho^{l-n-1} \sqrt{\frac{(1 - ss')\rho_1 + (1 + ss')\rho_2 - 2\rho}{(1 + ss')\rho_1 + (1 - ss')\rho_2 - 2\rho}}$$
$$\left[\sqrt{\frac{n!}{(l-1)!}}\left(1 + s\frac{m}{M_l}\right)\left(1 - s'\frac{m}{M_n}\right) L_n^{l-n-1}(\rho) - ss' \sqrt{\frac{(n-1)!}{l!}}\left(1 - s\frac{m}{M_l}\right)\left(1 + s'\frac{m}{M_n}\right)\rho\, L_{n-1}^{l-n+1}(\rho)\right]^2$$

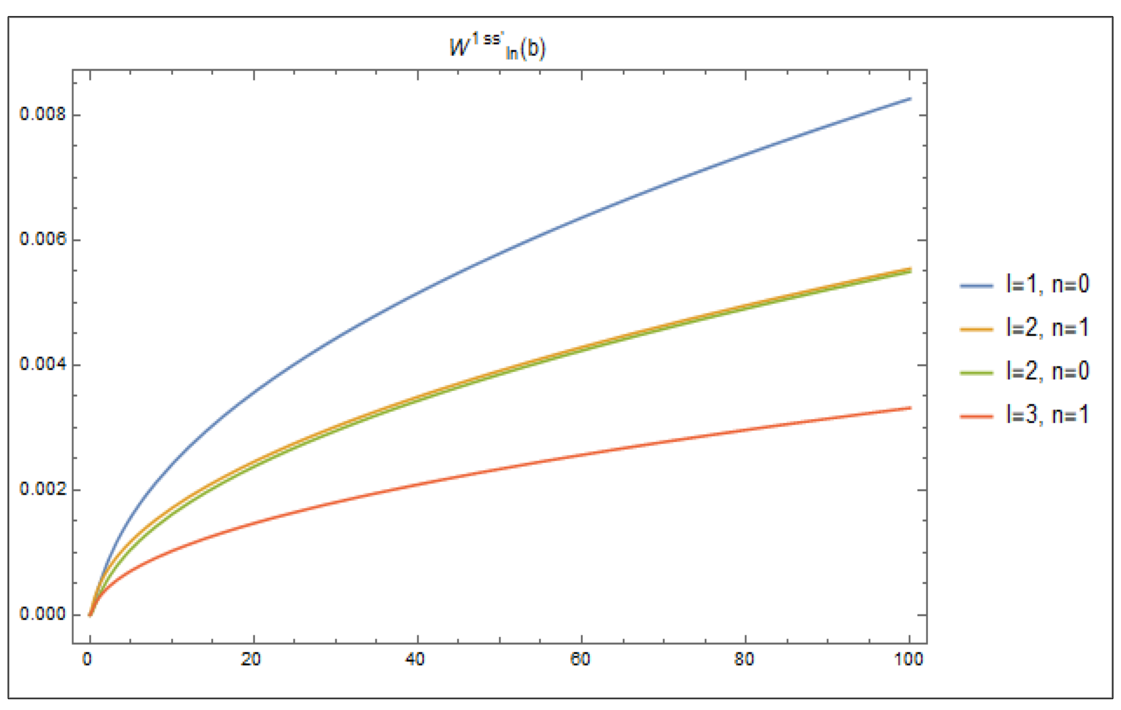
$$W^{2ss'}_{ln} = \frac{\alpha m^2 b}{4E}\left(\frac{1 - ss'}{2}\rho_1 + \frac{1 + ss'}{2}\rho_2\right) \int_0^{\rho_1} d\rho\, e^{-\rho} \rho^{l-n-1} \sqrt{\frac{(1 + ss')\rho_1 + (1 - ss')\rho_2 - 2\rho}{(1 - ss')\rho_1 + (1 + ss')\rho_2 - 2\rho}}$$
$$\left[\sqrt{\frac{n!}{l!}}\left(1 - s\frac{m}{M_l}\right)\left(1 - s'\frac{m}{M_n}\right) L_n^{l-n}(\rho) + ss' \sqrt{\frac{(n-1)!}{(l-1)!}}\left(1 + s\frac{m}{M_l}\right)\left(1 + s'\frac{m}{M_n}\right) L_{n-1}^{l-n}(\rho)\right]^2$$

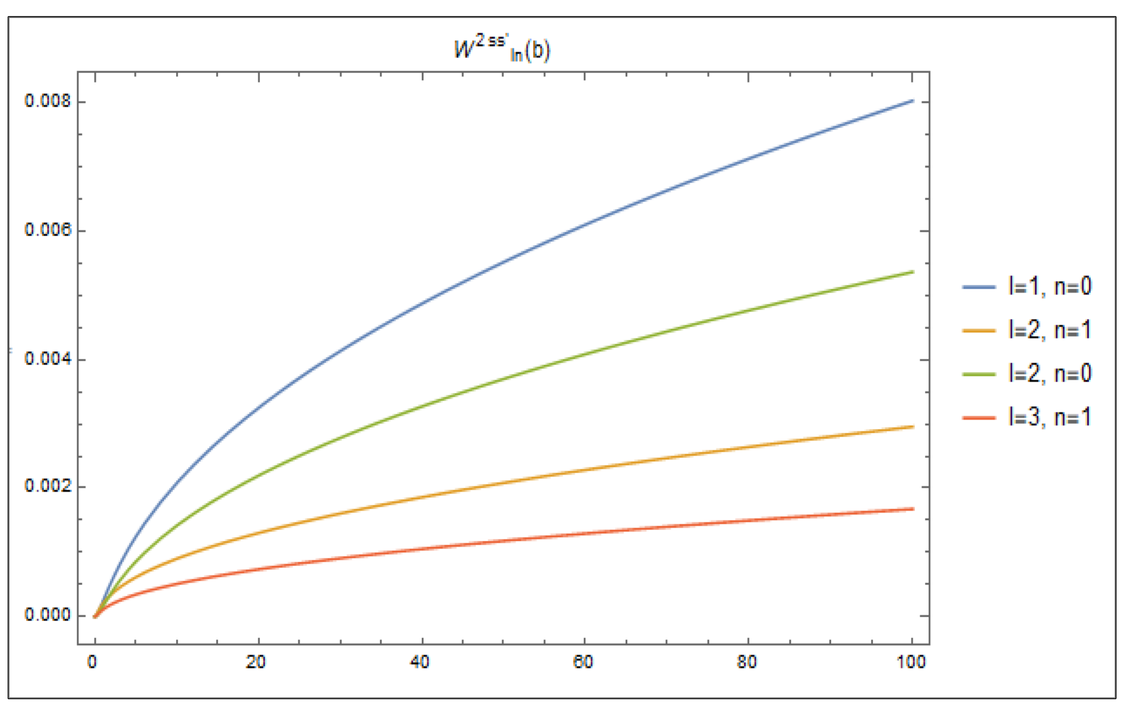
Designations:

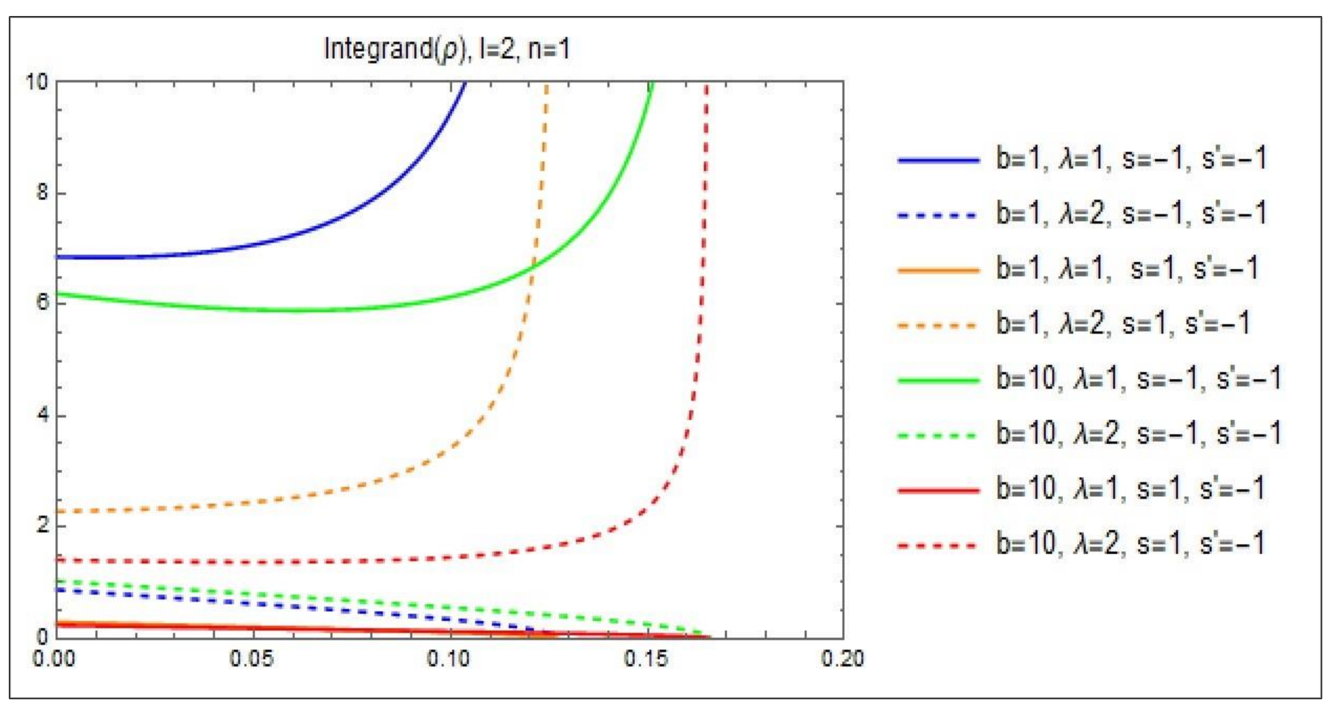
$$b = B/B_e$$
$$M_n = m\sqrt{2nb + 1}$$
$$\rho = q_\perp^2/(2\beta)$$
$$q_\perp = \sqrt{q_x^2 + q_y^2}$$
$$\rho_1 = (l - n)\frac{M_l - M_n}{M_l + M_n}$$
$$\rho_2 = (l - n)\frac{M_l + M_n}{M_l - M_n}$$

Here, α is the fine structure constant, E is the initial electron energy, $L^s_n(\rho)$ are the generalized Laguerre polynomials with a completion of $L^s_{-1}(\rho) = 0$, $q_\perp$ is the photon momentum projection on the plane orthogonal to the magnetic field direction.

Below, the plots 1-2 show the field dependence of the process probability averaged over the initial and summarized over the final polarization states in some occasions; on the plot 3, the photon spectra over the transverse momentum magnitude are presented.







Conclusion

The integrands in four of the eight probabilities have square-root singularities. It is interesting to compare these formulas with the corresponding results obtained in the limit of a superstrong field $B \gg B_e$, where it was found that all the probabilities coincide. This result is confirmed by taking the limit, when $\rho_1 \simeq \rho_2 \simeq 1$. However, this assertion is valid only in an asymptotically strong magnetic field. In the case of a moderately strong field, exact formulas should be used for the analysis of the processes $e_l \to e_n + \gamma$.

References

1. Sokolov A A and Ternov I M 1968 *Synchrotron Radiation* (Oxford: Pergamon)
2. Melrose D B and Parle A J 1983 *Aust. J. Phys.* **36** 755
3. Kuznetsov A and Mikheev N 2013 *Electroweak Processes in External Active Media* (Berlin, Heidelberg: Springer-Verlag)
4. Kuznetsov A V, Rumyantsev D A and Savin V N 2014 *Int. J. Mod. Phys. A* **29** 1450136
5. Kuznetsov A V, Rumyantsev D A and Savin V N 2016 *J. Phys.: Conf. Ser.* **675** 032019