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Description of processes passing at finite space-time intervals in the framework of quantum field theory

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Outline

- Introduction: the difficulties of the standard description, the idea of the novel approach
- Decay of an unstable particle
- Neutrino oscillations
- Conclusion

Introduction

The standard S-matrix formalism is not appropriate for describing finite space-time interval processes: $t \to \pm\infty$.

Neutrino oscillations: the standard QM description in terms of plane waves ➔ violation of energy-momentum conservation; the QM and QFT description in terms of wave packets ➔ complicated calculations.

- C. Giunti, C.W. Kim, J.A. Lee and U.W. Lee, «On the treatment of neutrino oscillations without resort to weak eigenstates,» Phys. Rev. D **48** (1993) 4310.

The idea is to modify the standard S-matrix formalism: consider a process as a whole; start with the Feynman diagram technique in the coordinate representation; pass to the momentum representation in a way corresponding to the experimental setting.

The approach was put forward in the paper

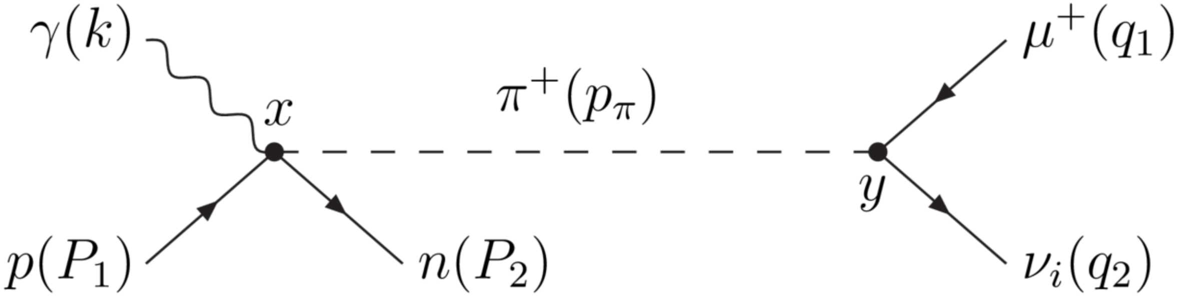
- I.P. Volobuev, «Quantum field-theoretical description of neutrino and neutral kaon oscillations,» Int. J. Mod. Phys. A **33** (2018) no.13, 1850075

and developed in the papers

- V.O. Egorov and I.P. Volobuev, «Neutrino oscillation processes in a quantum field-theoretical approach,» Phys. Rev. D **97** (2018) no.9, 093002,
- V.O. Egorov and I.P. Volobuev, «Neutrino oscillation processes with a change of lepton flavor in quantum field-theoretical approach,» arXiv:1712.04335 [hep-ph].

Decay of an unstable particle

Let us consider a process of pion photoproduction with its subsequent decay to a lepton pair:



Suppose x and y are separated by a macro-scopic interval.

W. Grimus and P. Stockinger [Phys. Rev. D **54** (1996) 3414]: the virtual particles propagating at macroscopic distances are almost on the mass shell. It means that $\left|p^2 - m_i^2\right| / \vec{p}^{\,2} << 1$.

The exact scalar field propagator:

$$D_e(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{m^2 + M\left(k^2\right) - k^2} \underset{\substack{\uparrow \\ \text{Mass shell}}}{\approx} \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{m^2 - k^2 - im\Gamma}$$

The amplitude in the coordinate representation ➔ in accordance with the usual Feynman rules. If we integrate with respect to x and y, we lose the information about the space-time interval between them. In order to save it we introduce the delta-function $\delta\left(y^0 - x^0 - T\right)$ into the integral. It is equivalent to the replacement:

$$D_e(y-x) \to D_e(y-x)\delta\left(y^0 - x^0 - T\right)$$

We arrive at the time-dependent propagator of the unstable scalar particle in the momentum representation:

$$D(p,T) = \int d^4x\, e^{ipx} D_e(x)\delta\left(x^0 - T\right) \approx \frac{i}{2p^0} e^{-i\frac{m^2 - p^2 - im\Gamma}{2p^0}T}$$

When almost on the mass shell

The amplitude in the momentum representation for the case $y^0 - x^0 = T$:

$$M^{(i)} = i\frac{G_F}{2\sqrt{2}p_\pi^0}\cos\theta f_\pi m_{(\mu)} U_{2i}^* e^{-i\frac{m_\pi^2 - p_\pi^2 - im_\pi\Gamma}{2p_\pi^0}T}\bar{\nu}_i(q_2)\left(1-\gamma^5\right)v(q_1)M_P(k, P_1, p_\pi, P_2)$$

The squared modulus of the amplitude:

$$\left\langle \left|M^{(i)}\right|^2 \right\rangle = \left\langle \left|M_1\right|^2 \right\rangle \left\langle \left|M_2^{(i)}\right|^2 \right\rangle \frac{1}{4\left(p_\pi^0\right)^2} e^{-\frac{m_\pi \Gamma}{p_\pi^0}T}$$

The squared modulus of the amplitude of the photoproduction process

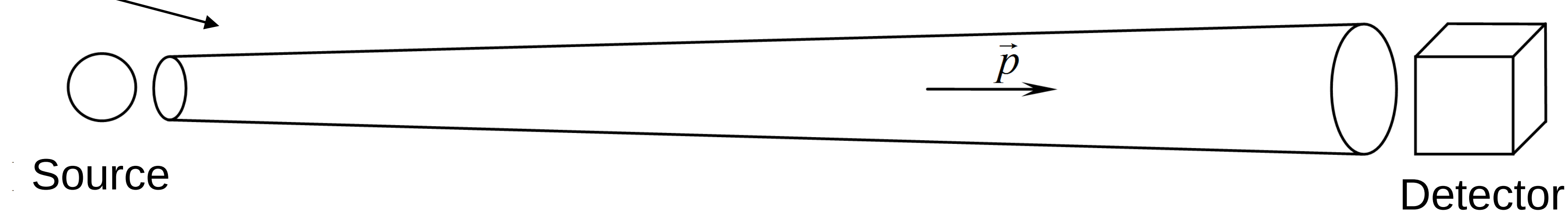
The squared modulus of the amplitude of the pion decay process

Here we have only one amplitude, thus there is no interference.

Let us find the probability of the process. |The amplitude|^2 is multiplied by the delta-function of energy-momentum conservation $(2\pi)^4\delta(k+P_1-P_2-q_1-q_2)$ and is integrated with respect to the momenta of the final particles.

The integration → the variation in the pion momenta → contradiction with the experimental situation, since the pions propagate from the production to the detection point → one must calculate the differential probability of the process, where p_π is fixed.

It can be obtained by the additional multiplication of |the amplitude|^2 by $2\pi\delta(p_\pi - p)$, or, equivalently, by the replacement of p_π by p there and by its multiplication by $\underbrace{2\pi}\delta(k+P_1-P_2-p)$.



Summing over the final neutrino type i and integrating with respect to the final particles' momenta, we arrive at the differential probability. It factorizes as follows:

$$\frac{dW}{d\vec{p}} = \frac{dW_1}{d\vec{p}} W_2 e^{-\frac{m_\pi \Gamma}{|\vec{p}|} L}$$

Here we substituted $T = Lp^0 / |\vec{p}|$, since the π-meson is almost on the mass shell

which reproduces the anticipated result.

The additional delta-function fixes not only the direction of pion momentum, but also its modulus $|\vec{p}_\pi| = |\vec{p}|$ ➔ we must also integrate with respect to $|\vec{p}|$. The final result for the probability of the process:

$$\int \underbrace{\frac{dW}{d\vec{p}}}_{\substack{\text{Singular} \\ \text{(extra } \delta)}} |\vec{p}|^2 d|\vec{p}| = \frac{dW_1}{d\Omega_{\vec{p}}} W_2 \Big|_{\underbrace{|\vec{p}| = |\vec{p}|^*}} e^{-\frac{m_\pi \Gamma}{|\vec{p}|^*} L}$$

the solution of the equation

$$k^0 + P_1^0 - P_2^0|_{\vec{P}_2 = \vec{k} + \vec{P}_1 - \vec{p}} - p^0 = 0$$

Neutrino oscillations

We work in the framework of the minimal extension of the SM by the right neutrino singlets. The charged-current interaction Lagrangian of the leptons:

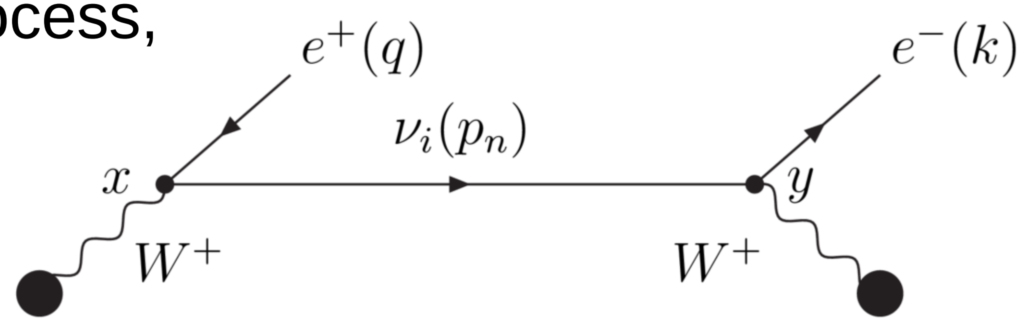
$$L_{cc} = -\frac{g}{2\sqrt{2}} \left(\sum_{i,k=1}^{3} \bar{l}_i \gamma^\mu (1 - \gamma^5) U_{ik} \nu_k W_\mu^- + h.c. \right),$$

The field of the charged lepton of the i-th generation

The neutrino mixing matrix (PMNS matrix)

The field of the neutrino mass eigenstate

Let us consider a process, where a neutrino is emitted and detected in the charged-current interaction with nuclei.



The time-dependent propagator of i-th neutrino mass eigenstate in the momentum representation:

$$S_i^c(p,T)=\int d^4x\, e^{ipx} S_i^c(x)\delta\left(x^0-T\right)\approx i\,\frac{\hat{p}+m_i}{2p^0}\,e^{-i\frac{m_i^2-p^2}{2p^0}T}$$

The amplitude of the process in the momentum representation for the case $y^0-x^0=T$:

$$M=-i\frac{G_F^2}{4p_n^0}\sum_{i=1}^{3}|U_{1i}|^2\,e^{-i\frac{m_i^2-p_n^2}{2p_n^0}T}\,j_\rho^{(2)}\bar{u}(k)\,\gamma^\rho\left(1-\gamma^5\right)\hat{p}_n\gamma^\mu\left(1-\gamma^5\right)v(q)\,j_\mu^{(1)}$$

It includes three terms, which interfere with each other, leading to the oscillations.

The squared modulus of the amplitude:

$$\left\langle|M|^2\right\rangle = \underbrace{\left\langle|M_1|^2\right\rangle}_{\text{Production process}}\underbrace{\left\langle|M_2|^2\right\rangle}_{\text{Detection process}}\frac{1}{4\left(p_n^0\right)^2}\left[1-4\sum_{\substack{i,k=1\\ i<k}}^{3}|U_{1i}|^2|U_{1k}|^2\sin^2\left(\frac{m_i^2-m_k^2}{4p_n^0}T\right)\right]$$

In order to find the differential probability of the process, one must multiply |the amplitude|^2 by $(2\pi)^4\delta\left(P^{(1)}+P^{(2)}-P^{(1')}-P^{(2')}-q-k\right)$ and by $2\pi\,\delta\left(P^{(1)}-P^{(1')}-q-p\right)$, substitute p instead of p_n and integrate with respect to the final particles' momenta. The result is as follows:

$$\frac{dW}{d\vec{p}} = \frac{dW_1}{d\vec{p}} W_2 \left[1 - 4\sum_{\substack{i,k=1\\ i<k}}^{3} |U_{1i}|^2 |U_{1k}|^2 \sin^2\left(\frac{m_i^2 - m_k^2}{4|\vec{p}|} L\right)\right]$$

The neutrino is almost on the shell → $T = Lp^0/|\vec{p}|$

This reproduces the well-known formula.

The final probability of detecting an electron is given by the last expression integrated with respect to $|\vec{p}|$:

$$\int\limits_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{dW}{d\vec{p}} |\vec{p}|^2 \, d|\vec{p}| = \int\limits_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{dW_1}{d\vec{p}} W_2 \left[1 - 4\sum_{\substack{i,k=1\\ i<k}}^{3} |U_{1i}|^2 |U_{1k}|^2 \sin^2\left(\frac{m_i^2 - m_k^2}{4|\vec{p}|} L\right)\right] |\vec{p}|^2 \, d|\vec{p}|$$

If one neglects the nuclear form-factors and considers only the ground state of the final nucleus, then

$$\int\limits_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{dW}{d\vec{p}} |\vec{p}|^2 \, d|\vec{p}| = \int\limits_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{dW_1}{d\vec{p}} W_2 \left[1 - 4 \sum_{\substack{i,k=1 \\ i<k}}^{3} |U_{1i}|^2 |U_{1k}|^2 \sin^2 \left(\frac{m_i^2 - m_k^2}{4|\vec{p}|} L \right) \right] |\vec{p}|^2 d|\vec{p}|$$

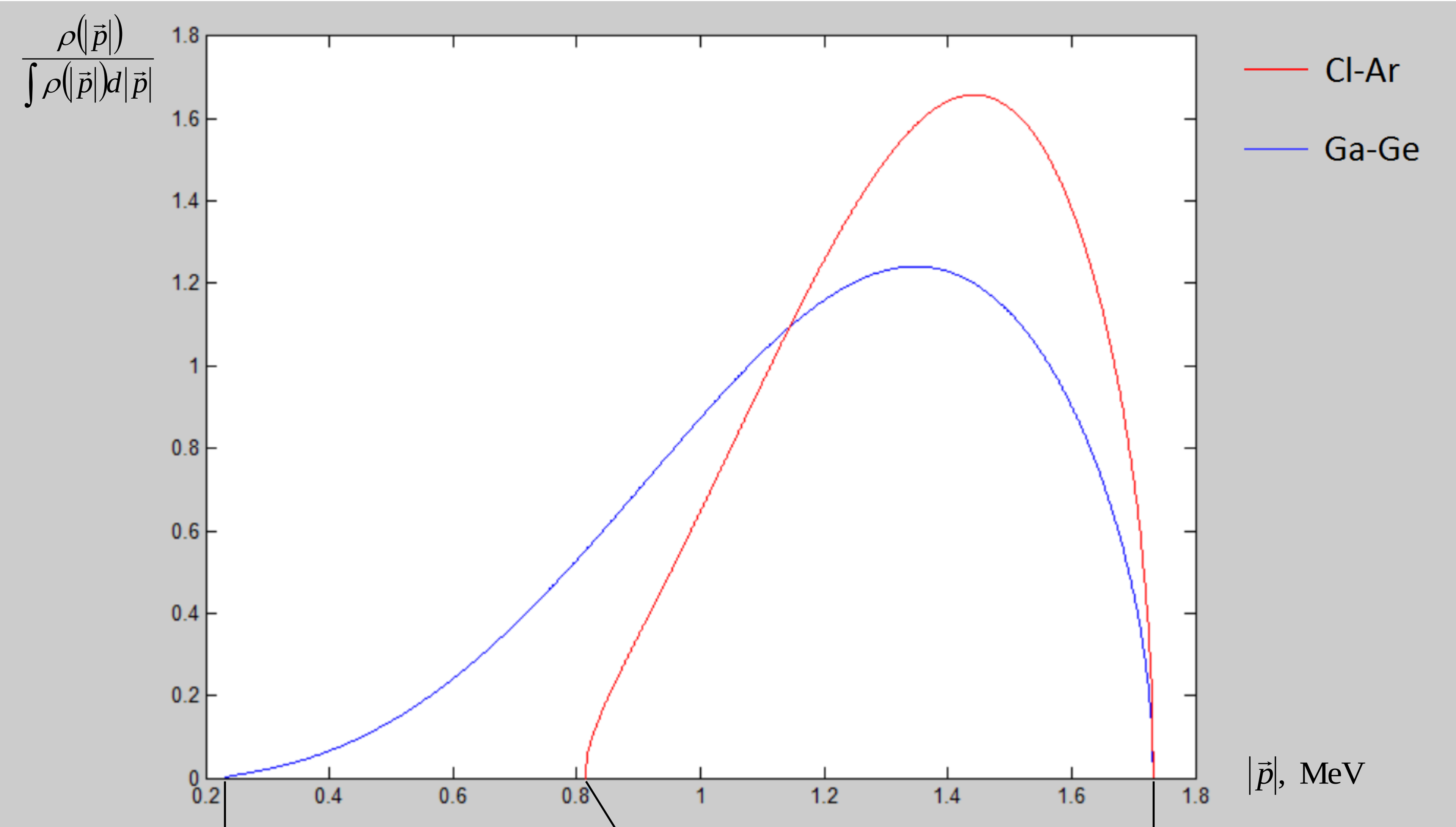
can be approximated by the function:

$$\rho(|\vec{p}|) = C\sqrt{\left(|\vec{p}|_{\max} - |\vec{p}|\right)\left(|\vec{p}|_{\max} - |\vec{p}| + 2m\right)}\left(|\vec{p}|_{\max} - |\vec{p}| + m\right)|\vec{p}|^2 \sqrt{\left(|\vec{p}| - |\vec{p}|_{\min}\right)\left(|\vec{p}| - |\vec{p}|_{\min} + 2m\right)}\left(|\vec{p}| - |\vec{p}|_{\min} + m\right)$$

Is determined by the production process.

Is determined by the detection process.

Let us consider the production process $^{15}\mathrm{O} \to {}^{15}\mathrm{N} + e^+ + \nu_i$ and the registration with the Cl-Ar or Ga-Ge detector.

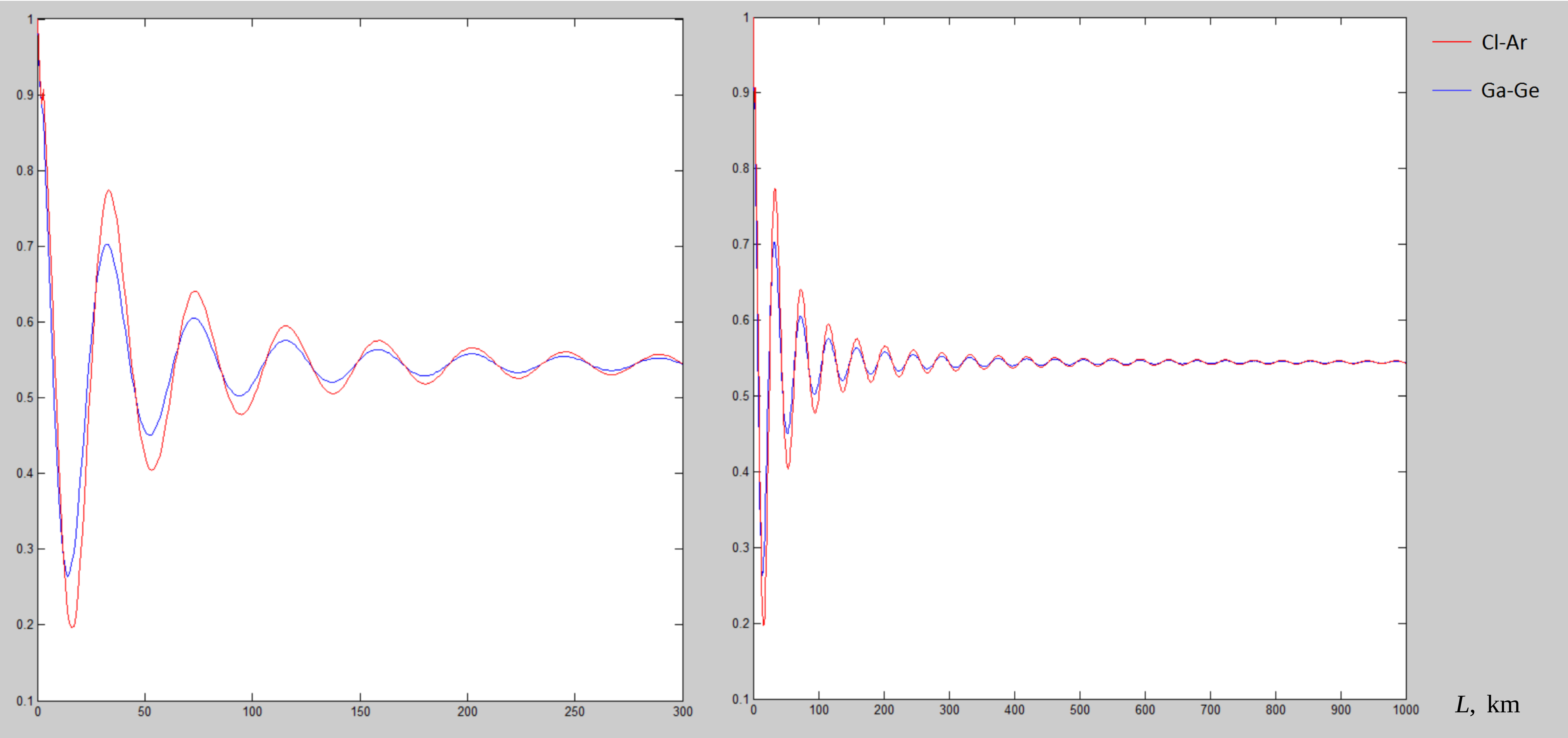


$|\vec{p}|_{\min}^{\text{Ga-Ge}} = 0.232\,\text{MeV}$ $\quad |\vec{p}|_{\min}^{\text{Cl-Ar}} = 0.814\,\text{MeV}$ $\quad |\vec{p}|_{\max} = 1.732\,\text{MeV}$

The results of numerical integration with the parameters:

$$m_2^2 - m_1^2 = 10^{-4}\ \text{eV}^2, \quad m_3^2 - m_1^2 = m_3^2 - m_2^2 = 10^{-3}\ \text{eV}^2,$$

$$\theta_{12} = 0.59, \quad \theta_{13} = 0.16, \quad \theta_{23} = 0.70.$$



The intermediate neutrinos have a momentum distribution ➔ the oscillations fade out with L. This fact gives rise to the coherence length.

By analogy with interference in optics → the visibility function:

$$V(L) = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}},$$

where $I_{\max/\min}$ stands for the relative neutrino registration probability in the max or min of the oscillation pattern.

Let us take the condition of the oscillations' visibility as $V(L) > 0.1$, then the coherence length reads:

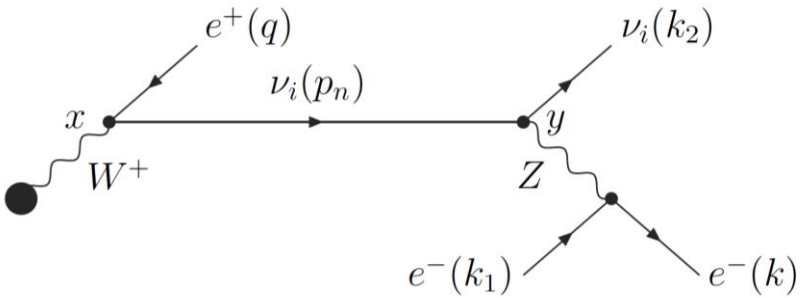
$$L_{\mathrm{coh}}^{\text{Cl-Ar}} \approx 103 \text{ km}, \quad L_{\mathrm{coh}}^{\text{Ga-Ge}} \approx 71 \text{ km}.$$

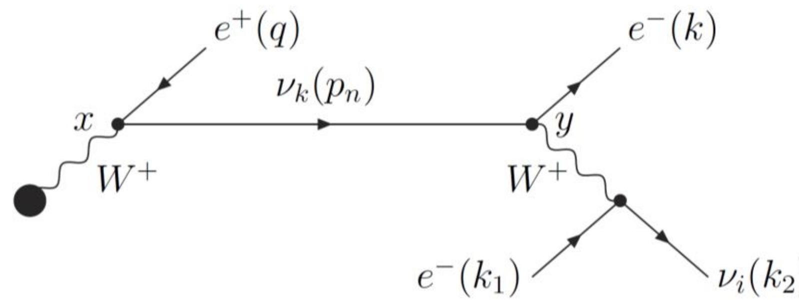
The QM treatment in terms of wave packets gives:

$$L_{kj}^{\mathrm{coh}} = \frac{4\sqrt{2}E^2}{\left|\Delta m_{kj}^2\right|}\sigma_x \sim 10 \div 100\,\text{km}.$$

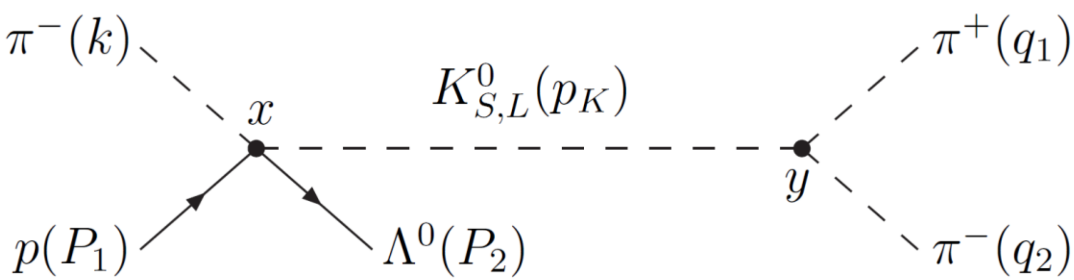
• C. Giunti and C. W. Kim, «Fundamentals of Neutrino Physics and Astrophysics,» Oxford, UK: Univ. Pr. (2007)

One can also apply the discussed approach to describe the neutrino oscillation process where the neutrinos are detected in both the charged- and neutral-current interactions





and neutral kaon oscillations.



The well-known results of the standard description are also reproduced.

Conclusion

- A novel QFT approach to the description of processes passing at finite space-time intervals is discussed. It is based on the Feynman diagram technique in the coordinate representation supplemented by the modified rules of passing to the momentum representation, which reflect the experimental situation at hand.
- It is explicitly shown that the approach allows to consistently describe such processes as decay of an unstable particle and neutrino oscillations. The main results of the standard approach are reproduced.
- The advantages of the approach are technical simplicity and physical transparency. Wave packets are not employed, we use only the description in terms of plane waves.
- The neutrino flavor states are redundant, we deal only with neutrino mass eigenstates.

Thank you for your attention!