Cosmological and astrophysical magnetic fields in turbulent matter

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Outline

- Overview of physics background: chiral magnetic effect (CME), magnetic helicity, relativistic magnetohydrodynamics (MHD) , and modified Faraday equation
- Model to account for the matter velocity in the study of magnetic field evolution
- Kinetic equations for magnetic field evolution accounting for turbulence and CME
- Comparison with previous studies
- Cosmological magnetic fields in turbulent matter
- Astrophysical magnetic fields in turbulent quark matter: magnetar bursts

References

- M. Dvornikov, V.B. Semikoz *The influence of the turbulent motion on the chiral magnetic effect in the early universe* Phys. Rev. D **95**, 043538 (2017), arXiv:1612.05897

- M. Dvornikov *Magnetic fields in turbulent quark matter and magnetar bursts* to be published in Int. J. Mod. Phys. D (2017), arXiv:1612.06540

CME in a nutshell

Helicity is strongly correlated with the momentum for massless particles

Left particle

Spin

Momentum

Right particle

Spin

Momentum
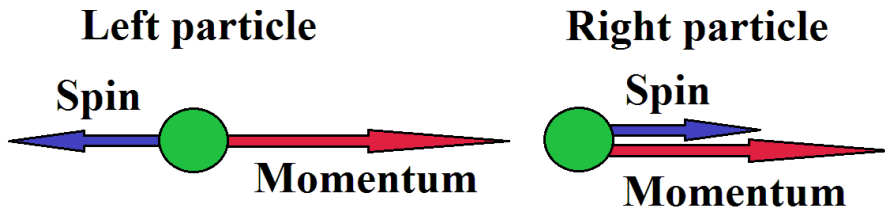
While interacting with a constant magnetic field **B**, the spin of a charged particle (e.g. an electron) is aligned opposite **B** and the spin of an antiparticle (a positron) along B, at zero Landau level

B p_L Spin   **B** p_R Spin

p_R Spin p_L
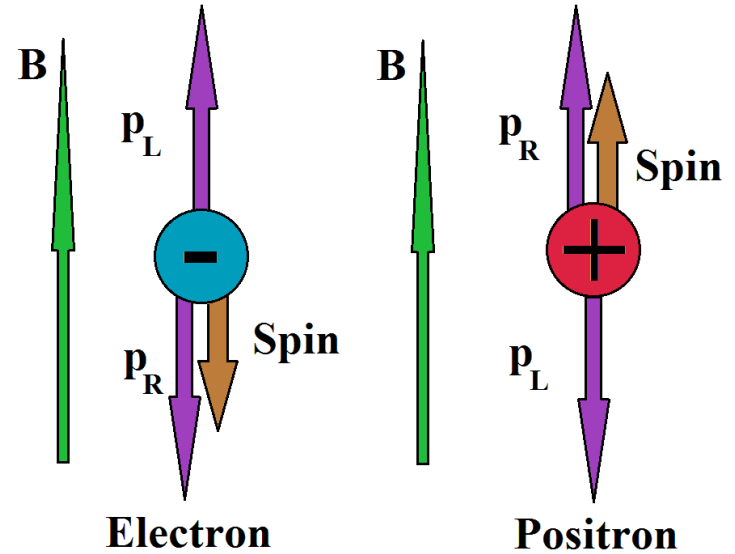
Electron   **Positron**

Left electrons move along **B**, whereas right ones opposite **B**

Thus we can expect a flux of charged particles, i.e. electric current, along **B**

The detailed calculation by Vilenkin (1980) shows that $\mathbf{J} = \dfrac{2\alpha_{em}}{\pi}\mu_5\mathbf{B},\ \mu_5 = \dfrac{1}{2}(\mu_R - \mu_L)$

If fermions electroweakly interact with background matter, Dvornikov & Semikoz (2015) found that the electric current has the form

$\mathbf{J} = \dfrac{2\alpha_{em}}{\pi}(\mu_5 + V_5)\mathbf{B},\ V_5 = \dfrac{1}{2}(V_L - V_R)$

$V_{L,R} \sim G_F$ are the effective potentials for the electroweak interaction of left and right fermions

Magnetic helicity
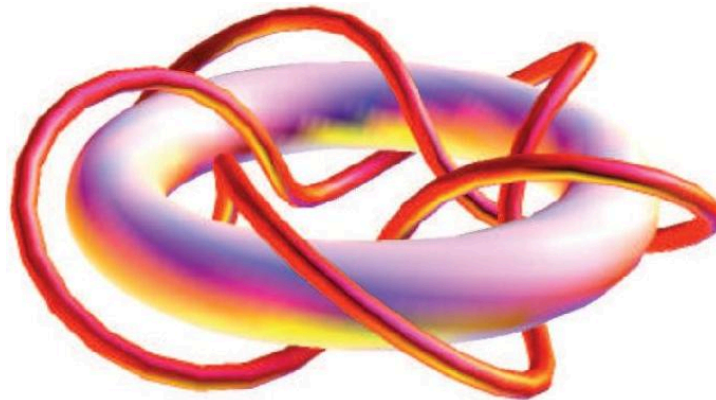
$$H = \int d^3x \left(\mathbf{A} \cdot \mathbf{B} \right)$$

- Magnetic helicity was first introduced by Gauss (1833)
- Magnetic helicity is conserved in the perfectly conducting fluid
- Magnetic helicity is gauge invariant
- In the system of two linked magnetic fluxes, magnetic helicity takes the form (Berger, 1999)

$$H = 2L\Phi_1\Phi_2$$
$$L = 0, \pm 1, \pm 2, \ldots$$

|L| = 5

Relativistic MHD in presence of CME

The relativistic MHD equations have the form

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda, \quad \partial_\mu F^{\mu\nu} = j^\nu$$

Multiplying eq. for $T^{\mu\nu}$ by u_ν and $(g_{\alpha\nu} - u_\alpha u_\nu)$, neglecting the electric field, and adding the viscous terms, we get in the linear approximation in velocity **v**

$$\left(\partial_t + \mathbf{v}\cdot\nabla\right)\varepsilon + \left(\varepsilon + P\right)\nabla\cdot\mathbf{v} = 0, \quad \left(\varepsilon + P\right)\left(\partial_t + \mathbf{v}\cdot\nabla\right)\mathbf{v} = -\nabla P + \mathbf{j}\times\mathbf{B} + \left(\varepsilon + P\right)\nu\nabla^2\mathbf{v}$$

Neglecting the displacement current d**E**/dt, that is valid in MHD approximation $\omega \ll \sigma_{cond}$, the Maxwell equations take the form

$$\mathbf{j} = \nabla\times\mathbf{B}, \quad \partial_t\mathbf{B} = -\nabla\times\mathbf{E}$$

Accounting for the CME contribution, the electric current reads

$$\mathbf{j} = \mathbf{j}_{Ohm} + \mathbf{j}_{CME} = \sigma_{cond}\left[\mathbf{E} + \left(\mathbf{v}\times\mathbf{B}\right)\right] + \mathbf{j}_{CME}$$

We get the modified Faraday equation

$$\partial_t\mathbf{B} = \nabla\times\left(\mathbf{v}\times\mathbf{B}\right) + \eta_m\nabla^2\mathbf{B} + \frac{\Pi}{\sigma_{cond}}\left(\nabla\times\mathbf{B}\right), \quad \Pi = \frac{2\alpha_{em}}{\pi}\left(\mu_5 + V_5\right)$$

If we study large scale magnetic fields, i.e. neglecting **v**, then under certain conditions ($|\Pi| > k$) this eq. describes the enhancement of a seed field B_0

$$B(k,t) = B_0 \exp\left[\int_{t_0}^{t}\left(|\Pi|k - k^2\right)\frac{dt'}{\sigma_{cond}}\right]$$

# Model to account for the turbulent motion of matter

How to take into account the plasma motion in the generation of magnetic fields?

We shall study ultrarelativistic background matter

$$\varepsilon + P = \frac{4}{3}\varepsilon$$

We assume that the Lorentz force in Navier-Stokes equation is dominant

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{1}{\varepsilon + P}(\mathbf{j} \times \mathbf{B})$$

This approximation is valid since we study turbulent motion with high Reynolds number Re = VL/v >> 1 and consider relatively large length scales, i.e. we omit nonlinear terms

We use the drag time approximation. The phenomenological drag time parameter τ_d is equal to the time of the Coulomb scattering in plasma

$$\mathbf{v} = \frac{\tau_d}{\varepsilon + P}(\mathbf{j} \times \mathbf{B})$$
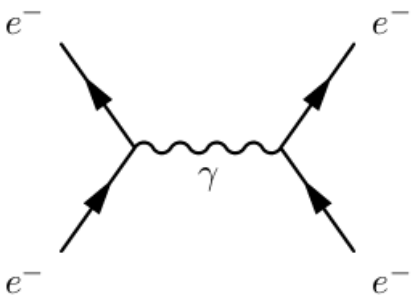
We shall assume that τ_d >> Larmor radius. In this case the plasma mean velocity is driven mainly by the Lorentz force.

Magnetic field evolution equation

The master equation for the magnetic field reads

$$\left[\partial_t + \eta_m k^2\right]B_i(\mathbf{k},t) = \varepsilon_{ijk}k_j\frac{\tau_d}{P+\varepsilon}\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}q_r B_s(\mathbf{q})\left[\varepsilon_{krs}B_n(\mathbf{p}-\mathbf{q})B_n(\mathbf{k}-\mathbf{p})-\varepsilon_{rsm}B_k(\mathbf{k}-\mathbf{p})B_m(\mathbf{p}-\mathbf{q})\right]$$

One should take into account the two point correlator

$$\left\langle B_i(\mathbf{k},t)B_j(\mathbf{p},t)\right\rangle = \frac{(2\pi)^3}{2}\delta^3(\mathbf{k}+\mathbf{p})\left[(\delta_{ij}-\hat{k}_i\hat{k}_j)S(k,t)+i\varepsilon_{ijk}\hat{k}_k A(k,t)\right]$$

Form factors S(k,t) and A(k,t) are related to spectra of magnetic energy and magnetic helicity

$$\frac{B^2}{2}=\frac{1}{2V}\int B^2 d^3x=\int dk\rho_B(k,t),\ \rho_B(k,t)=k^2\frac{S(k,t)}{(2\pi)^2},\ \frac{H(t)}{V}=\frac{1}{V}\int(\mathbf{A}\cdot\mathbf{B})d^3x=\int dk h(k,t),\ h(k,t)=-k\frac{A(k,t)}{2\pi^2}$$

Multiplying the master equation by $A_i(\mathbf{k},t)$ and $B_i(\mathbf{k},t)$ and adding the CME contribution, we get the equations for the spectra

$$\frac{\partial h(k,t)}{\partial t}=-2k^2\left(\eta_m+\frac{4}{3}\frac{\tau_d}{P+\varepsilon}\int\rho_B(p,t)dp\right)h(k,t)+4\left(\frac{\Pi}{\sigma_{cond}}+\frac{2}{3}\frac{\tau_d}{P+\varepsilon}\int p^2 h(p,t)dp\right)\rho_B(k,t)$$

$$\frac{\partial\rho_B(k,t)}{\partial t}=-2k^2\left(\eta_m+\frac{4}{3}\frac{\tau_d}{P+\varepsilon}\int\rho_B(p,t)dp\right)\rho_B(k,t)+k^2\left(\frac{\Pi}{\sigma_{cond}}-\frac{2}{3}\frac{\tau_d}{P+\varepsilon}\int p^2 h(p,t)dp\right)h(k,t)$$

Comparison with previous studies

The total energy and magnetic helicity evolve as

$$\frac{d}{dt}\left(\frac{B^2}{2}\right) = \alpha_{CME}\int dk\,k^2 h(k,t) - 2\eta_m\int dk\,k^2\rho_B(k,t) - \frac{2\tau_d}{3(P+\varepsilon)}\int dk\,dp\,k^2\left[4\rho_B(k,t)\rho_B(p,t) + p^2 h(k,t)h(p,t)\right]$$

$$\frac{dH}{dt} = 4\alpha_{CME}\int dk\,\rho_B(k,t) - 2\eta_m\int dk\,k^2 h(k,t)$$

One can see that the turbulence only ($\alpha_{CME} = 0$) results in the decay of the magnetic field. The growth of the magnetic field can be owing to CME only

If the contribution of turbulence is dominant, kinetic eqs. can be rewritten in the form

$$h(k,t) = \frac{2\rho_0(k)}{k}\exp\left(-2k^2 l_{diss}^2\right)\left[q\cos\left(2kl_d\right) - \sqrt{\frac{\alpha_d + \alpha_{CME}}{\alpha_d - \alpha_{CME}}}\sin\left(2kl_d\right)\right], \quad l_{diss}^2 = \int_{t_0}^{t}\eta_{eff}(t')\,dt'$$

$$\rho_B(k,t) = \rho_0(k)\exp\left(-2k^2 l_{diss}^2\right)\left[\cos\left(2kl_d\right) + q\sqrt{\frac{\alpha_d - \alpha_{CME}}{\alpha_d + \alpha_{CME}}}\sin\left(2kl_d\right)\right], \quad l_d = \int_{t_0}^{t}\sqrt{\alpha_d^2(t') - \alpha_{CME}^2(t')}\,dt'$$

It is possible to see that, in this case, turbulence does not provide the magnetic field growth

This result contradicts the claim of Campanelli (2007,2014) and Sigl et al. (2016) that the turbulence can cause the dynamo amplification of magnetic field

$$h(k,t) = h_0(k)e^{-2k^2 l_{diss}^2}\left[\sinh\left(2kl_d\right) + q\cosh\left(2kl_d\right)\right], \quad \rho_B(k,t) = \rho_0(k)e^{-2k^2 l_{diss}^2}\left[\cosh\left(2kl_d\right) + q\sinh\left(2kl_d\right)\right]$$

Summary I

- Turbulent motion of matter is accounted for by replacement $\mathbf{v}$ -> $\mathbf{F}_L \sim \tau_d$ ($\mathbf{J}$ x $\mathbf{B}$).

- Turbulence is responsible for suppression of large-k (small scale) modes.

- Turbulence in this model cannot enhance a seed magnetic field since the Lorentz force does not linearly accelerate charged particles in plasma. Thus self-sustained electric currents, which could generate an unstable magnetic field, cannot be excited in such plasma.

- The only source of magnetic field instability is CME.

Cosmic magnetic field (CMF) of cosmological origin

- Our universe is permeated by CMF which are dynamo amplified from a seed field

- CMF is a source of galactic magnetic field with B = 10^{-6} G

• Seed fields can be produced by MHD mechanisms during epoch of galaxy formation, or ejected by first supernovae or active galactic nuclei

• Another scenario suggests that a seed field can originate from much earlier epoch of the Universe expansion: inflation era, phase transitions in radiation era

• We consider a stage after EWPT

Strength of CMF

- Upper bound on CMF can be obtained from Faraday rotation measure: $B_{CMF} < 10^{-9}$ G

- CMF influence the propagation of intergalactic cosmic rays, e.g. from blazars (compact quasars) to Milky Way

- Lower bound (Neronov et al., 2010) – $B_{CMF} > 10^{-16}$ G – from non-observation of secondary photons with E = 1 GeV in the initial flux with E = 1 TeV: $\gamma\gamma \rightarrow e^+e^-$; $e^-\gamma_{CMB} \rightarrow e^-\gamma(GeV)$

Kinetic equations for magnetic field in hot plasma of the early universe

We shall study the evolution of cosmological magnetic fields after electroweak phase transition, i.e $T \ll M_W = 100$ GeV

It is convenient to use dimensionless (conformal) variables

$$t \to \eta = M_0 / T, \; \tilde{k} = k / T, \; \tilde{\rho}_B(\tilde{k},\eta) = \rho_B(k,t)/T^3,$$

$$\tilde{h}(\tilde{k},\eta) = h(k,t)/T^2, \; \tilde{\mu}_5 = \mu_5 / T, \; M_0 = M_{Pl} / 1.66\sqrt{g^*}$$

Since Maxwell equations are conformal invariant, the kinetic equations take the form

$$\frac{\partial \tilde{h}(\tilde{k},\eta)}{\partial \eta} = -2\tilde{k}^2 \tilde{\eta}_{eff} \tilde{h}(\tilde{k},\eta) + 4\tilde{\alpha}_- \tilde{\rho}_B(\tilde{k},\eta), \; \tilde{\eta}_{eff} = \sigma_c^{-1} + \frac{30\alpha_{em}^{-2}}{\pi^2 g^*} \int d\tilde{p} \tilde{\rho}_B(\tilde{p},\eta)$$

$$\frac{\partial \tilde{\rho}_B(\tilde{k},\eta)}{\partial \eta} = -2\tilde{k}^2 \tilde{\eta}_{eff} \tilde{\rho}_B(\tilde{k},\eta) + \tilde{k}^2 \tilde{\alpha}_+ \tilde{h}(\tilde{k},\eta), \; \tilde{\alpha}_\pm = \frac{\Pi}{\sigma_c} \mp \frac{15\alpha_{em}^{-2}}{\pi^2 g^*} \int d\tilde{p} \tilde{p}^2 \tilde{h}(\tilde{p},\eta)$$

Here $\sigma_{cond} = \sigma_c T$, where $\sigma_c = 100$, is the conductivity of hot plasma (Pitaevskii & Lifshitz, 2002) and g* = 106.75 is the number of relativistic degrees of freedom in considered epoch (Gorbunov & Rubakov, 2008)

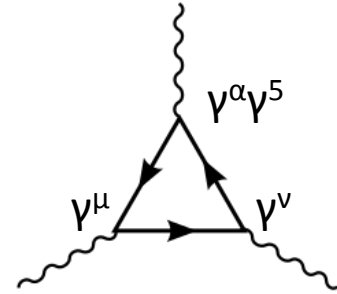We shall use the initial Batchelor spectrum for magnetic energy $\tilde{\rho}_B(\tilde{k},\eta_0) = C\tilde{k}^4$,

since we study relatively small k (Davidson, 2015): $0 \le \tilde{k} \le \tilde{k}_{max} = 10^{-6}$

The initial magnetic helicity is $\tilde{h}(\tilde{k},\eta_0) = 2\tilde{\rho}_B(\tilde{k},\eta_0) / \tilde{k}$

Evolution of the chiral imbalance μ_5

The evolution equation for μ_5 is required since $\Pi \sim \mu_5$.
It can be obtained on the basis of Adler anomaly in QED

$$\partial_\mu \left(j_R^\mu - j_L^\mu \right) = \frac{2\alpha_{em}}{\pi} \left(\mathbf{E} \cdot \mathbf{B} \right)$$

$\gamma^\alpha \gamma^5$

γ^μ γ^ν

Evolution of the magnetic helicity results from Maxwell equations

$$\frac{dh(t)}{dt} = -\frac{2}{V} \int d^3 x \left(\mathbf{E} \cdot \mathbf{B} \right)$$

Integrating this eq. over isotropic space, we get the conservation law

$$\frac{d}{dt} \left[n_R - n_L + \frac{\alpha_{em}}{\pi} h(t) \right] = 0$$

Finally we obtain eq. for μ_5 in conformal variables

$$\frac{d\tilde{\mu}_5(\eta)}{d\eta} = -\frac{6\alpha_{em}}{\pi} \int d\tilde{k} \frac{\partial \tilde{h}}{\partial \eta} - \tilde{\Gamma}_f \tilde{\mu}_5$$

The helicity flip rate Γ_f is owing to ee collisions in the broken phase (Boyarsky et al., 2012)

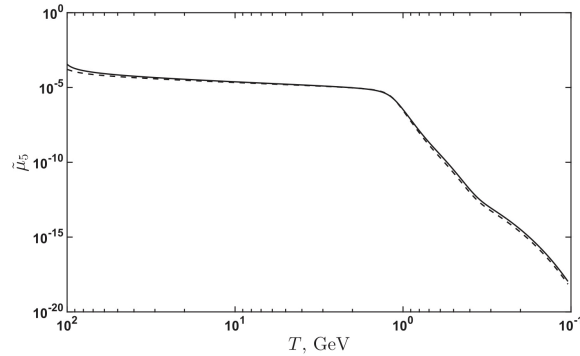$$\tilde{\Gamma}_f = \alpha_{em} \left(\frac{m_e}{3M_0} \right)^2 \eta^2$$

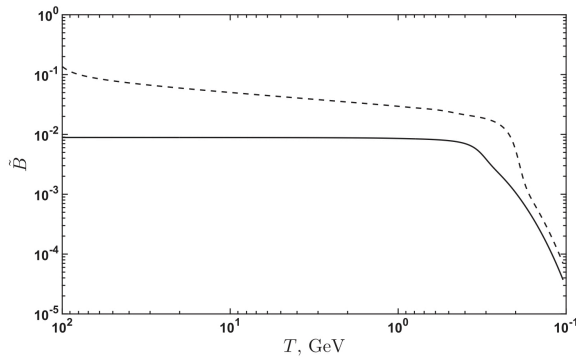The initial condition for μ_5 (Boyarsky et al., 2012): $\tilde{\mu}_5(\eta_0) = 4 \times 10^{-5}$
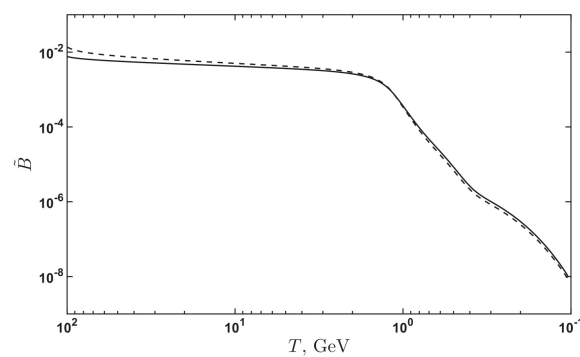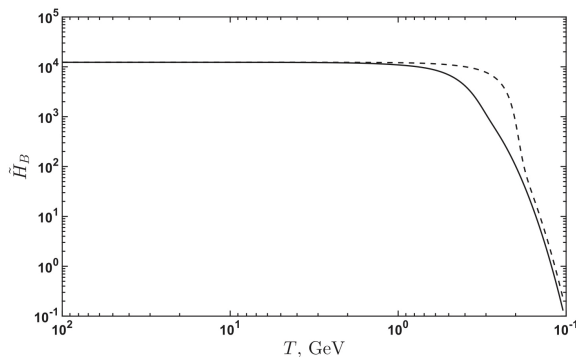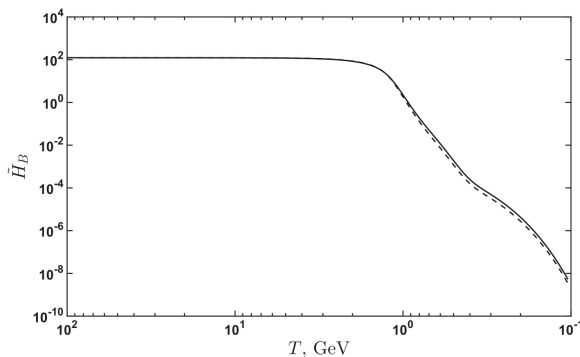
Results of numerical simulations
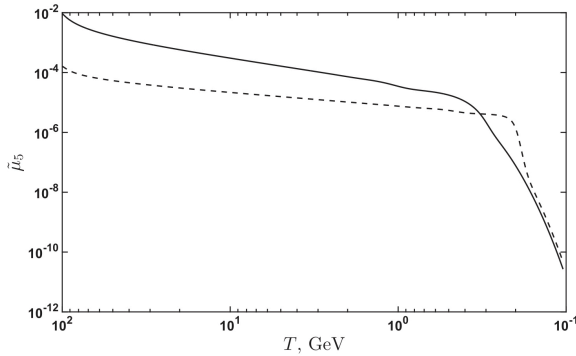
(a)

(b)

(c)

(d)

(e)

(f)

Dashed lines – only CME is accounted for
(Boyarsky et al., 2012);
Solid lines – both CME and turbulence are taken into account

(a), (c) and (e) – for $\tilde{B}_0 = 10^{-1}$

(b), (d) and (f) – for $\tilde{B}_0 = 10^{-2}$

Note that $\tilde{B}_0 = 10^{-1}$ corresponds to $B_{crit} = 10^{11}$ G at $T_{BBN} = 0.1$ MeV, which is a critical strength for BBN nucleosynthesis
(Cheng et al., 1994)

Summary II

- The effect of turbulence is bigger for greater k_{max} (smaller scales). Smaller scales decay faster. It can be the indication on the existence of the inverse cascade

- The influence of turbulence is more significant for stronger B_0. Indeed, the contribution of turbulence is quadratic in spectra

- The behavior of B differs from that for μ_5 because of opposite signs in $\alpha_{\pm}$

- At greater evolution time (lower temperatures) the effect of turbulence is washed out

Highly magnetized compact stars: magnetars

- Starting from 1979, anomalous X-ray pulsars (AXP) and soft gamma-ray repeaters (SGR) are observed

- According to modern astrophysics, AXP and SGR are highly magnetized B > 10^{15} G compact stars or *magnetars*

- Note that a typical pulsar can have a magnetic field up to 10^{12} G, i.e. one should explain the enhancement of magnetic field by 3 orders of magnitude

- Despite the existence of numerous models of magnetars none of them can explain all the observed features of these stars, i.e. neither the origin of strong magnetic field nor the mechanism of magnetar bursts are known
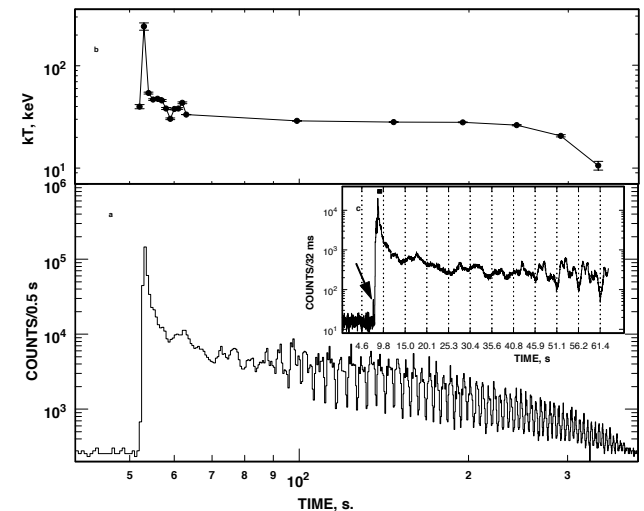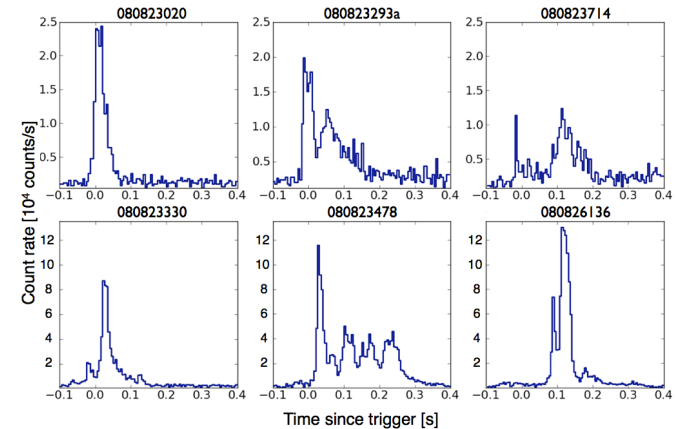
# Generation of magnetic fields in magnetars driven by electoweak interaction between fermions

- CME can take place only if chiral symmetry is unbroken (Vilenkin 1980, Dvornikov 2016), i.e. when fermions are effectively massless
- Restoration of chiral symmetry is possible in dense quark matter in hybrid and/or quark stars (Buballa & Carignano 2016)
- Anomalous current along B, including the electroweak correction, has the form $\mathbf{J} = (2\alpha_{em}/\pi) (\mu_5 + V_5) \mathbf{B}$, where $V_5 \sim G_F n_f$
- Basing on this current, Dvornikov (2016) predicted the generation of strong, $B = (10^{14} - 10^{15})$ G, and large-scale, $\Lambda_B = (0.1 - 10)$ km, magnetic fields in dense quark matter of a compact star starting with seed field $B_0 = 10^{12}$ G typical in a pulsar
- Generation of these magnetic fields is driven by the electroweak interaction between u and d quarks
- We suggest that the generated fields can model magnetic fields in magnetars

Magnetar bursts

- Magnetar flashes are caused by a twist of magnetic lines in the compact star magnetosphere
- This twist should be associated with a motion of a stellar crust
- Beloborodov & Levin (2014) suggested that plastic deformation of a crust is driven by a thermoplastic wave (TPW)
- Lander (2016) found that TPW can be excited if
  $B > 10^{13}$ G
- Li et al. (2016) obtained that, to generate a magnetar flare, the initial magnetic field fluctuation for TPW should have
  Λ_B = several meters
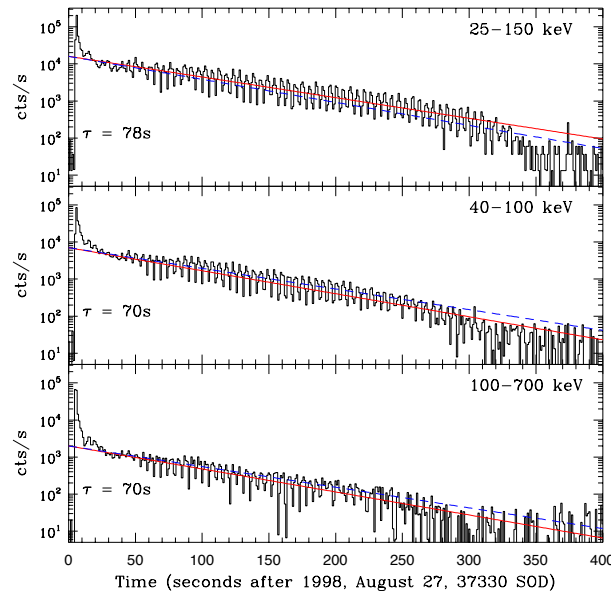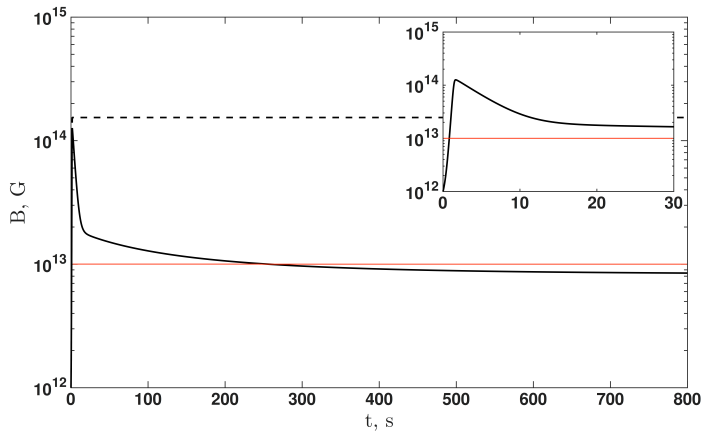- The physical process which trigger TPW is unknown

Small scale magnetic fields in turbulent quark matter

- $T_0 = 10^8$ K, 1 cm $< \Lambda_B <$ 10 cm
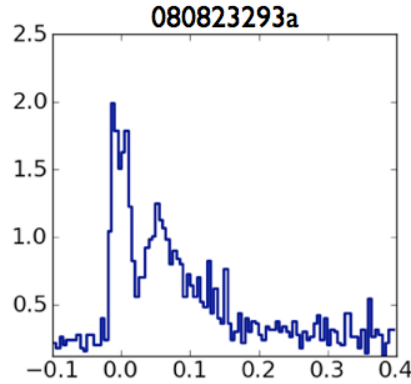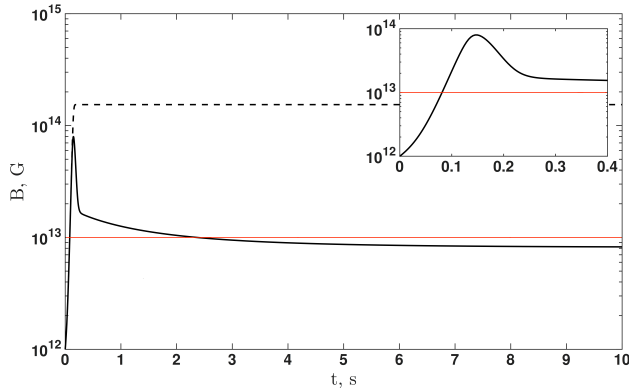
Short burst of magnetar
SGR J0501+4516
recorded by Fermi/GBM;
Huppenkothen et al. (2013)

- $T_0 = 10^8$ K, 10 cm $< \Lambda_B <$ 1 m

The combined analysis
of the giant flare
of magnetar
SGR 1900+14
on August 27 1998
recorded by BeppoSAX
and Ulysses;
Feroci et al. (2001)

Summary III

- In frames of our model we predict the generation of small scale magnetic fields in turbulent quark matter

- The seed field $B_0 = 10^{12}$ G is amplified to $B > 10^{14}$ G. Thus $B_{crit} = 10^{13}$ G, necessary to excite a TPW, is reached

- The scale of magnetic fields can be up to 1 m, which is close to the expectations for TPW

- Smaller scale fields correspond to short bursts, and larger scale fields − to giant flares. Indeed larger scale fields should have more total energy since $E_B \sim B^2 V$.

- If these fields are created in the core of compact star, where there are appropriate conditions for the existence of CME (unbroken chiral symmetry), they can trigger the propagation of TPW, which then passes through the stellar crust and causes bursts/flares of magnetars

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