Relic neutrino asymmetry in a hot plasma of early Universe

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Two Boltzmann equations for neutrinos

Standard model, $T \ll T_{EWPT}$, **known Boltzmann equation** for *massless* neutrinos (antineutrinos) (V.S. 1987, Silva et al.1999, Oraevsky & V.S. 2002):

$$\frac{\partial f^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)}{\partial t} + \mathbf{n}\frac{\partial f^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)}{\partial \mathbf{x}} \pm [\mathbf{E}_e(\mathbf{x},t) + \mathbf{n}\times\mathbf{B}_e(\mathbf{x},t)]$$

$$\times\frac{\partial f^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)}{\partial \mathbf{k}} = J^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t), \quad \text{unit velocity} \;\; \mathbf{n} = \frac{\mathbf{k}}{k},$$

where $\mathbf{E}_e(\mathbf{x},t) = G_F\sqrt{2}c_V^a[-\nabla\delta n^{(e)}(\mathbf{x},t) - \partial_t\delta\mathbf{j}^{(e)}(\mathbf{x},t)]$,

$$\mathbf{B}_e(\mathbf{x},t) = G_F\sqrt{2}c_V^a\nabla\times\delta\mathbf{j}^{(e)}(\mathbf{x},t),$$

$\delta j_\mu^{(e)} = (\delta n^{(e)}, \delta\mathbf{j}^{(e)})$, $\delta n^{(e)} = n_e - n_{\bar{e}}$, $\delta\mathbf{j}^{(e)} = \mathbf{j}_e - \mathbf{j}_{\bar{e}}$, $G_F = 10^{-5}/m_p^2$ - Fermi constant, $c_V^a = 2\xi \pm 0.5$, $\xi = \sin^2\theta_W = 0.23$.

The neutrino (antineutrino) four-current,

$$j_\mu^{(\nu_a,\bar{\nu}_a)}(\mathbf{x},t) = \int \frac{d^3k}{(2\pi)^3}\frac{k_\mu}{\varepsilon_\mathbf{k}} f^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t), \qquad \varepsilon_\mathbf{k} = k,$$

is conserved,

$$\frac{\partial}{\partial x_\mu} j_\mu^{(\nu_a,\bar{\nu}_a)}(\mathbf{x},t) = 0,$$

due to the Lorentz form of Boltzmann equation, and zero contribution of the collision integral,

$$\int d^3k J_{coll}^{(\nu e)} = 0$$

New Boltzmann equation accounting for the Berry curvature $\mathbf{\Omega}_{\mathbf{k}} = \pm\hat{\mathbf{k}}/2k^2$ (M. Dvornikov & V.S. 2016)

$$\frac{\partial f_{\mathbf{k}}^{(\nu_a)}}{\partial t} + \frac{1}{\sqrt{\omega}}\left(\tilde{\mathbf{v}} + \tilde{\mathbf{E}}_e \times \mathbf{\Omega}_{\mathbf{k}} + (\tilde{\mathbf{v}} \cdot \mathbf{\Omega}_{\mathbf{k}})\mathbf{B}_e\right)\frac{\partial f_{\mathbf{k}}^{(\nu_a)}}{\partial \mathbf{x}}$$

$$+\frac{1}{\sqrt{\omega}}\left(\tilde{\mathbf{E}}_e + \tilde{\mathbf{v}} \times \mathbf{B}_e + (\tilde{\mathbf{E}}_e \cdot \mathbf{B}_e)\mathbf{\Omega}_{\mathbf{k}}\right)\frac{\partial f_{\mathbf{k}}^{(\nu_a)}}{\partial \mathbf{k}} = J^{(\nu_a)}(f_{\mathbf{k}}^{(\nu_a)}),$$

Here $\omega = (1 + \mathbf{B}_e \cdot \mathbf{\Omega}_{\mathbf{k}})^2$, $\tilde{\mathbf{v}} = \partial\varepsilon_{\mathbf{k}}/\partial\mathbf{k}$, $\tilde{\mathbf{E}}_e = \mathbf{E}_e - \partial\varepsilon_{\mathbf{k}}/\partial\mathbf{k}$, **where modified neutrino spectrum**

$$\varepsilon_{\mathbf{k}} = k[1 - \mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{B}_e(\mathbf{x}, t)].$$

Neutrino number density and neutrino 3-current density

$$n^{(\nu_a)}(\mathbf{x},t) = \int \frac{d^3k}{(2\pi)^3}\sqrt{\omega} f_{\mathbf{k}}^{(\nu_a)},$$

$$\mathbf{j}^{(\nu_a)}(\mathbf{x},t) = \int \frac{d^3k}{(2\pi)^3}\left(\tilde{\mathbf{v}} + \tilde{\mathbf{E}}_e \times \boldsymbol{\Omega}_{\mathbf{k}} + (\tilde{\mathbf{v}}\cdot\boldsymbol{\Omega}_{\mathbf{k}})\mathbf{B}_e\right) f_{\mathbf{k}}^{(\nu_a)}$$

obey a new anomaly at $T \ll T_{EWPT}$

$$\frac{\partial j_\mu^{(\nu_a)}(\mathbf{x},t)}{\partial x_\mu} = \partial_t n^{(\nu_a)} + \nabla\cdot\mathbf{j}^{(\nu_a)} = -C^{(\nu_a)}(\mathbf{E}_e\cdot\mathbf{B}_e) \neq 0$$

$$C^{(\nu_a)} = [4\pi^2(1+e^{-\mu_{\nu_a}/T})]^{-1}$$

Neutrino asymmetry evolution at $T \ll T_{EWPT}$ (new result)

$$\frac{\mathrm{d}(n_{\nu_a} - n_{\bar{\nu}_a})}{\mathrm{dt}} = -\frac{1}{4\pi^2} \int \frac{d^3x}{V} (\mathbf{E}_e \cdot \mathbf{B}_e),$$

where using Maxwell equations $\mathbf{j}_{em} = -e\delta\mathbf{j}^{(e)} = \nabla \times \mathbf{B}$, $\dot{\mathbf{B}} = -(\nabla \times \mathbf{E})$, $\nabla \cdot \mathbf{E} = -e\delta n_e$, $(\nabla \cdot \mathbf{B}) = 0$, the effective (weak) fields take the form :

$$\mathbf{E}_e(\mathbf{x}, t) = A\nabla^2\mathbf{E}(\mathbf{x}, t), \quad \mathbf{B}_e(\mathbf{x}, t) = A\nabla^2\mathbf{B}(\mathbf{x}, t)$$

$$A = G_F\sqrt{2}c_V^a/e, \qquad e = \sqrt{4\pi\alpha} \sim 0.3$$

Toy model. In the Fourier representation

$$\frac{\mathrm{d}(n_{\nu_a} - n_{\bar{\nu}_a})}{\mathrm{dt}} = \frac{A^2}{8\pi^2}\int \frac{d^3k}{(2\pi)^3} k^4 \frac{\partial}{\partial t} h(k,t) dk, \qquad (A)$$

for the monochromatic magnetic helicity spectrum $h(k,t) = h(t)\delta(k - k_0)$, where $k_0 = r_D^{-1}$, $r_D = v_T/\omega_p$ is the Debye radius, $h(t) = V^{-1}\int d^3x(\mathbf{A}\cdot\mathbf{B})$ is the magnetic helicity density, one gets the conservation law:

$$\frac{\mathrm{d}}{\mathrm{dt}}\left[(n_{\nu_a} - n_{\bar{\nu}_a}) - \frac{\alpha_{ind}^a}{2\pi} h(t)\right] = 0, \qquad (AA)$$

where $\alpha_{ind}^a = [e_{ind}^{(\nu_a)}]^2/4\pi$ is given by the induced charge of neutrino in plasma (V.S. 1987, Nieves & Pal, 1994):

$$e_{ind}^{(\nu_a)} = -G_F c_V^a (1-\lambda)/\sqrt{2} e r_D^2.$$

This conservation law is analogous to the known conservation law for the chiral magnetic effect (CME) in the case of charged (right & left)electrons:

$$\frac{\mathrm{d}}{\mathrm{dt}}\left[(n_{eR} - n_{eL}) + \frac{\alpha}{\pi}h(t)\right] = 0. \qquad (B)$$

From the conservation law (AA) for neutrino asymmetry $n_{\nu_a} - n_{\bar{\nu}_a} \approx \xi_{\nu_a} T^3/6$, assuming zero initial $\xi_{\nu_a}(T_0) = 0$ one gets neutrino asymmetry parameter $\xi_{\nu_a} = \mu_{\nu_a}/T$

$$\xi_{\nu_a}(T) = -5.7(c_V^a)^2 \times 10^{-15} \left(\frac{T_0}{m_p}\right)^4 \left(\frac{T_0}{T}\right)^3, \qquad (C)$$

or for $T_0 = 1$ GeV, $T = O(\mathrm{MeV})$, $\xi_{\nu_a} = -7.3(c_V^a)^2 \times 10^{-6}$, while for $T_0 = 10$ GeV $\ll T_{EWPT} \simeq 100$ GeV one gets

$$\xi_{\nu_e}(T = \mathrm{MeV}) = 73!!! \text{ for } \nu_e, \qquad c_V^e \sim 1$$

Continuous (Kolmogorov's) spectrum of magnetic energy density. For the initial $\tilde{\rho}_B(\tilde{k},\eta) = C\tilde{k}^{n_B}, \quad n_B = -5/3$ we solve 3 self-consistent equations in conformal variables

$$\frac{\partial}{\partial\eta}\tilde{h}(\tilde{k},\eta) = -\frac{2\tilde{k}^2}{\sigma_c}\tilde{h}(\tilde{k},\eta) + \frac{4\tilde{\Pi}}{\sigma_c}\tilde{\rho}_B(\tilde{k},\eta), \tag{1}$$

$$\frac{\partial}{\partial\eta}\tilde{\rho}_B(\tilde{k},\eta) = -\frac{2\tilde{k}^2}{\sigma_c}\tilde{\rho}_B(\tilde{k},\eta) + \frac{\tilde{\Pi}}{\sigma_c}\tilde{k}^2\tilde{h}(\tilde{k},\eta), \tag{2}$$

where $\tilde{\Pi} = 2\alpha\tilde{\mu}_5/\pi$, $\tilde{\mu}_5 = (\mu_{eR} - \mu_{eL})/2T$ is governed by CME in Eq. (B) accounting for chirality flip, $\tilde{\Gamma}_f \sim m_e^2$

$$\frac{\partial\tilde{\mu}_5}{\partial\eta} + \frac{6\alpha}{\pi}\int d\tilde{k}\frac{\mathrm{d}\tilde{h}(\tilde{k},\eta)}{\mathrm{d}\eta} = -\tilde{\Gamma}_f\tilde{\mu}_5. \tag{3}$$

The solution of Eq. (A) in conformal variables,

$$\frac{\mathrm{d}\xi_{\nu_a}}{\mathrm{d}\eta} = \frac{3A^2}{4\pi^2 a^2}\int \tilde{k}^4 \frac{\mathrm{d}}{\mathrm{d}\eta}\left[\frac{\tilde{h}(\tilde{k},\eta)}{a^2}\right]\mathrm{d}\tilde{k}$$

using Eqs. (1-3) for the maximum initial helicity $\tilde{h}(\tilde{k},\eta_0) = 2\tilde{\rho}(\tilde{k},\eta_0)/\tilde{k}$ gives a negligible growth of (negative) neutrino asymmetry for the ZERO INITIAL $\xi_{\nu_a}(\eta_0) = 0$, in plot built for $\xi_{\nu_e}(T)$.

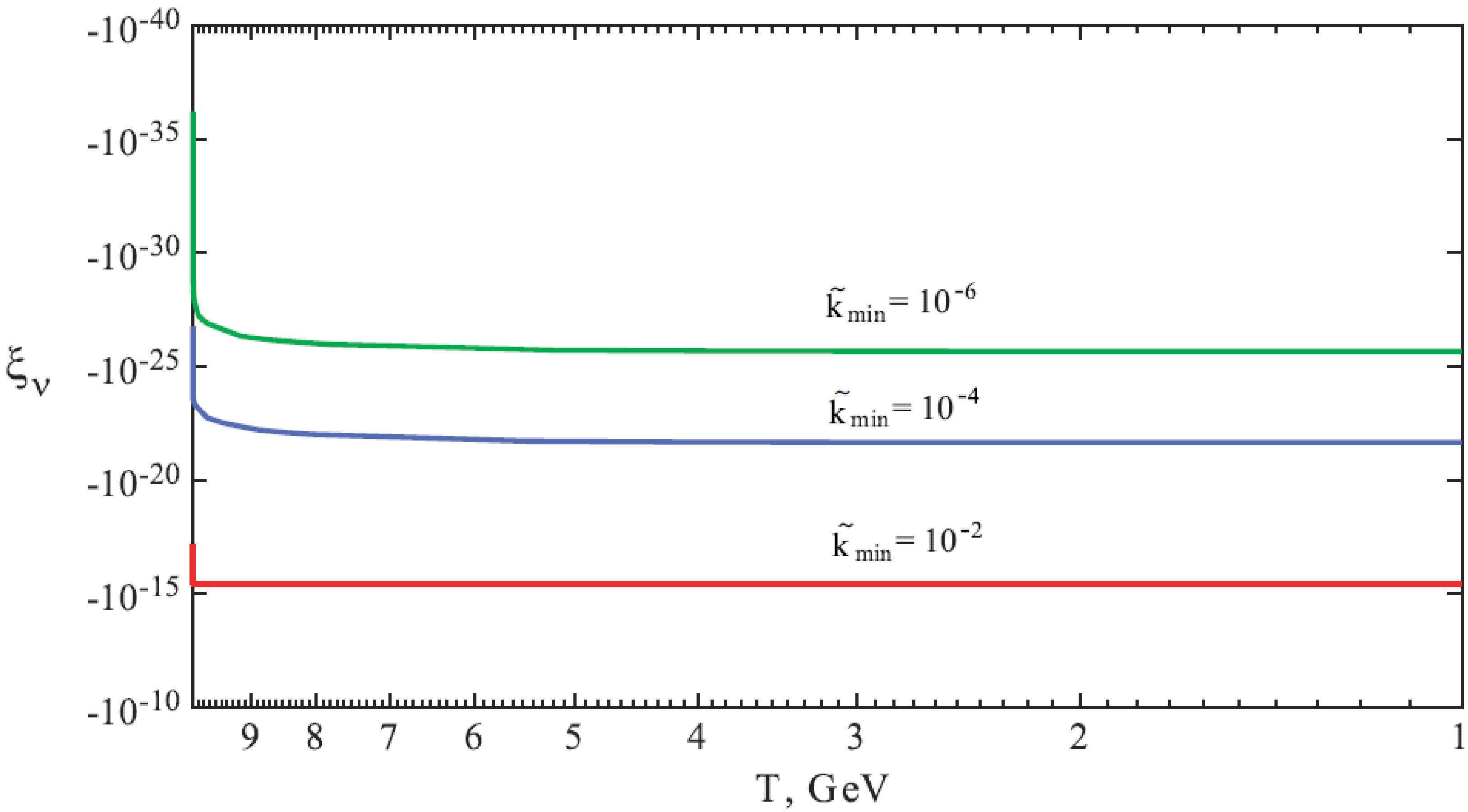

Figure 1: Asymmetry of the electron neutrino in the hot universe plasma for the Kolmogorov′s initial spectrum, $n_B = -5/3$

On the other hand, modifying Boltzmann equation in hypercharge fields at $T > T_{EWPT}$,

$$\frac{\partial f^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)}{\partial t} + \mathbf{n}\frac{\partial f^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)}{\partial \mathbf{x}} \pm g_L\left[\mathbf{E}_Y(\mathbf{x},t) + \mathbf{n}\times\mathbf{B}_Y(\mathbf{x},t)\right] \times\frac{\partial f^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t)}{\partial \mathbf{k}} = J^{(\nu_a,\bar{\nu}_a)}(\mathbf{k},\mathbf{x},t),$$

where $g_L = g^{'}y_L/2$, $y_L = -1$ is the hypercharge for the left doublet $L_a = (\nu_a\ l_a)^T$, $g^{'} = e/\cos\theta_W$, or switching on Berry curvature in spectrum, $\varepsilon_{\mathbf{k}} = k(1 - g_L\mathbf{\Omega}\cdot\mathbf{B}_Y)$, we recover well-known Abelian anomaly in hypercharge fields (not using Feynman's triangle diagram!):

$$\frac{\mathrm{d}(n_{\nu_a} - n_{\bar{\nu}_a})}{\mathrm{d}t} = -\frac{g_L^2}{4\pi^2}\int\frac{d^3x}{V}(\mathbf{E}_Y\cdot\mathbf{B}_Y) = -\frac{g^{'2}}{16\pi^2}\int\frac{d^3x}{V}(\mathbf{E}_Y\cdot\mathbf{B}_Y)$$

Conclusions

• The neutrino current is not conserved both before EWPT and after it. This happens (even after neutrino decoupling) due to the Berry curvature in momentum space.

• If before EWPT at $T > T_{EWPT} \simeq 100$ GeV the asymmetry grows due to Abelian anomaly up to $\xi_{\nu_a} \sim 10^{-10}$ (we found such anomaly through Boltzmann equation modified due the Berry curvature), after EWPT neutrino asymmetry grows too even for zero initial $\xi_{\nu_a}(\eta_0) = 0$.

• Nevertheless, the effect after EWPT is negligible and , of course, asymmetry obeys well-known BBN limit $\mid \xi_{\nu_a} \mid < 0.07$ (Dolgov et al., 2002).

Faraday equation in electroweak plasma

Anomalous MHD (AMHD) Faraday equation takes the form

$$\frac{\partial \mathbf{B}}{\partial t} = \alpha \nabla \times \mathbf{B} + \eta \nabla^2 \mathbf{B},$$

where $\alpha = \Pi_2/\sigma_{cond}$- **magnetic helicity parameter** coming from the Chern-Simons (CS) term in effective Lagrangian $L_{CS} = \Pi_2 \mathbf{A} \cdot \mathbf{B}$ (M.Dvornikov & V.S., 2014)

$$\alpha(T) = \frac{\alpha_{em} G_F \sqrt{2} T^2 F(\omega/T)}{6\pi\sigma_c}[\xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau}],$$

$\xi_{\nu_a} = \mu_{\nu_a}/T$- **the neutrino asymmetry parameter**, $a = e, \mu, \tau$;
$\eta = (\sigma_{cond})^{-1}$ -**the magnetic diffusion coefficient**, $\sigma_{cond} = \sigma_c T \approx 100T$ is the electric conductivity in hot plasma. The dynamo solution for $k =\mid \alpha \mid /2\eta$

$$B(k,t) = B_0 \exp\left[\int_{t_0}^{t} (\mid \alpha \mid k - \eta k^2) dt'\right] \Longrightarrow B_0 \exp\left[\int_{t_0}^{t} \left(\frac{\alpha^2(t')}{4\eta(t')}\right) dt'\right]$$

Lower bound on neutrino asymmetry

Assuming magnetic field amplification due to neutrino asymmetries, denoting $\Xi_\nu = \xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau}$, one gets the lower bound

$$\int_{t_0}^{t} \left[\frac{\alpha^2(t^{'})}{4\eta(t^{'})}\right] dt^{'} > 1 \Longrightarrow \Xi_\nu^2 > \frac{10^3 \sqrt{g^*/106.75}}{[(T_0/\mathrm{GeV})^3 - T/(\mathrm{GeV})^3]},$$

or in cooling universe, $T \ll T_0 \equiv T_{EWPT} = 100$ GeV,

$$\mid \Xi_\nu \mid > \frac{1}{32}\left(\frac{g^*}{106.75}\right)^{1/4} \qquad (A)$$

From (A) after equilibration $\xi_{\nu_e} \sim \xi_{\nu_\mu} \sim \xi_{\nu_\tau}$ due to neutrino oscillations at $T \sim O(\mathrm{MeV})$ when $g^* = 10.75$, one obtains finally

V.S. (2016) $\Longrightarrow 0.0173 < \mid \xi_{\nu_e} \mid < 0.07 \Longleftarrow$ Dolgov et. al. (2002)

Cosmological bound on Dirac neutrino magnetic moment

Using BBN bound on excess of neutrino species, $\Delta N_\nu < 0.3$, arising due to Dirac neutrino spin oscillations via the magnetic moment $\mu^{(D)}_{\nu_e}$, $\nu_{eL} \leftrightarrow \nu_{eR}$, one claims to restrict conversion rate comparing that with the Hubble parameter,

$$\frac{\Gamma_{L\to R}}{H} = \frac{\langle P_{R\to L}\rangle \Gamma_W}{H} \leq 1, \quad \text{where} \quad \langle P_{R\to L}\rangle = \frac{1}{2}\frac{(2\mu^{(D)}_\nu B_\perp)^2}{(2\mu^{(D)}_\nu B_\perp)^2 + V^2}, \qquad (*)$$

V is the neutrino potential obeying $V \gg 2\mu^{(D)}_\nu B_\perp$, $\Gamma_W = 4G_F^2 T^5$ is the weak interaction rate. From $(*)$ one gets (P. Olesen, K. Enqvist & V.S., 1992):

$$\mu^{(D)}_\nu \leq \frac{6.5 \times 10^{-34}\mu_B}{B_{CMF}(t_{now})/1\ \mathrm{G}} \leq 6.5 \times 10^{-18}\mu_B,$$

where we use the lower bound (from γ-ray-observations by Fermi satellite) , $B_{CMF}(t_{now}) > 10^{-16}$ G at scales $L \sim$ Mps, (A. Neronov, I. Vovk, 2010).