The double Compton process in a strongly magnetized plasma

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Outline

- Introduction
- Photon dispersion in the magnetized medium
- Kinematic analysis
- Amplitude of double Compton process
- Photon absorption rate
- Cross section
- Conclusion

Introduction

The NS are very intresting astrophysical objects.
But, the many problems exist in the models of NS (cooling, radio emission, et.)
Here we consider one problem: the influence of different quantum processes on the photon polarisation states production in the magnetospheres of NS and its influence on the spectra formation. (see, for example, Suleimanov V. et. al. A&A 2012)
The Compton scattering, $\gamma e \to \gamma e$, is a basic process of the photon absorption in the solution of radiative transfer equation.
But the number of photons does not change in this process.
The previous investigations of radiation transfer problem in strongly magnetized hot plasma have shown that the photon splitting $\gamma \to \gamma\gamma$ could play a significant role as mechanism of photon production. (M. Chistyakov, D.R. et. al. PRD 2012, M. Chistyakov, D.R. et. al. EPJ Web Conf. 2016)

Introduction

However, as will be shown below, the production of photons of a certain polarization due to the reaction $\gamma \to \gamma\gamma$ will be kinematically forbidden.

Alternative mechanisms need to be considered.

In this talk we analyse the photon absorption rate of process $e\gamma \to e\gamma\gamma$ with taking into account the change of the photon dispersion properties.

As far as we know, previously, the process $e\gamma \to e\gamma\gamma$ in a plasma without a magnetic field was studied in the paper (A.P. Lightman, ApJ, 1981)

Introduction

The conditions in a strongly magnetized NS (magnetars) are a very exotic.

The characteristics of outer crust of magnetar

$$B \sim 10^{14} - 10^{16} \text{ G.}, \; B \gg B_e,$$

$$B_e = m^2/e \simeq 4.41 \times 10^{13} \text{ G},$$

$$T \sim 10^8 - 10^9 \text{ K}, \; T \ll \mu - m,$$

$$\frac{p_F}{m} \simeq 0.34 \frac{B_e}{B} \frac{\rho}{\rho_6}, \quad \rho \gtrsim \rho_6 = 10^6 \text{g/cm}^3$$

In these conditions we will investigate the double Compton process and photon splitting process.

Some notations

p^μ and p'^μ is the momentum of the plasma electrons,
q^μ, q'^μ and q''^μ are the momenta of initial and final photons,
The four-vectors with indices $\perp$ and $\parallel$ belong to the Euclidean {1, 2}-subspace and the Minkowski {0, 3}-subspace correspondingly in the frame were the magnetic field is directed along third axis;
$(ab)_\perp = (a\varphi\varphi b) = a_\alpha \varphi^{\rho}_{\alpha} \varphi_{\rho\beta} b_\beta$, $(ab)_\parallel = (a\tilde{\varphi}\tilde{\varphi} b) = a_\alpha \tilde{\varphi}^{\rho}_{\alpha} \tilde{\varphi}_{\rho\beta} b_\beta$.
The tensors $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly.

Photon dispersion in the magnetized medium

We begin to consider the processes $e\gamma \to e\gamma\gamma$ and $\gamma \to \gamma\gamma$ with investigation of the photon dispersion properties.

It is convenient to describe the propagation of the electromagnetic radiation in any active medium in terms of normal modes (eigenmodes). In turn, the polarization and dispersion properties of normal modes are connected with eigenvectors $\varepsilon_\alpha^{(\lambda)}(q)$ and eigenvalues of polarization operator $\varkappa^{(\lambda)}(q)$ correspondingly.

In the cold, quasidegenerate, moderately relativistic plasma:

$p_F/m \simeq v_F \ll 1$, (v_F is the Fermi velocity)

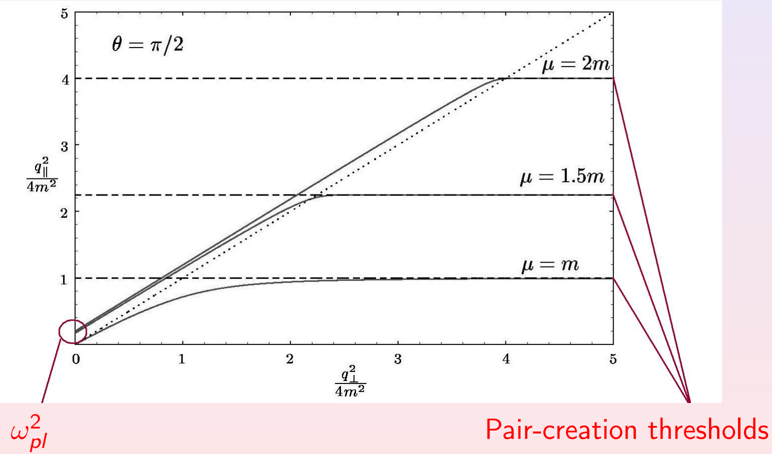
the physical polarization vectors of the photons

$$\varepsilon_\alpha^{(1)}(q) = -b_\alpha^{(1)} = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)}(q) = -b_\alpha^{(2)} = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}$$

are just as in the pure magnetic field.

Photon dispersion in the magnetized medium

The dispersion laws of the mode-2 photon for $B \simeq 10^{16}$ G and $T \simeq 50$ keV.



Photon dispersion in the magnetized medium

Two features of the photon dispersion in a cold $(T \ll \mu - m)$ magnetized plasma

- As can be seen from the Figure, in the cold plasma the pair-creation threshold is moved

$$4m^2 \to 4\mu^2$$

under the condition $k_z = 0$ $(\theta = \pi/2)$. In general case the shift of the pair-creation threshold is

$$4m^2 \to 2\left(\mu^2 - p_F|k_z| + \mu\sqrt{(p_F - |k_z|)^2 + m^2}\right)$$

under the condition $|q_z| < 2p_F$.
This result is in agreement with simple kinematical analysis of the process $\gamma_2 \to e^+e^-$ in degenerate plasma.

Kinematic analysis

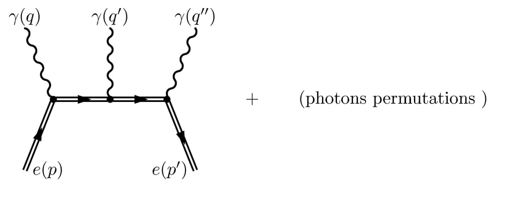
- The second features is connected with the appearance of the plasma frequency in the presence of real electrons which can be defined from the equation

$$\omega_{pl}^2 - \varkappa^{(2)}(\omega_{pl}, \mathbf{k} \to 0) = 0.$$

In our conditions $\omega_{pl}^2 = (2\alpha eB/\pi) v_F$.
These facts lead to new polarization selection rules for the photon splitting: in the region $q^2 > 0$ the splitting channels $\gamma_2 \to \gamma_2\gamma_2$, $\gamma_1 \to \gamma_2\gamma_2$ and $\gamma_1 \to \gamma_1\gamma_2$ are forbidden. Only the channel $\gamma_2 \to \gamma_1\gamma_1$ is kinematically allowed.
In this case the leading channel of mode 2 photons productions is $e\gamma_2 \to e\gamma_2\gamma_2$.

Amplitude of double Compton process

Feynman diagrams for the process $e\gamma \to e\gamma\gamma$.



In the cold plasma the scale of photons energies is $\omega \sim T \ll \mu - m$.
In this limit, the amplitude can be represented as

$$\mathcal{M}_{2\to 22} = -2m^2 (4\pi\alpha)^{3/2} \sqrt{q_{\|}^2 q_{\|}'^2 q_{\|}''^2}\, \mathcal{F}(q_{\|}, q_{\|}', q_{\|}'') \,,$$

where
$\mathcal{F}(q_{\|}, q_{\|}', q_{\|}'')$ is some function of photon momenta.

Photon absorption rate

To analyze the efficiency of the process, $e\gamma \to e\gamma\gamma$ under consideration and to compare it with other competitive reactions we calculate the photon absorption rates which can be defined in the following way (M. Chistyakov and D.R. IJMPA 2009):

$$W_{e\lambda \to e\lambda'\lambda''} = \frac{eB}{64(2\pi)^7 \omega_\lambda} \int | \mathcal{M}_{\lambda \to \lambda'\lambda''} |^2 Z_\lambda Z_{\lambda'} Z_{\lambda''} \times$$

$$\times f_E (1 + f_{\omega'}) (1 + f_{\omega''}) \delta(\omega + E - \omega' - \omega'' - E') \frac{dp_z \, d^3k' \, d^3k''}{EE'\omega'\omega''}$$

The eigenvalue of the polarization operator $\varkappa^{(2)}$ becomes large near the electron-positron pair production threshold. This suggests that the renormalization of the wave function for a photon of this polarization should be taken into account:

$$\varepsilon_\alpha^{(2)}(q) \to \varepsilon_\alpha^{(2)}(q)\sqrt{Z_2}, \quad Z_2^{-1} = 1 - \frac{\partial \varkappa^{(2)}(q)}{\partial \omega^2} \simeq 1.$$

Cross section

In our conditions ($T \ll \mu - m$) the absorption rates can be expressed in term of partial cross sections $W_{e\lambda \to e\lambda'\lambda''} = n_e \sigma_{\lambda \to \lambda'\lambda''}$. For the leading channel

$$\sigma_{2\to 22} \simeq \frac{1}{2^6 (2\pi)^5 m^2 \omega} \int d\Omega' \int d\Omega'' \int\limits_{\omega_{pl}/2}^{\omega-\omega_{pl}/2} d\omega'' \omega'' (\omega - \omega'') \mid \mathcal{M}_{2\to 22} \mid^2 ,$$

where

$$\mathcal{M}_{2\to 22} \simeq -2 \frac{(4\pi\alpha)^{3/2}}{m} \sin\theta \sin\theta' \sin\theta'' \times$$
$$\left[\left(2 - \frac{\omega''}{\omega} \right) \cos\theta' + \left(1 + \frac{\omega''}{\omega} \right) \cos\theta'' \right]$$

θ, θ' and θ'' are angles between photon momenta and magnetic field direction.

Cross section

After integration over omega, we obtain an expression for the differential cross section, which is convenient for use in solving the radiation transfer problem

$$\frac{d\sigma_{2\to 22}}{d\Omega' d\Omega''} \simeq \frac{\alpha^3}{240\pi^2 m^4} (\omega - \omega_{pl})\Theta(\omega - \omega_{pl}) \times$$
$$\sin^2\theta \sin^2\theta' \sin^2\theta'' \Big[(\omega - \omega_{pl})(23\cos^2\theta' + 44\cos\theta'\cos\theta'' +$$
$$23\cos^2\theta'') - \frac{\omega_{pl}^2}{2\omega}(29\cos^2\theta' + 32\cos\theta'\cos\theta'' + 29\cos^2\theta'') +$$
$$\frac{3\omega_{pl}^3}{4\omega^3}(4\omega - \omega_{pl})(\cos\theta' - \cos\theta'')^2\Big]$$

$\Theta(x)$ is the Theta-function.

Conclusion

- We have considered the double Compton process, $e\gamma \to e\gamma\gamma$, in the presence of a strongly magnetized, charge asymmetric, cold plasma.
- The changes of the photon dispersion properties in a magnetized medium are investigated. It has been shown, that in the conditions of the cold, quasidegenerate, moderately relativistic plasma the photons polarization vectors are just as in the pure magnetic field.

Conclusion

- The obtaining results show, that plasma influence modifies the polarization selection rules in comparison with pure magnetic field. In particular, the presence of plasma suppresses the probabilities of the photon splitting channels $\gamma_1 \to \gamma_1\gamma_2$ and $\gamma_1 \to \gamma_2\gamma_2$ in comparison with pure magnetic field.
- It is shown that in this case the process of double Compton scattering can be an effective mechanism for the production of polarized photons in cold plasma ($T \ll \mu - m$). As a result, it could lead to the modification in the mechanism of the spectra formation of SGR and AXP.

Thank you!!!

Photon dispersion in the magnetized medium

- Magnetic field without plasma.

In this case the eigenvectors are $r_\mu^{(\lambda)} = b_\mu^{(\lambda)}$ (A. Shabad 1988), where

$$b_\mu^{(1)} = \frac{(\varphi q)_\mu}{\sqrt{q_\perp^2}}, \qquad b_\mu^{(2)} = \frac{(\tilde{\varphi} q)_\mu}{\sqrt{q_\parallel^2}},$$

$$b_\mu^{(3)} = \frac{q^2\,(\Lambda q)_\mu - q_\mu\, q_\perp^2}{\sqrt{q^2 q_\parallel^2 q_\perp^2}}, \qquad b_\mu^{(4)} = \frac{q_\mu}{\sqrt{q^2}}.$$

The photon has the linear polarization.

Photon dispersion in the magnetized medium

- Strongly magnetized plasma

$$r_\mu^{(\lambda)} = \sum_{\lambda'=1}^{3} A_{\lambda'}^{\lambda}(q)\, b_\mu^{(\lambda')} \,.$$

Here $A_{\lambda'}^{\lambda}(q)$ are some complex coefficients, and the photon has the elliptical polarization.
The photon polarization operator in this case can be presented in the following form

$$\mathcal{P}_{\alpha\beta} = \sum_{\lambda} \varkappa^{(\lambda)} \frac{r_\alpha^{(\lambda)} (r_\beta^{(\lambda)})^*}{(r^{(\lambda)})^2}$$

Photon dispersion in the magnetized medium

In the cold, quasidegenerate, moderately relativistic plasma:
$p_F/m \simeq v_F \ll 1$, v_F is the Fermi velocity.
The physical polarization vectors of the photons

$$\varepsilon_\alpha^{(1)}(q) = -b_\alpha^{(1)} = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)}(q) = -b_\alpha^{(2)} = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}$$

are just as in the pure magnetic field.
The corresponding eigenvalues are

$$\varkappa^{(1)} \simeq -\frac{\alpha}{3\pi}\, q_\perp^2\,, \quad \varkappa^{(2)} \simeq -\frac{2\alpha}{\pi}\, eB \left[\mathcal{J}(q_\parallel) + H\left(\frac{q_\parallel^2}{4m^2}\right)\right]$$

$H(z)$ is the field contributions in the polarization operator.
$\mathcal{J}(q_\parallel)$ is the field and plasma contributions in the polarization operator.