Radiation transfer in a strong magnetic field with resonance effects taken into account

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Введение

The process of photon propagation in a magnetized equilibrium e^+e^- plasma taking into account the resonance in the Compton scattering reaction is considered.

$$\beta = B/B_e \lesssim 1, \;\; B_e = m^2/e.$$

The natural system of units is used: $c = \hbar = k_b = 1$.

Formulation of the problem. Find a solution of the kinetic equation for the distribution function of photons in a magnetized non relativistic plasma for the process of Compton scattering, taking into account the resonance on a virtual electron.

Previously, a similar problem was set in the work:
- Mushtukov A.A. et al. Compton scattering S-matrix and cross section in strong magnetic field Phys. Rev. D. 2016. Vol. 93.
- Mushtukov A.A. et al. Statistical features of multiple Compton scattering in a strong magnetic field at arxiv 2204.12271v1 2022

An equilibrium plasma is considered at a temperature $T \ll m$ and field $B \lesssim B_e$ directed along the z axis. The photon distribution function is non-equilibrium.

Stationary case $\frac{\partial f_\omega^{(\lambda)}}{\partial t} = 0$

Then the kinetic equation given by:

$$(\vec{n}, \vec{\bigtriangledown}_r f_\omega^{(\lambda)}) = \sum_{\lambda'=1}^{2} \int dW_{\lambda\to\lambda'} \times$$
$$\times \{ f_{E'}(1 - f_E) f_{\omega'}^{(\lambda')}(1 + f_\omega^{(\lambda)}) - f_E(1 - f_{E'}) f_\omega^{(\lambda)}(1 + f_{\omega'}^{\lambda'}) \, . \}$$

$\lambda\lambda' = 1, 2$ – polarization states of photons.

$f_\omega, f_{\omega'}$ – distribution functions of the final and initial photons.

$f_E, f_{E'}$ – distribution equilibrium functions of the final and initial electrons.

$dW_{\lambda\to\lambda'}$ – differential photon absorption rate (Chistyakov M. V., Rumyantsev D. A., Yarkov A. A. J.Phys: Conf. Ser. IOP Publishing, 2020. P. 1690 012015).

Under these conditions, the energies of the initial and final photon will be close to each other. Let Expande the right side of the kinetic equation in terms of $\Delta\omega = \omega - \omega' \ll \omega$, where

$$\omega' = \frac{1}{1-x'^2}\left(m + \omega(1-xx') - \sqrt{(m+\omega(1-xx'))^2 - 2eB(1-x'^2)}\right),$$

where $x = \cos\theta$, $x' = \cos\theta'$. θ, θ' - angle between photon momentum and magnetic field. We can use the technique developed in the works of A.S. Kompaneets 1956 and Y. E. Lyubarsky 1988 for Magnetic Field:

$$\frac{\partial f^{(\lambda)}(z,x)}{\partial z} = \frac{1}{x}\sum_{\lambda'=1}^{2}\int_{-1}^{1} dx' \varphi_{\omega}^{\lambda\lambda'}(x,x')(f_{\omega}^{(\lambda')}(z,x') - F_{\omega}^{(\lambda)}(z,x))$$

where

$$F_{\omega}^{(\lambda')}(z',x') = f_{\omega}^{(\lambda')}(z',x') - \frac{\Delta\omega}{T}\left[T\frac{\partial f_{\omega}^{(\lambda')}(z',x')}{\partial\omega} + f_{\omega}^{(\lambda')}(z',x')\right] +$$
$$+\frac{1}{2}\frac{\Delta^2\omega}{T^2}\left[T^2\frac{\partial^2 f_{\omega}^{(\lambda')}(z',x')}{\partial\omega^2} + 2T\frac{\partial f_{\omega}^{(\lambda')}(z',x')}{\partial\omega^2} + f_{\omega}^{(\lambda')}(z',x')\right],$$

$$\varphi_\omega^{\lambda\lambda'}(x, x') = \frac{n_e}{32\pi m\omega} \sum_{s''=\pm 1} \int_0^{2\pi} \frac{d\eta}{2\pi} \times$$

$$\times \frac{\left|\mathcal{M}_{e_1 \to e_0 \gamma^{(\lambda')}}\right|^2 \left|\mathcal{M}_{e_0 \gamma^{(\lambda')} \to e_1^{(s'')}}\right|^2}{[\omega^2(1 - x^2) + 2\omega m - 2eB]^2 + (\Gamma_1^{s''} P_0/2)^2} \times$$

$$\times \frac{(m + \omega - \omega x x' - \sqrt{m + \omega - \omega x x')^2 - x'^2 2eB}}{\sqrt{(m + \omega - \omega x x')^2 - x'^2 2eB}}$$

Where $\Gamma_1^{s''}$ – total electron absorption width.

$$E_1'' \Gamma_1^{\pm} \simeq \frac{e^2 (eB)^2}{\pi M_1} \frac{1}{M_1 \pm m} \int_0^{\zeta} dx e^{-x} \frac{1 - \zeta \cdot x}{\sqrt{x^2 - \zeta \cdot x + 1}}$$

Here $M_n = \sqrt{m^2 + 2 \cdot eBn}$ and $\zeta = \frac{M_1^2 + m^2}{eB}$

The solution of the equation can be formally represented as follows

$$f_\omega^{(\lambda)}(z,x) = f_{0\omega} e^{-\chi_\omega^{(\lambda)}(x)\cdot z} + \frac{1}{x}\int_0^z dz' \int_{-1}^{1} dx' e^{-\chi_\omega^{(\lambda)}(x)\cdot(z-z')} \times$$
$$\times\, \varphi_\omega^{\lambda\lambda'}(x,x') F_\omega^{(\lambda')}(z',x')\,,$$

where $f_{0\omega} = [\exp(\omega/T) - 1]^{-1}$ – photon equilibrium function.

$$\chi_\omega^{(\lambda)}(x) \equiv \frac{1}{x}\int_{-1}^{1} dx' \left\{ \varphi_\omega^{\lambda 1}(x,x') + \varphi_\omega^{\lambda 2}(x,x') \right\}\,.$$

Using the expansion in Legendre polynomials

$$f_{\omega}^{(\lambda)}(z,x) = \sum_{\ell=0}^{\infty} A_{\ell}^{(\lambda)}(z,\omega)\mathcal{P}_{\ell}(x)\,,$$

and Laplace transform, we obtain the system of differential equations:

$$\frac{2}{2\ell+1}\overline{A}_{\ell}^{(\lambda)}(s,\omega) = \int_{-1}^{1} \frac{f_{0\omega}^{(\lambda)}}{s+\chi_{\omega}(x)}\mathcal{P}_{\ell}(x)dx +$$
$$+ \sum_{\ell'=0}^{\infty} \frac{1}{x}\int_{-1}^{1} dx' \int_{-1}^{1} dx \frac{P_{\ell}(x)P_{\ell'}(x')}{s+\chi_{\omega}^{(\lambda)}(x)}\varphi_{\omega}^{\lambda\lambda'}(x,x')\overline{\mathcal{F}}_{\ell'}^{(\lambda')}(s,\omega)\,,$$

where

$$\overline{\mathcal{F}}_{\ell'}^{(\lambda')}(s,\omega) = \overline{A}_{\ell'}^{(\lambda')}(s,\omega) - \frac{\Delta\omega}{T}\left[T\frac{\partial\overline{A}_{\ell'}^{(\lambda')}(s,\omega)}{\partial\omega} + \overline{A}_{\ell'}^{(\lambda')}(s,\omega)\right] +$$
$$+ \frac{1}{2}\frac{\Delta^2\omega}{T^2}\left[T^2\frac{\partial^2\overline{A}_{\ell'}^{(\lambda')}(s,\omega)}{\partial\omega^2} + 2T\frac{\partial\overline{A}_{\ell'}^{(\lambda')}(s,\omega)}{\partial\omega^2} + \overline{A}_{\ell'}^{(\lambda')}(s,\omega)\right].$$

$$\overline{A}_{\ell'}^{(\lambda)}(s,\omega) = \int_0^\infty A_{\ell'}^{(\lambda)}(z,\omega)e^{-sz}dz\,,$$

Finally, the distribution functions of photons for two possible polarization states $\lambda = 1, 2$ can be represented as follows:

$$f_\omega^{(\lambda)}(z,x) = \frac{1}{2\pi i}\sum_{\ell=0}^{\infty} P_\ell(x)\int_{\sigma-i\infty}^{\sigma+i\infty} ds\cdot e^{sz}\overline{A}_\ell^{(\lambda)}(s,x)$$

Выводы

- The solution of the kinetic equation for finding the distribution function of photons of two possible polarizations in an equilibrium e^+e^- plasma in a relatively strong magnetic field in the cold plasma approximat.ion and taking into account resonance on a virtual electron is considered.
- Using the Laplace transform and the expansion of the distribution function in Legendre polynomials, the problem is reduced to a system of differential equations.
- The resulting distribution function is represented as inverse Laplace Transform.