Pasta phases in neutron stars: Extented Thomas-Fermi vs Compressible Liquid Drop Model

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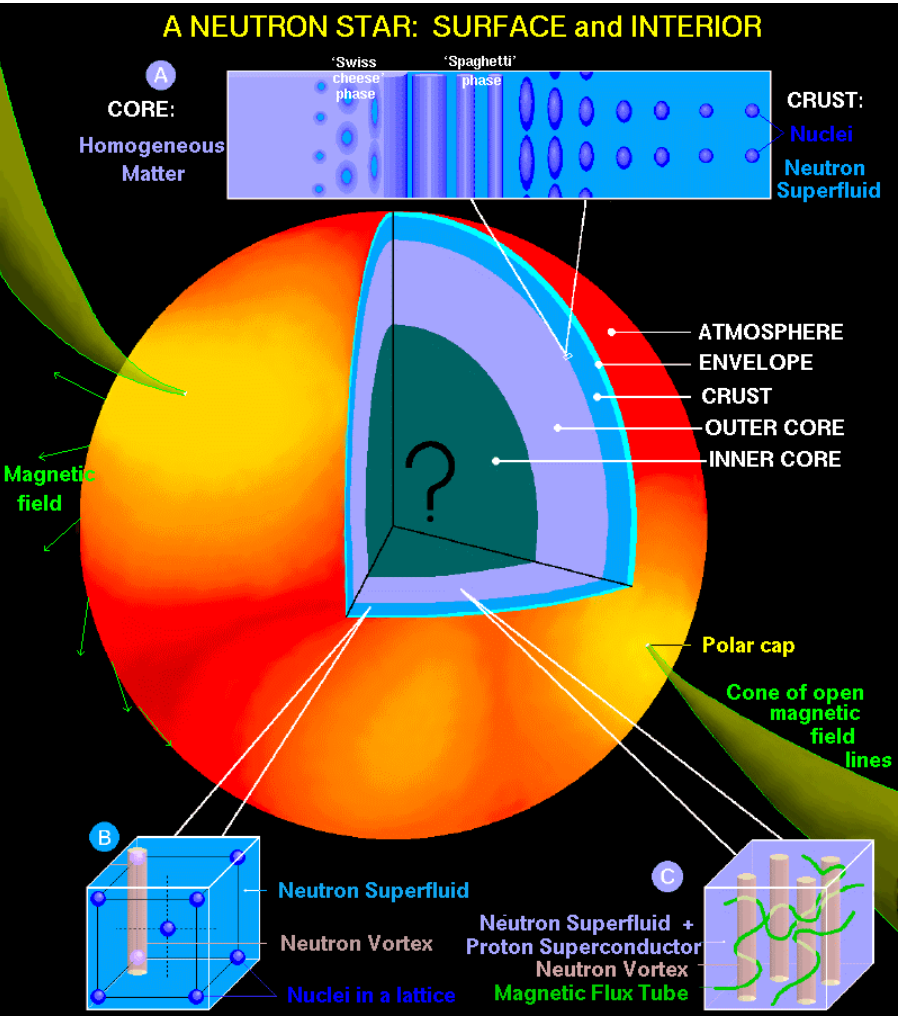
Motivation I: neutron stars and superdense matter



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$$\rho \sim 10^{15}\ \mathrm{g/cm^3}$$

$$T \lesssim 10^{9}\ \mathrm{K}$$

$$B \sim 10^{12}\ \mathrm{G}$$

$$g \sim 10^{14}\ \mathrm{cm/s^2}$$

$$R \sim 2R_{\mathrm{g}} = 4GM/c^2$$

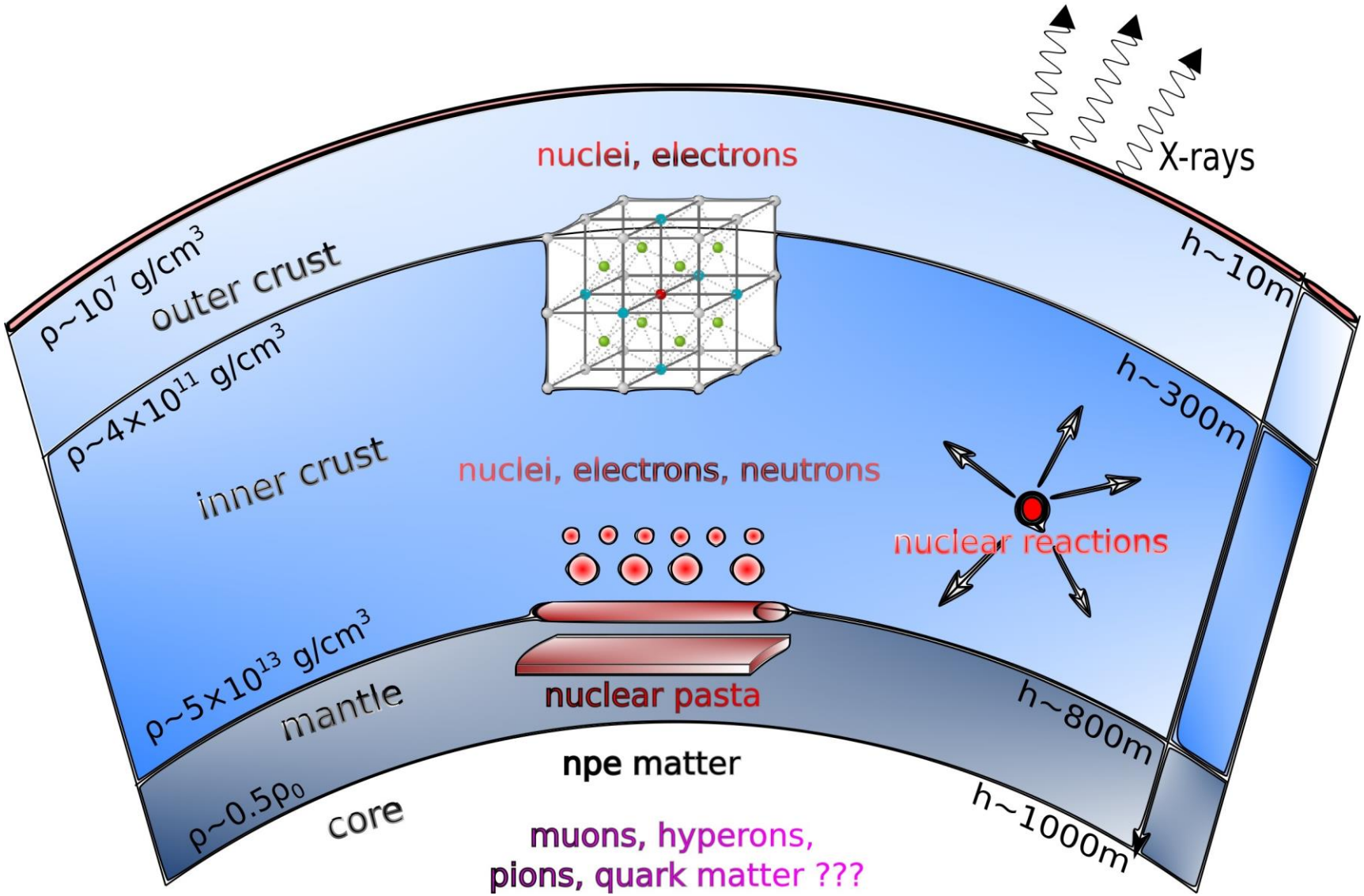
$$T_{\mathrm{cp}} \sim 10^{9}\ \mathrm{K}$$

$$T_{\mathrm{cn}} \sim 10^{8}\ \mathrm{K}$$

$R \sim 12\ \mathrm{km},\ M \sim 1.4 M_{\odot}$

Information on the surface layers is required for adequate interpretation of observations

Motivation II: Pasta phases



N. N. Shchechilin ©

Pasta phases

- Ravenhall et al. Phys. Rev. Lett. 1983, 50, 2066–2069
- Hashimoto et al. Progress of Theoretical Physics 1984, 71, 320–326.

Neutron-proton cluster shape:

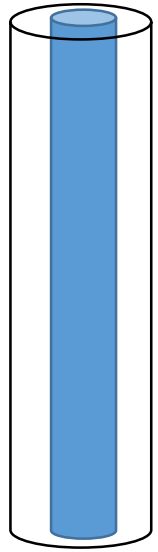
- Spheres (ordinary nuclei)
- Cylinders
- Plates
- Inverse cylinders (np outside, n inside)
- Inverse spheres (np outside, n inside)
- Core (microscopically uniform nuclear matter)

How to choose the pasta?

(Zero temperature) thermodynamic approach:

The pasta with the lowest energy is the best one

- Choose baryon number density
- Calculate energy for each of the cluster shapes:
  - ✓ Consider WZ cell, with the same shape as a cluster
  - ✓ Calculate energy density for this cell, adjusting parameters:
    - Size of the cell ~ distance between clusters;
    - Size of the cluster
    - Proton fraction (=beta-equilibrium)
    - Neutron and proton distribution
    - ….



- Compare energy density for all configurations
- Determine equilibrium phase

Extended Thomas-Fermi approach

Density functional (4th order Wigner-Kirkwood expansion)

$$e_{\mathrm{ETF}} = e_{\mathrm{nuc}} + e_{\mathrm{C}} + e_{\mathrm{e}}$$

nuclear Coulomb electron

$$e_{\mathrm{nuc}} = \frac{1}{A}\int \mathcal{E}(\rho_q(\mathbf{r}), \nabla\rho_q(\mathbf{r}), \triangle\rho_q(\mathbf{r}))\mathrm{d}^D\mathbf{r}.$$

Nucleon profile optimization: (accurate) Euler–Lagrange equations

$$\frac{\partial\delta e_{\mathrm{ETF}}}{\partial\delta n_p} = \mu_p; \quad \frac{\partial\delta e_{\mathrm{ETF}}}{\partial\delta n_n} = \mu_n; \quad \mu_n - \mu_p = \mu_e$$

Real life nucleon profile optimization: minimization over parametrized profiles

$$\rho_{\mathrm{q}}(r) = \rho_{\mathrm{B}q} + \frac{\rho_{\Lambda q}}{1 + \exp\left[\left(\frac{C_q - R}{r - R}\right)^2 - 1\right]\exp\left(\frac{r - C_q}{a_q}\right)}$$

Pearson et al., Mon. Not. R Astron. Soc. **2018**, 481, 2994

Compressible liquid drop model

$$\epsilon = w\,\epsilon^{\rm bulk}(n_{ni},\, n_{pi}) + (1-w)\,\epsilon^{\rm bulk}(n_{no},\, 0) + \frac{E_s(\nu_s, r_p)}{V_c} + \frac{E_C(n_{pi}, r_p, w)}{V_c} + \epsilon_e(n_e).$$

Explicit (algebraic) expression for the energy density

6 parameters:

$$n_{ni},\, n_{pi},\, n_{no},\, \nu_s,\, V_c,\, r_p$$

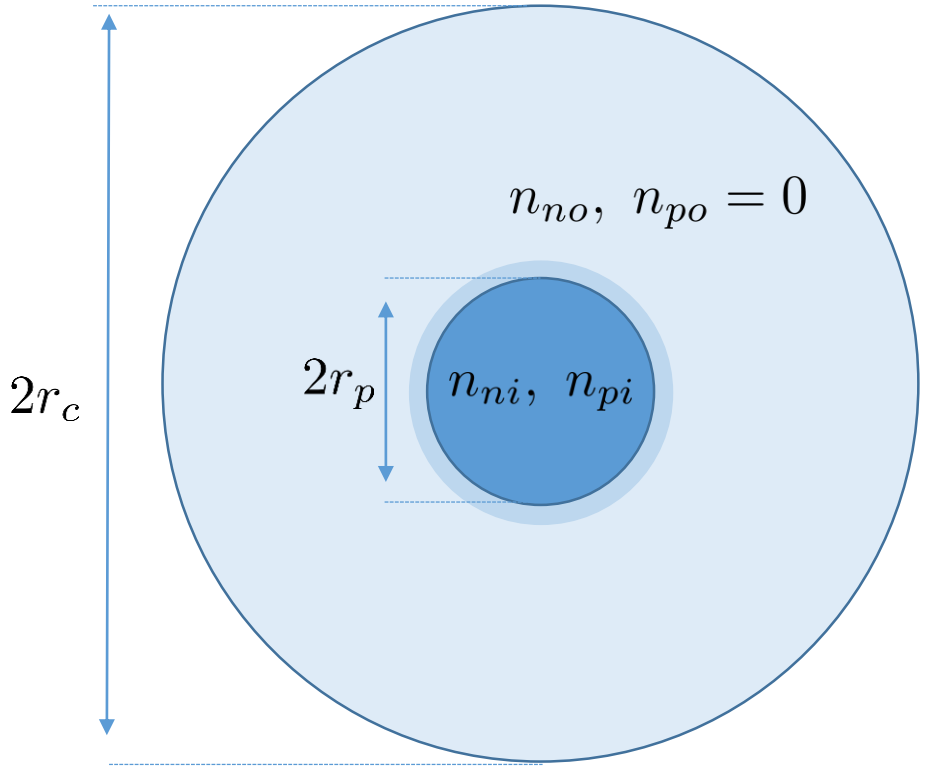
Minimization at fixed n_b

System of 5 algebraic (nonlinear) equations
With clear physical meaning:

- Chemical equilibrium inside the cell (2 eqs.)
- Beta-equilibrium (1 eq)
- Mechanical equilibrium (1 eq)
- Optimal size of the cell (1 eq)

Explicit formulae for thermodynamic quantities – useful for astrophysical applications!

$$\mu_n,\; P,\; \ldots$$

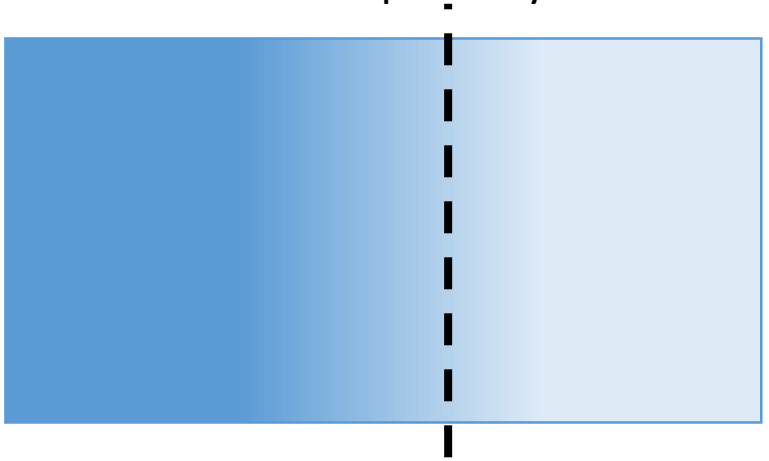


Surface energy can be straightforwardly calculated

Centelles et al. Nucl. Phys. A, 635 (1998), 193

Real two-phase system

Reference two-phase system



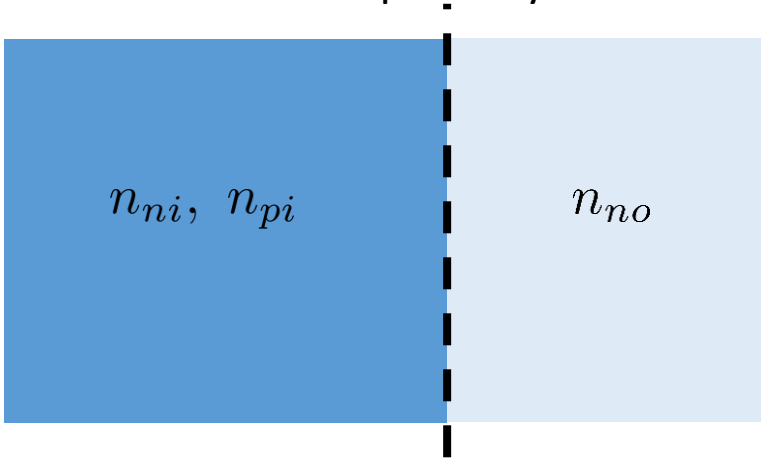
$$E_{\text{two phase}}, N_{\text{two phase}}, Z_{\text{two phase}}$$

Surface energy can be straightforwardly calculated

Centelles et al. Nucl. Phys. A, 635 (1998), 193

Real two-phase system

Reference two-phase system



$$E_{\text{two phase}}, N_{\text{two phase}}, Z_{\text{two phase}}$$

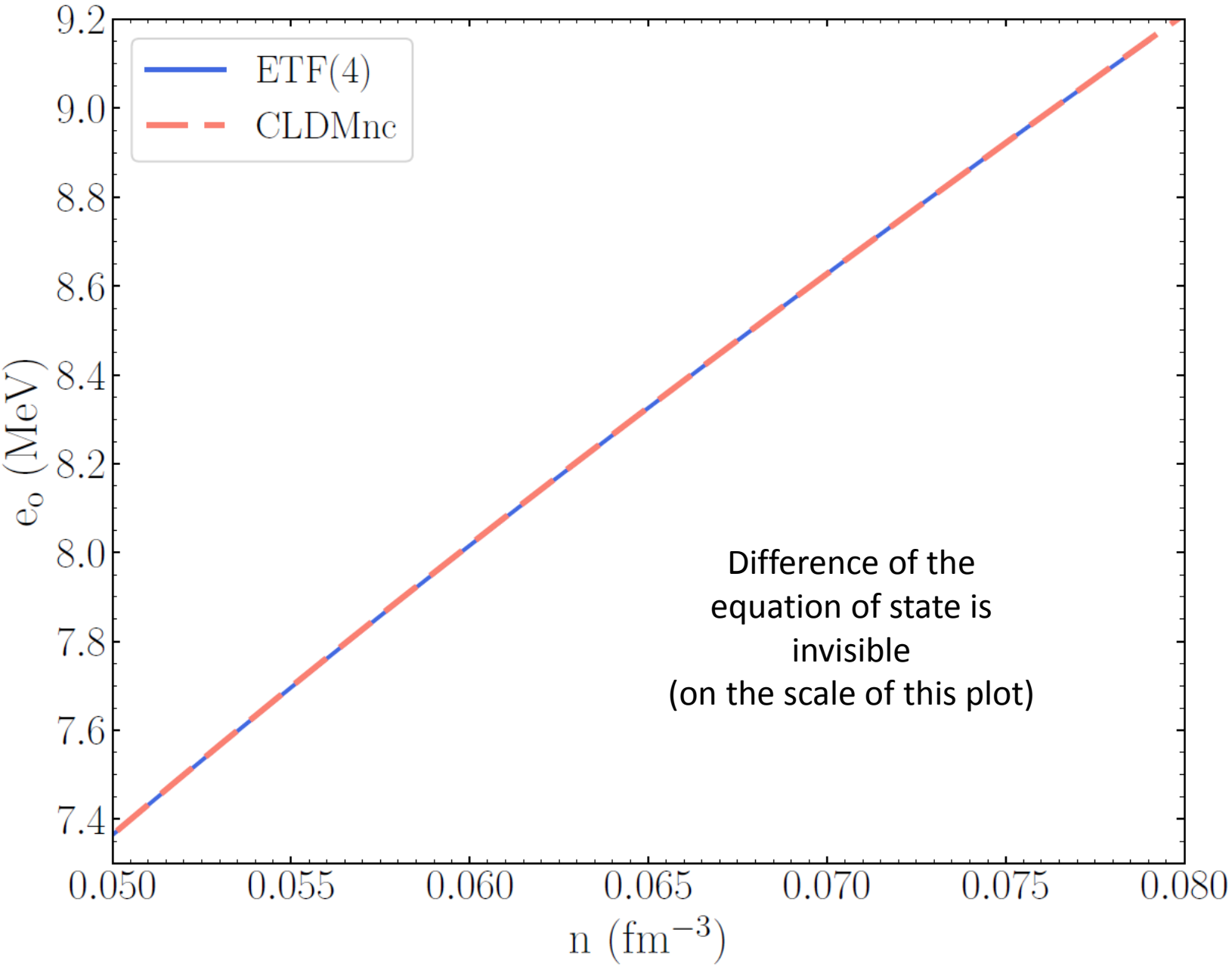
$$\begin{aligned} E_{\text{ref}} &= \epsilon_i V_i + \epsilon_o V_o, \\ N_{\text{ref}} &= n_{ni} V_i + n_{no} V_o, \\ Z_{\text{ref}} &= n_{pi} V_i \end{aligned}$$

$$\begin{aligned} E_{\text{surf}} &= E_{\text{two phase}} - E_{\text{ref}} \\ N_{\text{surf}} &= N_{\text{two phase}} - N_{\text{ref}} \\ Z_{\text{surf}} &= Z_{\text{two phase}} - Z_{\text{ref}} \end{aligned}$$

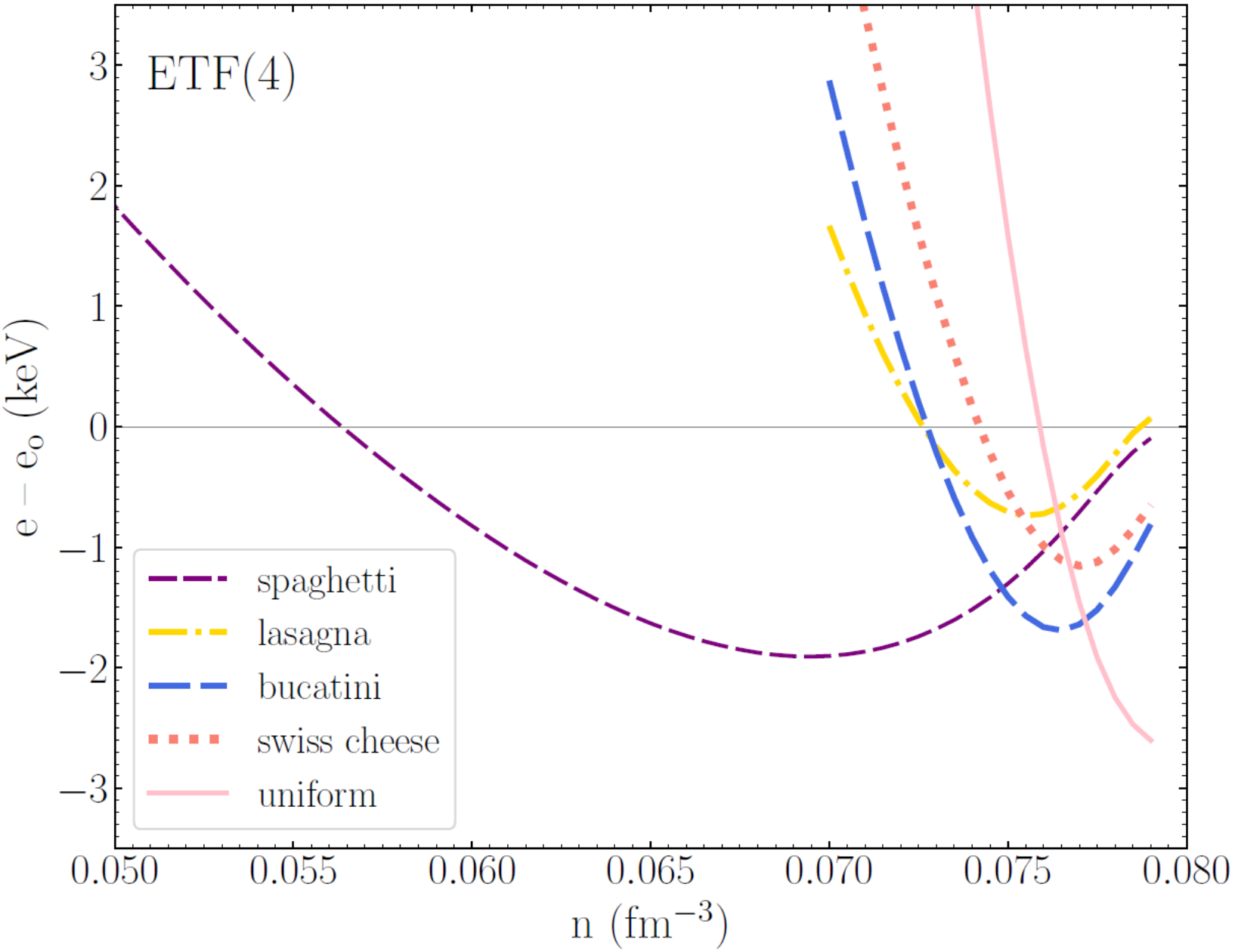
Can be nulled by choose of the reference system

- Surface energy is a correction, required to reproduce two phase system energy
- (Neutron) adsorption is required for thermodynamically consistent description of two phase boundary
- Useful thermodynamic relations

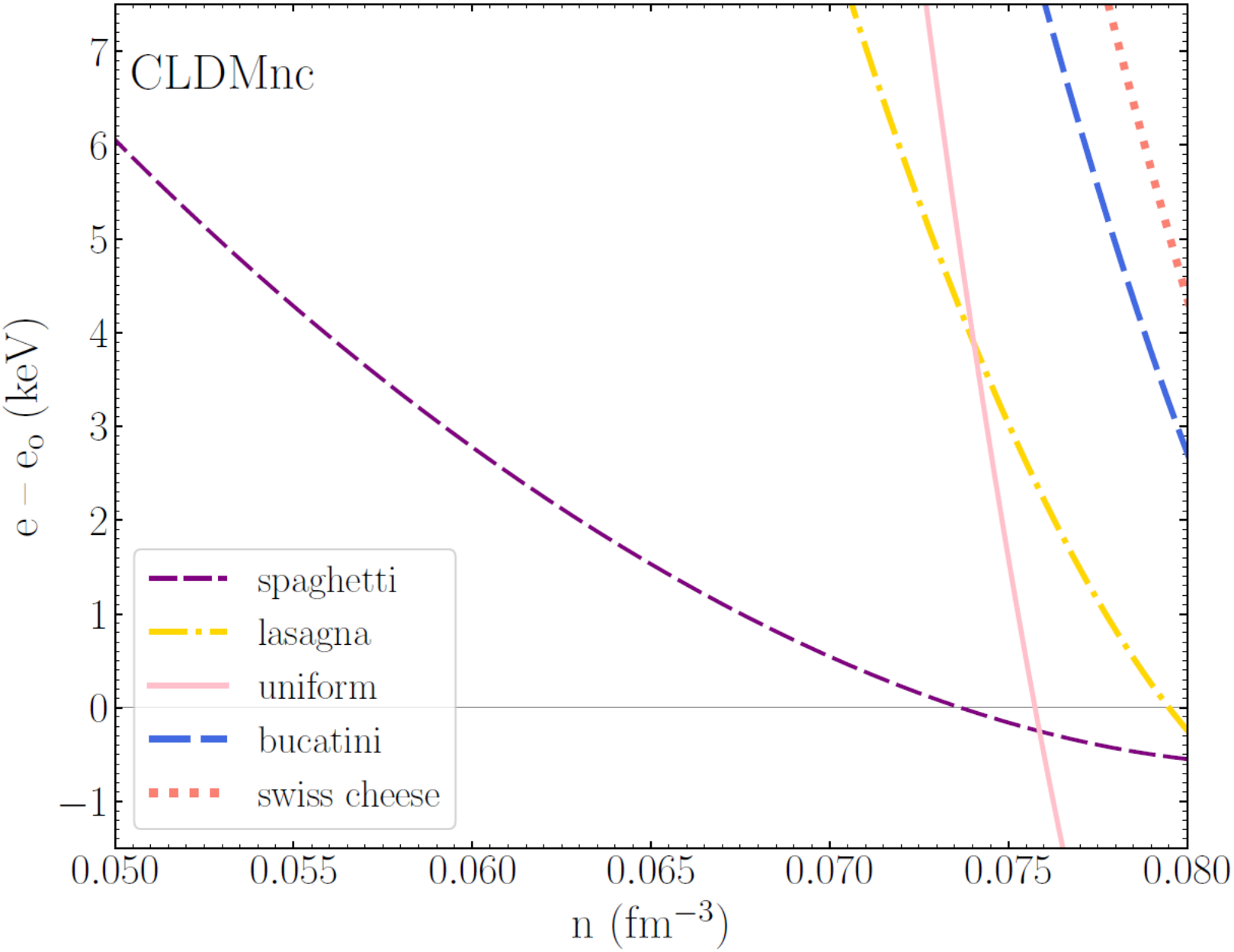
Equation of state



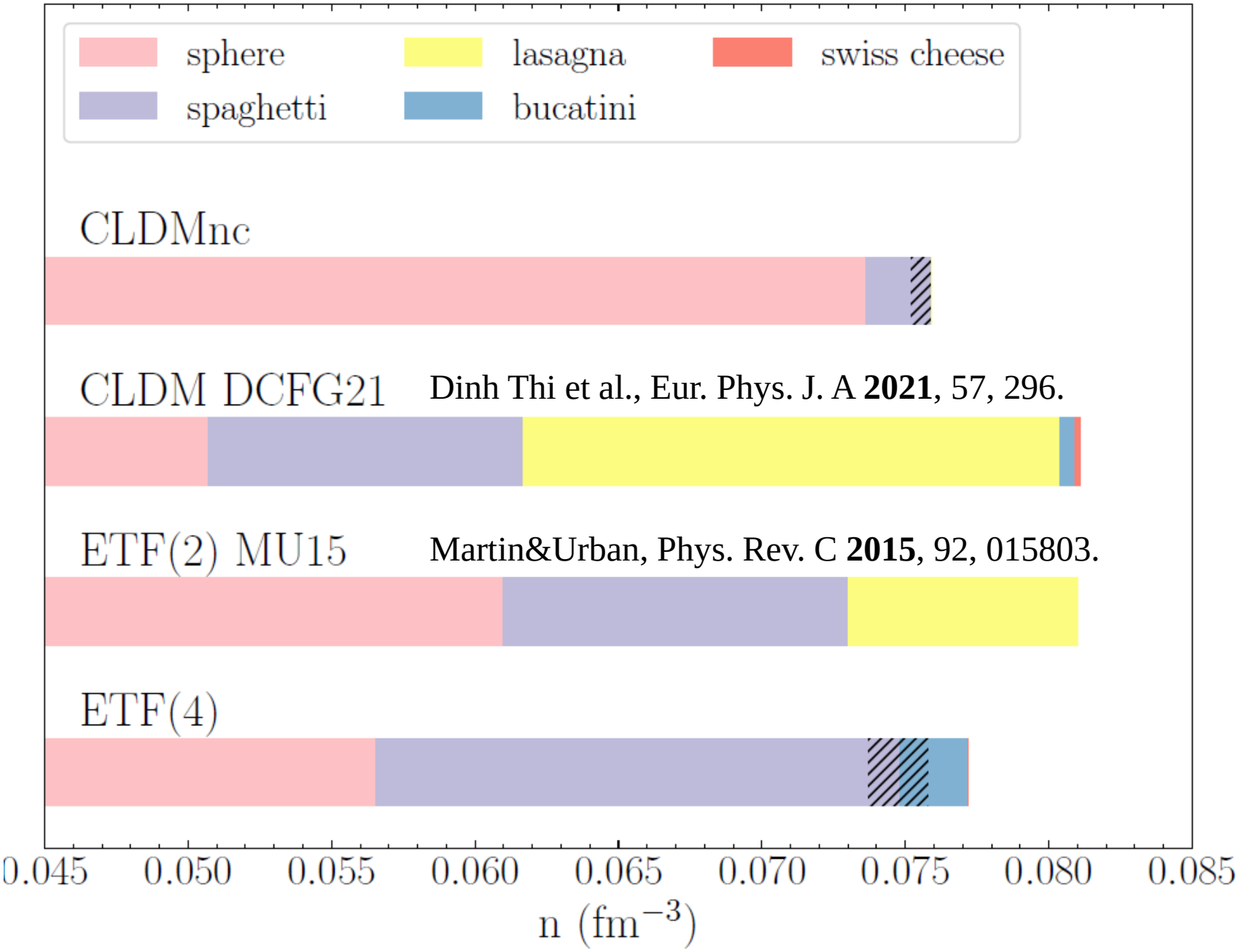
Pasta: Extended Thomas-Fermi with Sly4 potential



Pasta: Compressible liquid drop model with SLy4



SLy4 pastas: CLDM vs ETF



Results and conclusions

- Curvature corrections to the surface tension should be taken into account to improve agreement of the CLDM predictions of the pasta phase boundaries with ETF calculations
- Accurate prediction of the pasta phase boundaries is very delicate problem

$$\Delta\epsilon \sim 1 \frac{\text{keV}}{\text{nucleon}}$$

$$\epsilon \sim (7.5 \div 9) \frac{\text{MeV}}{\text{nucleon}}$$

(RMS for microscopical models for AME2020 nuclei ~ 0.7 MeV, e.g. Grams et al. arXiv:2211.03592)

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