Inflationary models with Gauss-Bonnet term

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The four-dimensional superstring corrections to Einstein gravity can be presented such as Gauss-Bonnet term $\xi(\phi)G$ [1]. At present inflationary scenarios in Einstein-Gauss-Bonnet gravity are rather popular [2]

[1] J. c. Hwang and H. Noh, Phys. Rev. D **61**, 043511 (2000),[astro-ph/9909480]; C. G. Callan, D. Friedan, E. J. Martinec and M. J. Perry, Nucl. Phys. B 262 593 (1985); B. Zwiebach, Phys. Lett.B 156, 315 (1985); S. Deser and A. N. Redlich, Phys. Lett.B 176, 350 (1986); D. J. Gross and J. H. Sloan, Nucl. Phys. B 291, 41 (1987)

[2] S. Nojiri, S. D. Odintsov, V. K. Oikonomou and A. Constantini, Nucl. Phys. B 985, 116011 (2022), [2210.16383 [gr-qc]]; Z. K. Guo and D. J. Schwarz,Phys. Rev. D **81**, 123520 (2010) [1001.1897 [hep-th]].; C. van de Bruck and C. Longden, Phys. Rev. D **93**, no.6, 063519 (2016), [1512.04768 [hep-ph]]; K. El Bourakadi, M. Ferricha-Alami, H. Filali, Z. Sakhi and M. Bennai, Eur. Phys. J. C 81, no.12, 1144 (2021),[2209.08581 [gr-qc]]; H. A. Khan and Yogesh, Phys. Rev. D 105, no.6, 063526 (2022), [2201.06439 [astro-ph.CO]]; E. O. Pozdeeva, Eur. Phys. J. C **80**, no.7, 612 (2020), [2005.10133 [gr-qc]];E. O. Pozdeeva and S. Y. Vernov, Eur. Phys. J. C **81**, no.7, 633 (2021), [2104.04995 [gr-qc]];

- We modify the Starobinsky inflation model by adding the Bel-Robinson tensor $T^{\mu\nu\lambda\rho}$ squared term proposed as the leading quantum correction inspired by superstring theory.
- The $(R + \frac{1}{6m^2}R^2 - \frac{\beta}{8m^6}T^2)$ model under consideration has two parameters: the inflaton mass m and the string-inspired positive parameter β.
- We derive the equations of motion in the Friedmann-Lemaitre-Robertson-Walker universe and investigate its solutions.
- We find the physical bounds on the value of the parameter β by demanding the absence of ghosts and consistency of the derived inflationary observables with the measurements of the cosmic microwave background radiation.

- The Starobinsky model of inflation [3] is described by the modified gravity action

$$S_{\rm Star.}[g_{\mu\nu}] = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left(R + \frac{1}{6m^2}R^2\right) \qquad (1)$$

$M_{\rm Pl} = 1/\sqrt{8\pi G_N}$-reduced Planck mass, m-inflaton (scalaron) mass.
- This model has an attractor-type solution describing a quasi-de Sitter expansion of the universe with the slow-roll inflation.
- Being proposed the long time ago, the Starobinsky inflationary model is in perfect agreement with the recent measurements of the cosmic microwave background (CMB) radiation [4]
- The only free parameter m is fixed by CMB measurements (COBE normalization) as

$$m = 1.3\left(\frac{55}{N}\right)10^{-5}M_{\rm Pl} = 3.2\left(\frac{55}{N}\right)10^{13}\,{\rm GeV}, \qquad (2)$$

N is the number of e-foldings describing the duration of inflation.

[3] A.A. Starobinsky, Phys. Lett. B 91 (1980) 99

[4] Planck collaboration, Astron. Astrophys.641 (2020) A10 [1807.06211]; BICEP, Keck collaboration, Phys.Rev. Lett. 127 (2021) 151301 [2110.00483]; M. Tristram et al., Phys. Rev. D 105 (2022) 083524 [2112.07961]

The Bel-Robinson (BR) tensor in 4D

- We consider the gravity action

$$S_{\rm SBR}[g_{\mu\nu}] = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}\left[R + \frac{1}{6m^2}R^2 - \frac{\beta}{8m^6}T^{\mu\nu\lambda\rho}T_{\mu\nu\lambda\rho}\right] , \tag{3}$$

  where we have introduced dimensionless coupling constant $\beta > 0$ and the Bel-Robinson (BR) tensor in four spacetime dimensions.
- The Bel-Robinson (BR) tensor in four spacetime dimensions [5]

$$\begin{aligned} T^{\mu\nu\lambda\rho} &\equiv R^{\mu\alpha\beta\lambda}R^{\nu}{}_{\alpha\beta}{}^{\rho} + {}^*R^{\mu\alpha\beta\lambda}\,{}^*R^{\nu}{}_{\alpha\beta}{}^{\rho} \\ &= R^{\mu\alpha\beta\lambda}R^{\nu}{}_{\alpha\beta}{}^{\rho} + R^{\mu\alpha\beta\rho}R^{\nu}{}_{\alpha\beta}{}^{\lambda} - \frac{1}{2}g^{\mu\nu}R^{\alpha\beta\gamma\lambda}R_{\alpha\beta\gamma}{}^{\rho} , \end{aligned} \tag{4}$$

- We define dual tensors with the help of Levi-Civita tensors, e.g.,

$${}^*R_{\mu\nu\lambda\rho} = \frac{1}{2}E_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\lambda\rho} , \quad E_{\mu\nu\lambda\rho} = \sqrt{-g}\,\epsilon_{\mu\nu\lambda\rho} , \tag{5}$$

  where $\epsilon_{\mu\nu\lambda\rho}$ is the constant Levi-Civita symbol.

[5] L. Bel, Colloq. Int. CNRS 91 (1962) 119.; I. Robinson, Bull. Acad. Pol. Sci. Ser. Sci. Math. Astron. Phys. 7 (1959) 351.; S. Deser, Iberian Gravity Symposium, 1, 1999 [gr-qc/9901007]

Euler density and Gauss-Bonnet-term

- The BR tensor squared can be rewritten in terms of the Euler and Pontryagin densities squared by using the identities [6]

$$T^{\mu\nu\lambda\rho} T_{\mu\nu\lambda\rho} = \frac{1}{4}\left(P_4^2 - E_4^2\right) = \frac{1}{4}\left(P_4 + E_4\right)\left(P_4 - E_4\right) \ , \tag{6}$$

where the Euler and Pontryagin (topological) densities have been introduced in $D = 4$ dimensions as

$$E_4 = {}^*R_{\mu\nu\lambda\rho}{}^*R^{\mu\nu\lambda\rho} \quad \text{and} \quad P_4 = {}^*R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} \ , \tag{7}$$

- The Euler density coincides with the Gauss-Bonnet (GB) term $E_4 = \mathcal{G}$.

[6] S. Deser,[gr-qc/9901007]

Therefore, we can rewrite the SBR action (3) to the form

$$S_{\rm SBR}[g_{\mu\nu}] = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R + \frac{1}{6m^2}R^2 + \frac{\beta}{32m^6}\left(\mathcal{G}^2 - P_4^2\right)\right]~, \quad (8)$$

thus establishing a connection to the modified $f(R,\mathcal{G})$ gravity theories. [7] In particular, the positive sign of β is consistent with the physical requirement in the $F(\mathcal{G})$ modified theories of gravity, demanding the second derivative of the F-function to be positive [8]

[7] To the best of our knowledge, the P_4-terms were never considered in the modified gravity literature.

[8] A. De Felice and S. Tsujikawa, Construction of cosmologically viable f(G) dark energy models, Phys. Lett. B 675 (2009) 1 [0810.5712].

The linearization of the SBR action

The classical actions (3) and (8) can also be rewritten to the following form:

$$S_{\mathrm{SBR}}[g_{\mu\nu}, \phi, \chi, \xi] = \frac{M_{\mathrm{Pl}}^2}{2} S_R - \frac{M_{\mathrm{Pl}}^2 \beta}{32 m^6} \left(S_{\mathcal{G}} + S_P\right), \tag{9}$$

where we have introduced the auxiliary scalar fields ϕ, χ and ξ, together with

$$S_R[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[R\left(1 + \frac{\phi}{3m^2}\right) - \frac{\phi^2}{6m^2}\right], \tag{10}$$

$$S_{\mathcal{G}}[g_{\mu\nu}, \chi] = \int d^4x \sqrt{-g} \left(\frac{\chi^2}{2} - \mathcal{G}\chi\right), \tag{11}$$

$$S_P[g_{\mu\nu}, \xi] = \int d^4x \sqrt{-g} \left(\xi P_4 - \frac{\xi^2}{2}\right). \tag{12}$$

Varying the action (9) with respect to the scalar fields, we get the equations

$$\chi = \mathcal{G}, \quad \phi = R, \quad \xi = P_4 \,, \tag{13}$$

while their substitution into Eq. (9) yields back the action (8).

Friedman-Lemaitre-Robertson-Walker (FLRW) universe

- We apply the SBR theory to inflation in a flat Friedman-Lemaitre-Robertson-Walker (FLRW) universe

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right), \tag{14}$$

t is the cosmic time, $a(t)$ is a scale factor.

- We find that the P_4 term in the SBR action does *not* contribute to the equations of motion in the FLRW case.
- Then the equations of motion in the SBR theory are given by

$$\left(R_{\rho\nu} - \frac{g_{\rho\nu}}{2}R\right)\left(1 + \frac{\phi}{3m^2}\right) + \frac{\phi^2}{12m^2}g_{\rho\nu} - \frac{1}{3m^2}\left(\frac{\nabla_\rho\nabla_\nu + \nabla_\nu\nabla_\rho}{2} - g_{\rho\nu}\Box\right)\phi$$

$$+\frac{\beta}{64m^6}\left[\chi^2 g_{\rho\nu} + 8\left\{(Rg_{\rho\nu} - 2R_{\rho\nu})\Box\chi - R\nabla_\rho\nabla_\nu\chi\right.\right.$$

$$\left.\left. + 2(R^\alpha_\nu\nabla_\alpha\nabla_\rho\chi + R^\alpha_\rho\nabla_\alpha\nabla_\nu\chi) - 2(g_{\rho\nu}R_{\alpha\beta} + R_{\alpha\rho\nu\beta})\nabla^\beta\nabla^\alpha\chi\right\}\right] = 0,$$

$$\chi = \mathcal{G}, \ \phi = R \ .$$

The $(0,0)$-component of the equations of motion in the FLRW universe reads

$$3H^2\left(1+\frac{R}{3m^2}\right)-\frac{R^2}{12m^2}+\frac{H\dot{R}}{m^2}=\frac{\beta}{64m^6}\left[\mathcal{G}^2-48H^3\dot{\mathcal{G}}\right]\ . \tag{15}$$

We rewrite Eq. (15) in terms of the Hubble function $H(t)$ and its time derivatives as

$$\begin{aligned}2\left(m^4+3\beta H^4\right)H\ddot{H} \quad &- \quad \left(m^4-9\beta H^4\right)\dot{H}^2 \\ &+ \quad 6\left(m^4+3\beta H^4\right)H^2\dot{H}-3\beta H^8+m^6H^2=0\end{aligned} \tag{16}$$

Slow-roll solution with $\beta = 0$

In the case $\beta = 0$ we can be written in terms of the Hubble function $H(t)$ as the non-linear ODE of the 2nd order (dubbed the *Starobinsky equation*)

$$2H\ddot{H} - \left(\dot{H}\right)^2 + H^2\left(6\dot{H} + m^2\right) = 0 \ , \tag{17}$$

In the slow-roll approximation defined by the conditions

$$\left|\ddot{H}\right| \ll \left|H\dot{H}\right| \quad \text{and} \quad \left|\dot{H}\right| \ll H^2 \ , \tag{18}$$

Eq. (17) is greatly simplified to

$$6\dot{H} + m^2 \approx 0 \ , \tag{19}$$

and has the well-known solution

$$H(t) \approx \frac{m^2}{6}(t_0 - t) \ , \tag{20}$$

where t_0 is the integration constant that apparently corresponds to the end of inflation, so that this leading term in $H(t) > 0$ should be a good approximation for $t \ll t_0$.

Taking into account the slow-roll conditions allows us to simplify Eq. (16) to the non-linear ordinary differential equation

$$6\left(m^4 + 3\beta H^4\right)\dot{H} - 3\beta H^6 + m^6 = 0\,. \tag{21}$$

When searching for a perturbative solution to this equation in the first order with respect to β, we find a simple answer,

$$H(t) \approx \frac{m^2(t_0 - t)}{6} - \beta\left(\frac{m}{6}\right)^6 (t_0 - t)^5 \left[\frac{m^2}{14}(t_0 - t)^2 + \frac{18}{5}\right]\,. \tag{22}$$

The first derivative of the Hubble function (22) with respect to time reads

$$\dot{H} = -\frac{m^2}{6} + \frac{\beta\, m^6\, (t - t_0)^4\left[(t - t_0)^2\, m^2 + 36\right]}{2^7 \cdot 3^6}\,. \tag{23}$$

Physical bounds on the value of β

• Cano, Fransen and Hertog [9] studied various scenarios of inflation in the neighborhood of the Starobinsky model modified by the higher-order curvature terms, depending upon the unknown effective function $F(H^2)$ entering the equations of motion in the FLRW universe.
• Though our modification of the Starobinsky model in Eq. (8) is outside their modified (higher-derivative) gravity theories because Eq. (16) includes $\ddot{H}$.
• We can apply their results in the slow-roll approximation under the conditions (18) after the identification of the parameters as $\alpha l^2 = 2/m^2$, where α is the dimensionless coupling constant [10] and $l = (\alpha')^{1/2}$ is the fundamental length in superstring theory.

[9] P.A. Cano, K. Fransen and T. Hertog, Phys. Rev. D 103 (2021) 103531 [2011.13933].
[10] P.A. Cano, K. Fransen and T. Hertog, Phys. Rev. D 103 (2021) 103531 [2011.13933].

The slow-roll conditions (18) allows to simplify Eq. (15)

$$R\left(\frac{R}{12}-H^2\right)-H\dot{R}=3\,m^2\left(H^2-\frac{3\,\beta\,H^8}{m^6}+\frac{18\,\beta\,H^6\,\dot{H}}{m^6}\right)\quad . \tag{24}$$

Equation (15) can be put to the form

$$R\left(\frac{R}{12}-H^2\right)-H\dot{R}=3\,m^2\left(H^2-\frac{3\beta}{m^4}H^6-\frac{3\beta}{m^6}H^8\right)\equiv 3m^2F(H^2)\ , \tag{25}$$

so that the effective $F(H^2)$ function [11] in our case is given by

$$F(H^2)=H^2-\frac{3\beta}{m^4}\left(H^2\right)^3-\frac{3\beta}{m^6}\left(H^2\right)^4\ . \tag{26}$$

Accordingly, we have

$$F'(H^2)=1-9\beta\left(\frac{H}{m}\right)^4-12\beta\left(\frac{H}{m}\right)^6 \tag{27}$$

where the primes here denote the differentiations with respect to H^2.

[11] P.A. Cano, K. Fransen and T. Hertog, Phys. Rev. D 103 (2021) 103531 [2011.13933].

The effective Newton constant in the higher-derivative gravities [12] must obey the condition (in the notation adapted to the F-function [13]

$$G_{\mathrm{eff.}} = \frac{1}{8\pi M_{\mathrm{Pl}}^2 \left[F'(H^2) + 4(H^2/m^2)\right]} > 0 \tag{28}$$

in order to avoid graviton ghosts. Given the F-function (26), we find the restriction

$$\beta < 6.941 \cdot 10^{-4}\,. \tag{29}$$

[12] P. Bueno, P.A. Cano, V.S. Min and M.R. Visser, Phys. Rev. D 95 (2017) 044010 [1610.08519].

[13] P.A. Cano, K. Fransen and T. Hertog, Phys. Rev. D 103 (2021) 103531 [2011.13933].

Inflationary parameters

We apply early obtained [14] formulas for inflationary parameters:

$$n_s = 1 - \frac{2}{N} - \frac{8\,\lambda_3 m^4\, N}{27} + \frac{\lambda_4 m^6\, N^2}{6} \tag{30}$$

$$r = \frac{12}{N^2} - \frac{16\,\lambda_3\, m^4}{9} + \frac{2\,\lambda_4\, N m^6}{3} \tag{31}$$

where λ_3, λ_4 are coefficients before corresponding power of the Hubble parameter in right part of the following equation:

$$R\left(\frac{R}{12} - H^2\right) - H\dot{R} = 3\,m^2\left(\lambda_4\, H^8 - \lambda_3\, H^6 + H^2\right) \tag{32}$$

[14] P.A. Cano, K. Fransen and T. Hertog, Phys. Rev. D 103 (2021) 103531 [2011.13933].

Yet another upper bound on β can be obtained from CMB measurements. The results of P.A. Cano, K. Fransen and T. Hertog, Phys. Rev. D 103 (2021) 103531 [2011.13933] for the observable CMB tilts, specified to our case, are given by

$$n_s = 1 - \frac{2}{N} - \frac{8\beta N}{9} - \frac{\beta N^2}{2} \tag{33}$$

and

$$r = \frac{12}{N^2} - \frac{16}{3}\beta - 2\beta N \ , \tag{34}$$

for scalar and tensor perturbations, respectively.

- According to the CMB observation data [15], we have

$$n_s = 0.9649 \pm 0.0042, \quad r < 0.036 \quad , \tag{35}$$

for the tilt n_s of scalar (curvature) perturbations and the tensor-to-scalar ratio r.

- For example,
(i) to get $n_s = 0.9691$ with $N = 65$, we need $\beta = 4.608 \cdot 10^{-8}$,
(ii) to get $n_s = 0.9607$ with $N = 55$, we need $\beta = 1.857 \cdot 10^{-6}$, and
(iii) to get $n_s = 0.9607$ with $N = 65$, we need $\beta = 3.9 \cdot 10^{-6}$.
- Therefore, in order to be consistent with the observed value of the spectral index n_s for all $55 < N < 65$, we should demand

$$\beta \leqslant 3.9 \cdot 10^{-6} \quad . \tag{36}$$

- The tensor-to-scalar-ratio r is under the upper bound of Eq. (35) for these values of β.

[15] Planck collaboration, Astron. Astrophys.641 (2020) A10 [1807.06211]; BICEP, Keck collaboration, Phys.Rev. Lett. 127 (2021) 151301 [2110.00483]; M. Tristram et al., Phys. Rev. D 105 (2022) 083524 [2112.07961]

We summarize our findings in Figures 1 and 2.

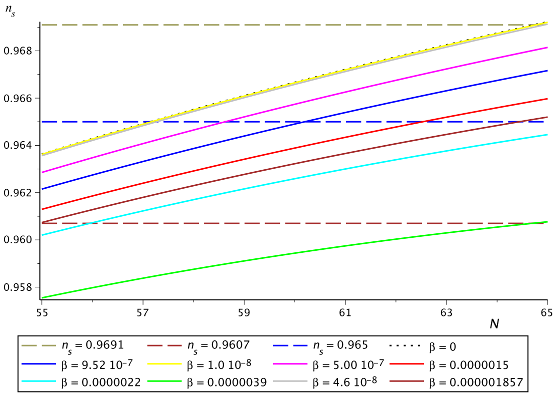

Figure: The spectral index n_s for $0 \leqslant \beta \leqslant 3.9 \cdot 10^{-6}$ with the e-foldings $55 \leqslant N \leqslant 65$. The dotted lines are the boundaries for the observed value of n_s set by the CMB data.

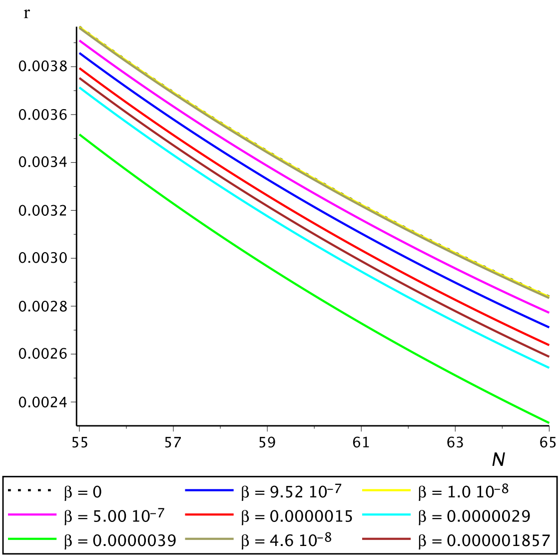

Figure: The tensor-to-scalar ratio r for $0 \leqslant \beta \leqslant 3.9 \cdot 10^{-6}$ with the e-folding number $55 \leqslant N \leqslant 65$.

In order to calculate the β-correction to the observable (CMB) amplitude A_s of scalar perturbations, we take the slow-roll parameter ϵ in terms of the function $H^2(N)$,

$$\epsilon = \frac{1}{2}\frac{d\ln H^2}{dN} = \frac{1}{2N} + \frac{\beta N}{3}\left(\frac{N}{8}+\frac{1}{3}\right), \tag{37}$$

$$A_s = \frac{(1+\zeta/9)h^2}{16\,\pi^2\,\epsilon}\frac{m^2}{M_{Pl}^2} = \left(\frac{N^2}{24\,\pi^2} + \frac{N^5\beta}{864\,\pi^2}\right)\frac{m^2}{M_{\mathrm{Pl}}^2} \quad , \tag{38}$$

where we have introduced the new parameter ζ as

$$\zeta = -\frac{9\,(4\,\epsilon + n_s - 1)}{8\,\epsilon} \approx \frac{\beta\,N^2\,(3\,N+4)}{4} \quad . \tag{39}$$

The first term of A_s is standard and has the value $\bar{A}_s = 2.1\cdot 10^{-9}$ for the best fit $N = 55$.

Substituting $m = 1.3(\frac{55}{N})10^{-5}M_{\mathrm{Pl}}$ into A_s gives the β-correction and

$$A_s \approx 2.1\cdot 10^{-9} + 5.5\cdot 10^{-11}N^3\beta \quad . \tag{40}$$

For instance, when $N = 65$ and $\beta = 10^{-6}$, we get the β-correction of the order $\mathcal{O}(10^{-3})\bar{A}_s$.

• We studied physical applications of the Bel-Robinson tensor $T^{\mu\nu\lambda\rho}$ squared term, proposed as the leading quantum correction inspired by superstring theory, to the inflationary stage of the early universe evolution. The proposed gravitational EFT action includes squares of two topological densities $E_4 = \mathcal{G}$ and P_4. The P_4^2 term does not contribute to the evolution equations in a spatially flat FLRW universe, so that the action reduces to the particular case of the $F(R, \mathcal{G})$ modified gravity on the FLRW background.

• Since we extended the Starobinsky inflation model by the new parameter β, the predictions for CMB observables are modified. We obtained the leading corrections in the first order with respect to β and the physical bounds of the parameter β.

• The next generation of CMB experiments e.g., the satellite missions LiteBIRD and CORE, as well as the ground-based experiments POLARBEAR, BICEP/Keck and Simons Observatory, will measure the values of the cosmological tilts and the CMB amplitude with higher precision, which may also probe quantum corrections to the Starobinsky inflation.

Thank you for attention