Spins of black holes SgrA* and M87* revealed from the size of dark spots at their EHT images

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MEPhI - ICPPA - 2022

Image of the object which is not a black hole ALMA: Star Betelgeuse arXiv:1706.06021


## SgrA* Astrophysical Case 1: Stationary background

 Classical black hole shadow (purple) $\stackrel{\text { def }}{=}$ capture photon cross-section by stationary luminous background, $\mathrm{GM}_{\mathrm{h}} / \mathrm{c}^{2}=1$, $\mathrm{a}=0, \mathrm{r}_{\mathrm{h}}=2$

Shadow radius $\mathrm{r}_{\text {sh }}=3 \sqrt{3} \simeq 5.2$
Blue disk - Euclidian image of the event horizon
Multicolored 3D-curve - photon trajectory with the return point $r_{\text {min }}=r_{p h}=3-$ circular photon orbit
Impact parameters: $\lambda=0-$ horizontal, $q=3 \sqrt{3},-$ vertical

SgrA*: Classical shadow (purple region) of the Kerr black hole: $\mathrm{a}=1, \mathrm{r}_{\mathrm{h}}=1$, $\theta_{0}=\pi / 2-$ polar angle of a distant observer


Photon trajectory (multicolored 3D-curve) at the shadow outline with the return point $\mathrm{r}_{\text {min }}=\mathrm{r}_{\mathrm{ph}}=1+\sqrt{2}$ (spherical photon orbit)
Impact parameters: $\lambda=0$ and $\mathrm{q}=\sqrt{11+8 \sqrt{2}} \simeq 4.72$

SgrA* Astrophysical Case 1: Compact star on the equatorial circular orbit with radius $r_{s}=20 \mathrm{M}_{\mathrm{h}} \mathrm{G} / \mathrm{c}^{2}$ around $\mathrm{SgrA}^{*}$, observed by a developed Cosmic Submillimeter Observatory (Millimetron) Radiation outside the photon spheres $\mathrm{r}_{\mathrm{ph}}$


For animation see YouTube: https://youtu.be/P6DneV0vk7U

## Astrophysical Case 2: GRMHD accretion simulations!!!

 Accretion disk is very luminous at the vicinity of event horizon! Radiation from both the outside and inside photon spheres $\mathrm{r}_{\mathrm{ph}}=$ constThe Blandford-Znajek process (quite different from the $\alpha$-disk!) is a suitable model for the General Relativistic Magnetohydrodynamics (GRMHD) accretion onto black holes, in which the inflowing plasma is strongly heated even in the vicinity of the event horizon by the radial electric current:

J.C.McKinney, A.Tchekhovskoy, R.D.Blandford (2012)
$\operatorname{SgrA}^{*}, \mathrm{a}=0:$ silhouette of the event horizon globe (dark and light blue regions) projected inside the classical black hole shadow (purple closed curve)


SgrA*: The face profiles of the event horizon silhouettes
The lensed image of event horizon (related with photons emitted toward a distant observer inside the photon spheres) is always projected at the celestial sphere inside the awaited position of the classical black hole shadow

$a=0.9982$

$a=0$

Lensed image of the horizon is viewed at once from both sides

## SgrA* Astrophysical Case 2: Outgoing photon from $\mathrm{r}=1.01 \mathrm{r}_{\mathrm{h}}$

Radiation inside the photon spheres $\mathrm{r}_{\mathrm{ph}}$
$\operatorname{SgrA}^{*}, \mathrm{a}=0.9982, \theta_{0}=82.2^{\circ}$ : dark spot (black region) is recovered by emission of the nonstationary inner part of thin accretion disk adjoining the event horizon in the black hole equatorial plane. Photon trajectory with impact parameters $\lambda=-1.493$ and $\mathrm{q}=3.629$ :



Astrophysical Case 2: GRMHD accretion simulation Radiation from both the outside and inside photon spheres $\mathrm{r}_{\mathrm{ph}}$

$\mathrm{Fe} \mathrm{K} \alpha$ line at 6.4 keV
Armitage \& Reynolds 2003

Astrophysical Case 2: Line emission from accretion disk Radiation from both the outside and inside photon spheres $\mathrm{r}_{\mathrm{ph}}$

B.C.Bromley, K.Chen, W.A.Miller ApJ 47557 (1997)

SgrA*: outgoing photons from $\mathrm{r}=1.01 \mathrm{r}_{\mathrm{h}}, \mathrm{a}=1, \theta_{0}=90^{\circ}$


Dark spot in the case of $\operatorname{SgrA}^{*}\left(\theta_{0}=82.2^{\circ}\right)$ projected inside the awaited position of (invisible) classical black hole shadow at the celestial sphere


$$
a=1
$$

$$
\mathrm{a}=0.65
$$


$\mathrm{a}=0$

Dark spot in the case of $\operatorname{SgrA}$ * is a lensed image of the northern hemisphere of the event horizon globe

SgrA*, $\mathrm{a}=0.9982$ : gravitational lensing of the compact emitting source falling into black hole
Distant observer is placed at $\cos \theta=0.1, \theta_{0}=82.2^{\circ}$


VD, Natalia Nazarova https://youtu.be/fps-3frL0AM

## Superposition:

The modeled dark spot and the EHT image of SgrA*


$$
a=0.9982
$$

$$
a=0.65
$$

## 3 D picture of $\mathrm{M} 87^{*}, \theta_{0}=17^{\circ}, \mathrm{a}=1$ :

Thin accretion disk and silhouette of the southern hemisphere of event horizon (internal part of the gray closed curve), which is projected inside the awaited position of (invisible) classical black hole shadow at the celestial sphere (purple closed curve)


Dark spot in the case of $\operatorname{M87} 7^{*}\left(\theta_{0}=17^{\circ}\right)$ projected inside an outline of the classical black hole shadow


$$
a=1
$$


$a=0.75$

$a=0$

Dark spot in the case of M87* is a lensed image of the southern hemisphere of the event horizon globe

## Superposition:

The modeled dark spot and the EHT image of M87*


$$
\mathrm{a}=0.9982
$$



$$
a=0.75
$$

Dark spot size $r_{\text {eh }}(a)$ in dependence of spin $a$ Green regions mark the range of possible spin parameters


SgrA*: $065 \leq \mathrm{a} \leq 0.9$


M87*: $0.7 \leq \mathrm{a} \leq 1$

## Results and Conclusions

We reconstruct the dark spots at the EHT images of supermassive black holes SgrA* and M87* by using the model of geometrically thin accretion disk, which highlights black hole in the vicinity of event horizon.
The existence of hot accreting matter in the vicinity of black hole event horizon is predicted by the Blandford-Znajek mechanism and confirmed by recent General Relativistic MHD numerical supercomputer simulations.
In the used model of thin accretion disk the dark spots at images of SgrA* and M87* are the lensed images of their event horizon globes. In the case of SgrA * the dark spot is a lensed image of the northern hemisphere of the event horizon. In the case of M87* the dark spot is a lensed image of the southern hemisphere of the event horizon
The lensed images of event horizons projected at the celestial sphere inside the awaited positions of the classical black hole shadows, which are invisible in both cases of M87* and SgrA*.
We use the sizes of dark spots at the images of SgrA* and M87* for inferring their spins, $0.65<\mathrm{a}<0.9$ and $\mathrm{a}>0.75$, accordingly.

## Publication list of author

- VD / Physical origin of the dark spot at the image of supermassive black hole SgrA* revealed by the EHT collaboration // Astronomy 2022, 1(2), 93-98
- VD, Nazarova, N.O. / Modeling the motion of a bright spot in jets from black holes M87* and SgrA* // Gen. Relativ. Gravit. 53, 83 (2021)
- VD, N.O. Nazarova / Silhouettes of invisible black holes // Physics-Uspekhi 63 (6) 583-600 (2020)
- VD and N. O. Nazarova / Visible shapes of black holes M87* and SgrA* // Universe 2020, 6(9), 154.
- VD, N.O. Nazarova / Event horizon image within black hole shadow // JETP. 128, 578-585 (2019)
- VD and N. O. Nazarova / Brightest point in accretion disk and black hole spin: Implication to the image of black hole M87* // Universe 2019, 5(8), 183
- VD, N. O. Nazarova and V. P. Smirnov / Event horizon silhouette: implications to supermassive black holes M87* and SgrA* // Gen. Relativ. Gravit. (2019) 51: 81
- VD / To see invisible: image of the event horizon within the black hole shadow // IJMPD 28, No. 13 (2019) 1941005


## Astrophysical Case 1: Classical black hole shadow

Contour (boundary) of the black hole shadow on the bright background
Radiation outside the photon spheres $r_{p h}$
Distant observer is in the black hole equatorial plane

$a=1$

$a=0.65$

Parametric equation for black hole shadow: $(\lambda, Q)=(\lambda(r), Q(r))$ :

$$
\lambda=\frac{(3-r) r^{2}-a^{2}(r+1)}{a(r-1)}, \quad q^{2}=\frac{r^{3}\left[4 a^{2}-r(r-3)^{2}\right]}{a^{2}(r-1)^{2}}
$$

Bardeen 1973, Chandrasekhar 1983
$\lambda$ - horizontal and $q=\sqrt{Q}-$ vertical photon impact parameters,
Q - Carter constant, arrow - black hole rotation axis
dashed circle - black hole event horizon $r_{h}=\left(1+\sqrt{1-a^{2}}\right)$

Two parameters for null geodesics: $\lambda=\Phi / \mathrm{E}, \quad \mathrm{q}=\mathrm{Q}^{1 / 2} / \mathrm{E}$
Two impact parameters on the celestial sphere with the distant observer at $\theta_{0}$ :

$$
\begin{aligned}
\alpha & =-\frac{\lambda}{\sin \theta_{0}} \\
\beta & =q+a^{2} \cos ^{2} \theta_{0}-\lambda^{2} \cot ^{2} \theta_{0}
\end{aligned}
$$



Equations of motion of test particles
The Hamilton-Jacobi equation for the Jacobi action S

$$
\frac{\partial \mathrm{S}}{\partial \lambda}=\frac{1}{2} \mathrm{~g}^{\mathrm{ij}}\left[\frac{\partial \mathrm{~S}}{\partial \mathrm{x}^{\mathrm{i}}}-\epsilon \mathrm{A}_{\mathrm{i}}\right]\left[\frac{\partial \mathrm{S}}{\partial \mathrm{x}^{\mathrm{j}}}-\epsilon \mathrm{A}_{\mathrm{j}}\right]
$$

If there is a separable solution:

$$
\begin{gathered}
\mathrm{S}=-\frac{1}{2} \mu^{2} \lambda-\mathrm{Eu}+\Phi \varphi+\mathrm{S}_{\theta}+\mathrm{S}_{\mathrm{r}} \\
\mathrm{p}_{\theta}=\frac{\partial \mathrm{S}}{\partial \theta}, \quad \mathrm{p}_{\mathrm{r}}=\frac{\partial \mathrm{S}}{\partial \mathrm{r}}
\end{gathered}
$$

$\mathrm{p}_{\theta}^{2}+\left(\mathrm{aE} \sin \theta+\frac{\Phi}{\sin \theta}\right)^{2}+\mathrm{a}^{2} \mu^{2} \cos ^{2} \theta=$
$=\Delta \mathrm{p}_{\mathrm{r}}^{2}-2\left[\left(\mathrm{r}^{2}+\mathrm{a}^{2}\right) \mathrm{E}-\mathrm{a} \Phi+\epsilon \mathrm{er}\right] \mathrm{p}_{\mathrm{r}}+\mu^{2} \mathrm{r}^{2} \quad \Rightarrow \quad=\mathcal{K}=\mathrm{const}$

$$
\mathrm{p}_{\theta}=\frac{\mathrm{dS}}{\mathrm{~d} \theta}=\sqrt{\mathrm{V}_{\theta}}, \quad \mathrm{p}_{\mathrm{r}}=\frac{\mathrm{dS}}{\mathrm{~d} \theta}=\frac{1}{\Delta} \sqrt{\mathrm{~V}_{\mathrm{r}}}, \quad \Delta=\mathrm{r}^{2}-2 \mathrm{r}+\mathrm{a}^{2}+\mathrm{e}^{2}
$$

Equations of motion for test particles in the integral form B. Carter 1968

$$
\begin{gathered}
\mathrm{S}=\frac{1}{2} \mu^{2} \tau-\mathrm{Et}+\Phi \varphi+\int^{\theta} \sqrt{\mathrm{V}_{\theta}} \mathrm{d} \theta+\int^{\mathrm{r}} \frac{\sqrt{\mathrm{~V}_{\mathrm{r}}}}{\Delta} \mathrm{dr} \\
\mathrm{~V}_{\theta}=\mathrm{Q}+\mathrm{a}^{2}\left(\mathrm{E}^{2}-\mu^{2}\right) \cos ^{2} \theta-\Phi^{2} \cot ^{2} \theta, \quad \Delta=\mathrm{r}^{2}-2 \mathrm{r}+\mathrm{a}^{2}+\mathrm{e}^{2} \\
\mathrm{~V}_{\mathrm{r}}=\mathrm{r}\left[\mathrm{r}\left(\mathrm{r}^{2}+\mathrm{a}^{2}\right)+2 \mathrm{a}^{2}\right] \mathrm{E}^{2}-4 \operatorname{arE} \Phi-\left(\mathrm{r}^{2}-2 \mathrm{r}\right) \Phi^{2}-\Delta\left(\mathrm{r}^{2} \mu^{2}+\mathrm{Q}\right) \\
f^{\mathrm{r}} \frac{\mathrm{dr}}{\sqrt{\mathrm{~V}_{\mathrm{r}}}}=f^{\theta} \frac{\mathrm{d} \theta}{\sqrt{\mathrm{~V}_{\theta}}}, \quad \tau=f^{\theta} \frac{\mathrm{a}^{2} \cos ^{2}}{\sqrt{\mathrm{~V}_{\theta}}} \mathrm{d} \theta+f^{\mathrm{r}} \frac{\mathrm{r}^{2}}{\sqrt{\mathrm{~V}_{\mathrm{r}}}} \mathrm{dr} \\
\mathrm{t}=f^{\theta} \frac{\mathrm{a}^{2} \mathrm{E}^{2} \cos ^{2} \theta}{\sqrt{\mathrm{~V}_{\theta}}} \mathrm{d} \theta+f^{\mathrm{r}} \frac{\mathrm{r}^{2}\left(\mathrm{r}^{2}+\mathrm{a}^{2}\right) \mathrm{E}+2 \mathrm{ar}(\mathrm{aE}-\Phi)}{\Delta \sqrt{\mathrm{V}_{\mathrm{r}}}} \mathrm{dr} \\
\varphi=f^{\theta} \frac{\Phi \cot ^{2} \theta}{\sqrt{\mathrm{~V}_{\theta}}} \mathrm{d} \theta+f^{\mathrm{r}} \frac{\mathrm{r}^{2} \Phi+2 \operatorname{ar}(\mathrm{aE}-\Phi)}{\Delta \sqrt{\mathrm{V}_{\mathrm{r}}}} \mathrm{dr} \\
\text { Andrew Strominger arXiv:1710.11112}
\end{gathered}
$$

Path integral equations of motion

$$
f^{\theta} \frac{\mathrm{d} \theta}{\sqrt{\mathrm{~V}_{\theta}}}=f^{\mathrm{r}} \frac{\mathrm{dr}}{\sqrt{\mathrm{~V}_{\mathrm{r}}}}, \quad \mathrm{~V}_{\theta}\left(\theta_{\min }\right)=0, \quad \mathrm{~V}_{\mathrm{r}}\left(\mathrm{r}_{\min }\right)=0
$$

The integrals are understood to be path integrals along the trajectory Integral equation with respect to $\lambda=\Phi / \mathrm{E}$ and $\mathrm{q}=\mathrm{Q}^{1 / 2} / \mathrm{E}$ for the trajectories of the first light echo:

$$
\sum_{\mathrm{N}} \int_{\theta_{\mathrm{N} 1}}^{\theta_{\mathrm{N} 2}} \frac{\mathrm{~d} \theta}{\sqrt{\mathrm{~V}_{\theta}}}=\sum_{\mathrm{N}} \int_{\mathrm{r}_{\mathrm{N} 1}}^{\mathrm{r}_{\mathrm{N} 2}} \frac{\mathrm{dr}}{\sqrt{\mathrm{~V}_{\mathrm{r}}}}
$$



2D photon trajectory $r(\theta)$


Solutions ( $\lambda, \mathrm{q}$ )

Path integral equations of motion

$$
f^{\theta} \frac{\mathrm{d} \theta}{\sqrt{\mathrm{~V}_{\theta}}}=f^{\mathrm{r}} \frac{\mathrm{dr}}{\sqrt{\mathrm{~V}_{\mathrm{r}}}}, \quad \mathrm{~V}_{\theta}\left(\theta_{\min }\right)=0, \quad \mathrm{~V}_{\mathrm{r}}\left(\mathrm{r}_{\min }\right)=0
$$

The integrals are understood to be path integrals along the trajectory Integral equation with respect to $\lambda=\Phi / \mathrm{E}$ and $\mathrm{q}=\mathrm{Q}^{1 / 2} / \mathrm{E}$ for the trajectories of the first light echo:
$\int_{\theta_{\mathrm{s}}}^{\theta_{\max }} \frac{\mathrm{d} \theta}{\sqrt{\mathrm{V}_{\theta}}}+\int_{\theta_{\min }}^{\theta_{\max }} \frac{\mathrm{d} \theta}{\sqrt{\mathrm{V}_{\theta}}}+\int_{\theta_{0}}^{\theta_{\min }} \frac{\mathrm{d} \theta}{\sqrt{\mathrm{V}_{\theta}}}=\int_{\mathrm{r}_{\mathrm{s}}}^{\mathrm{r}_{\text {min }}} \frac{\mathrm{dr}}{\sqrt{\mathrm{V}_{\mathrm{r}}}}+\int_{\mathrm{r}_{\text {min }}}^{\mathrm{r}_{0}} \frac{\mathrm{dr}}{\sqrt{\mathrm{V}_{\mathrm{r}}}}$


2D photon trajectory $\mathrm{r}(\theta)$


Solutions $(\lambda, \underline{q})$

3D photon trajectories
Prime image: no intersections of equatorial plane First light echo: 1 intersection of equatorial plane


3D photon trajectory
Second light echo: 2 intersections of equatorial plane

$$
\lambda=-1.78, \mathrm{q}=5.2, \mathrm{r}_{\mathrm{h}}=1, \mathrm{r}_{\mathrm{s}}=20, \mathrm{r}_{\mathrm{min}}=3.11
$$



