Noether symmetrical gravity Lagrangian as a fourth order polynomial in Ricci and Gauss-Bonnet scalars with no need to introduce neither a Cosmological Constant nor a Dark Matter

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There are reasons to **extend General Relativity** based on the metrics $g_{\mu\nu}$ of space-time with the Riemann curvature tensor $R_{\alpha\beta\mu\nu}$,

and characterized by the Ricci scalar ($R = R_{\mu\nu}{}^{\mu\nu}$) , and the Gauss-Bonnet topological invariant ($G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$).

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,R \Rightarrow \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,F(R,G)\ ,\ \kappa = 8\pi G_N\ ,$$ and function $F(R,G)$ **to be specified.**

There are some conditions and symmetries to restrict this uncertainty in $F(R,G)$:

1. **For Noether symmetry vector $X = \partial_t \quad \Rightarrow \quad f(R,G) = Rf_1(G) + f_2(G)$ - time translation invariance: $\mathcal{H} = \mathcal{E} = const$.**
2. **Linear in G term does not contribute because it is topological invariant.**
3. **Newton (weak field) limit: $f(R,G) = R + R\phi_1(G) + \phi_2(G)$.**
4. **In Minkovski space-time $R = 0, G = 0$** , and $\phi_{1,2}(G)$ may be presented by **polynomial in G.**
5. **Note: In the fundamental theory of the standard particle physics the Lagrangian is polynomial in the fundamental fields.**

Noether Symmetrical Polynomial (NSP) Lagrangian is suggested to be $\mathcal{L}_{NSP} = \frac{1}{2\,\kappa}\left[-2\Lambda + R\left(1 + \frac{G}{G_P}\right) - \frac{G^2}{G_P H_I{}^4}\right]$.

Here G_P and H_I – gravitational constants additional to G_N, which are concerned with the post inflation and inflation dynamics.

At $G_P \to \infty$ (ΛCDM model), **and in Newton limit** $V = -\frac{1}{6}\Lambda\, r^2 - \frac{G_N M}{r} + \frac{1}{G_P}(\dots)$. May be, together with other problems $\Lambda = 0$.

In FRW metrics Friedmann equations in dimensionless variables $\tau = H_0 t,\ h(\tau) = H(\tau/H_0)/H_0,\ g_P = G_P/H_0{}^4,\ \ h_I = H_I/H_0$
can be solved with "initial conditions" $h_0 = 1, \dot{h}_0 = -(1+q_0)h_0{}^2$ **, where q_0 defines the accelerated expansion instead of Λ :**

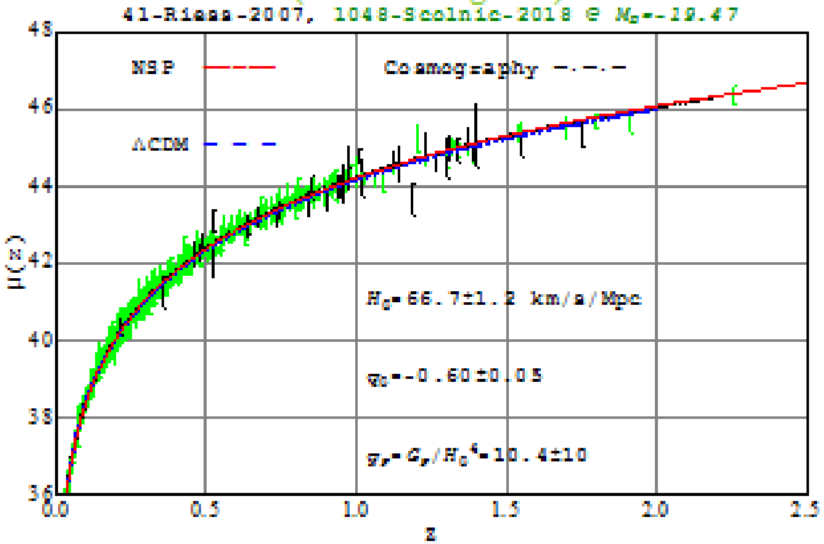
$$3h^2 = \frac{\kappa^2\rho_m}{H_0{}^2} + 72\frac{h^6}{g_P}\left[1 - \frac{h^2}{h_I^2} - 6\frac{\dot{h}}{h^2}\left(1 - \frac{h^2}{h_I^2}\right) - \frac{\dot{h}^2}{h^4}\left(1 - 3\frac{h^2}{h_I^2}\right) - 2\frac{\ddot{h}}{h^3}\left(1 - \frac{h^2}{h_I^2}\right)\right]$$ **- energy (0,0) equation.**

$$2\dot{h} = -\frac{\kappa^2 p_m}{H_0{}^2} + 144\frac{h^6}{g_P}\left[\frac{\dot{h}}{h^2}\left(1 - \frac{4}{3}\frac{h^2}{h_I^2}\right) - 4\frac{\dot{h}^2}{h^4}\left(1 - \frac{3}{2}\frac{h^2}{h_I^2}\right) - \frac{1}{3}\frac{\dot{h}^3}{h^6}\left(1 - 6\frac{h^2}{h_I^2}\right) - \frac{4}{3}\frac{\dot{h}}{h^2}\frac{\ddot{h}}{h^3}\left(1 - 2\frac{h^2}{h_I^2}\right) - \left(\frac{\ddot{h}}{h^3} + \frac{1}{3}\frac{\dddot{h}}{h^4}\right)\left(1 - \frac{h^2}{h_I^2}\right)\right]$$ **- pressure equation.**

Best fit to SN Ia data: $\chi^2(H_0, q_0, G_P, H_I \to \infty) = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\mu(z_i, H_0, q_0, G_P, H_I) - \mu_i^{exp}}{\sigma_i^{exp}}\right]^2$

Riess-2007 (41 data - black) or

Scolnic-2018 (1048 data - green)



Best fit to the Riess-2007 data

$\chi^2(H_0, q_0, G_P, H_I) = 1.15$ at

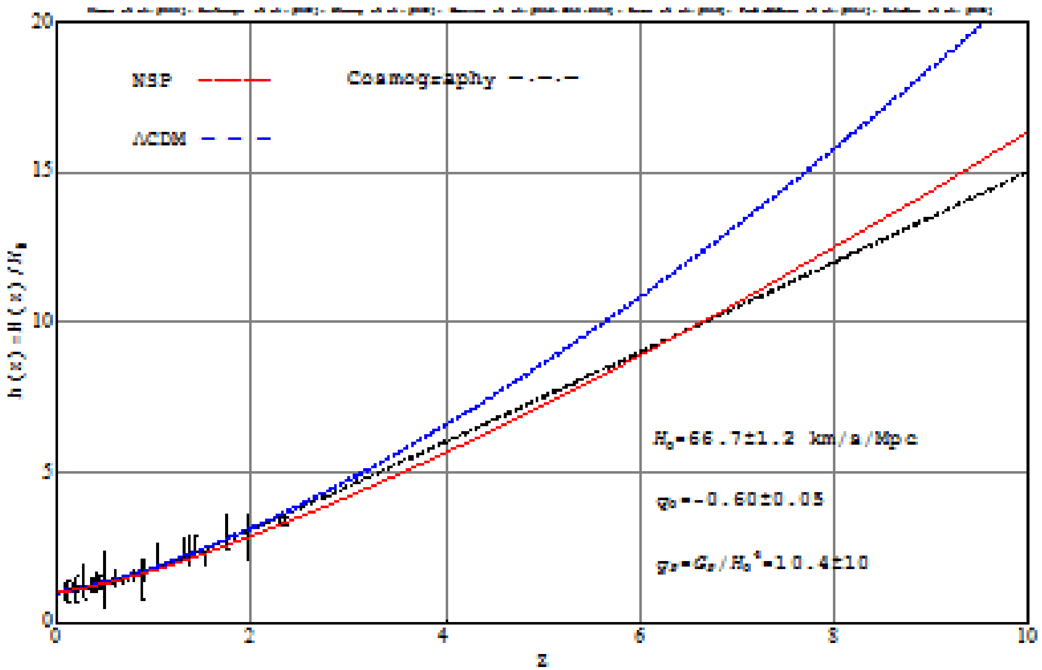
$H_0 = 66.7 \pm 1.2 \frac{km}{s\,Mpc}$

$q_0 = -0.60 \pm 0.05$

$G_P = (10.4 \pm 10) H_0{}^4$ (It may be changed by solar system and some astrophysical data)

$H_I \sim (10^{36} - 10^{42}) s^{-1}$. ($H_{Plank} = 1.8\ 10^{43} s^{-1}$)

Hubble parameter data with predictions of NSP, ΛCDM models, and Cosmographic Analysis: $a(\tau) = a_0\left(1 + \tau - \frac{1}{2!}q_0\tau^2 + \frac{1}{3!}j_0\tau^3 + \frac{1}{4!}s_0\tau^4\right)$ for the best fit to the Riess-2007 data:

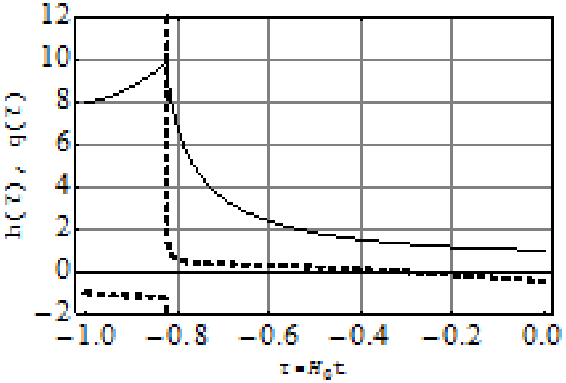


$\chi^2 = 0.61$ for NSP model

$\chi^2 = 0.96$ for ΛCDM model

$\chi^2 = 0.65$ for Cosmographic Analysis

Nonsingular inflation, and post inflation stages with accelerated expansion at the late time:



From these Friedmann equations $h(\tau) < h_I$

$$3h^2 = \frac{\kappa^2 \rho_m}{H_0{}^2} + 72\frac{h^6}{g_P}\left[1 - \frac{h^2}{h_I^2} - 6\frac{\dot h}{h^2}\left(1 - \frac{h^2}{h_I^2}\right) - \frac{\dot h^2}{h^4}\left(1 - 3\frac{h^2}{h_I^2}\right) - 2\frac{\ddot h}{h^3}\left(1 - \frac{h^2}{h_I^2}\right)\right]$$ - energy equation

At the initial (De Sitter) inflation stage:

$$h(\tau) = const \Rightarrow \{\dot h, \ddot h\} \to 0 \Rightarrow 3h^2 = 72\frac{h^6}{g_P}\left[1 - \frac{h^2}{h_I^2}\right] \Rightarrow h_{Initial} \to h_I - \frac{g_P}{48\, h_I^3} \text{ at } h_I \gg 1.$$

As well, these Friedmann equations are defined the inflation end: $h(\tau) = h_I^2$.

← Illustration at $h_I = 10$

Conclusions:

1. Noether Symmetrical Polynomial (NSP) Lagrangian: $\mathcal{L} = \frac{1}{2\,\kappa}\left[R\left(1 + \frac{G}{G_P}\right) - \frac{G^2}{4 G_P H_I{}^4}\right]$ (4^{th} order in $R_{\alpha\beta\mu\nu}$).

2. Noether symmetry ⇒ Point-like Geometric Lagrangian, Hamiltonian, effective kinetic, potential energies.

3. The 4^{th} order Friedmann equation leads q_0 to be the initial condition instead of to be defined by the Λ and Dark Matter.

4. Energy conservation: $E = 0$ in respect with the time translation invariance.

5. Best fit ($\chi^2 = 1.15$) to SN Ia cosmological data with $\Lambda = 0$, and $\rho_{Dark\ Matter} = 0$.

6. "Initial conditions" for the Universe: $H_0 = 66.7 \pm 1.2 \frac{km}{s}/Mpc$, $q_0 = -0.60 \pm 0.05$.

7. Rough estimation of additional gravitational constant: $G_P = (10.4 \pm 10) H_0^4$.

8. Solar system, and some astrophysical data may be taken into account to define G_P more exactly.

9. There is a solution for inflation without singularity in Hubble parameter.

Thank you for attention