Formation of primordial black hole clusters in early dark matter halos

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Introduction

Possible signatures of PBHs:

- Dark matter (Carr and Kuhnel, 2021): $M_{PBH} = 10^{17} \div 10^{23}\, g$
- Supermassive black holes at $z > 7$ (Banados et al., 2018)
- Early structures formation (Carr and Silk, 2018): JWST observation (Hütsi et al., 2022; Liu and Bromm, 2022)
- Gravitational waves: PBHs binaries forming in the early Universe $\rightarrow f_{PBH} \lesssim 10^{-3}$ (Sasaki et al., 2016)

This work:

- PBH clusters can be formed in early DM halos
- dynamical processes in such clusters suppresses the merger rate of early binaries $\rightarrow f_{PBH} \lesssim 0.1$ for $M_{PBH} = 10\, M_{\odot}$

The Poisson noise

Matter power spectrum:

$$P = T_{ad}^2(k)P_{ad}(k) + T_{iso}^2 P_{PBH},$$

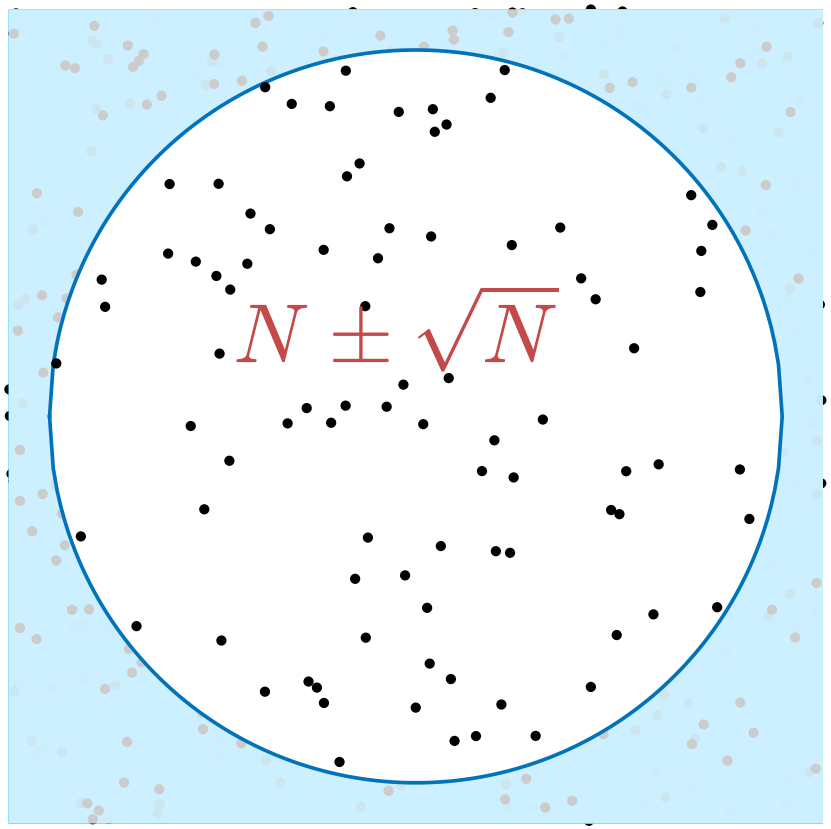
PBHs power spectrum:

$$P_{PBH} = T_{iso}^2 P_{PBH} = \frac{9}{4}(1+z_{eq})^2 \frac{f_{PBH} M_{PBH}}{\Omega_{DM}\rho_c}$$
$$\approx 8.3 \times 10^{-3} f_{PBH} \left(\frac{M_{PBH}}{10\, M_\odot}\right) \mathrm{Mpc}^3,$$

The variance of fluctuations:

$$\sigma^2(M_h) = \frac{D^2(z)}{2\pi^2} \int dk\, k^2 P(k) W^2(k, M_h)$$
$$\approx f_{PBH} \frac{17 M_{PBH}}{4 M_h} \left(\frac{1+z_{eq}}{1+z}\right)^2$$

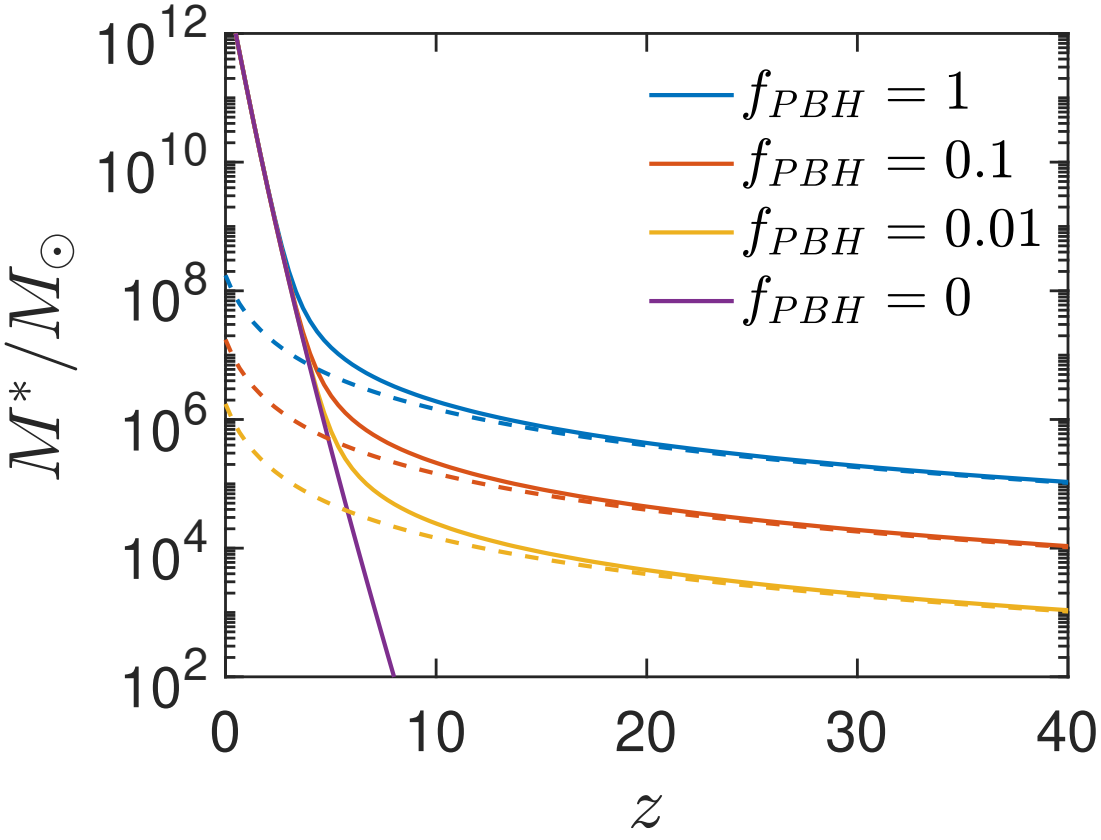
Initial distribution of PBHs



PBHs generate fluctuations
$\delta \sim 1/\sqrt{N}$

The characteristic halo mass

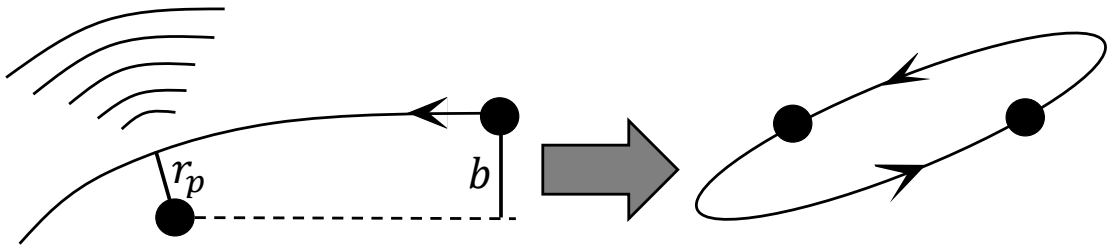
defined as $\sigma(M^*) = \delta_c \approx 1.69$:



dotted lines:

$$M^*(z) = f_{PBH} \frac{17 M_{PBH}}{4\delta_c^2} \left(\frac{1 + z_{eq}}{1 + z} \right)^2 \tag{1}$$

Mergers of PBHs



The cross-section of binary black hole formation (Mouri and Taniguchi, 2002):

$$\sigma_{BBH} = 2\pi \left(\frac{85\pi}{6\sqrt{2}}\right)^{2/7} \frac{G^2(m+m')^{10/7} m^{2/7} m'^{2/7}}{c^{10/7} v_{rel}^{18/7}}, \qquad (2)$$

Merger rate of late binaries:

$$\mathcal{R}_h = \frac{4\pi}{m^2} \int dr\, r^2 \rho^2 \sigma_{BBH} v_{\rm rel}$$
$$\sim 4 \times 10^{-13} f_{PBH}^2 \left(\frac{M_c}{10^4\, M_\odot}\right)^{17/14} \left(\frac{r_c}{10\,\mathrm{pc}}\right)^{-31/14} \mathrm{yr}^{-1}, \qquad (3)$$

Evolution of cluster parameters

simple model for single mass cluster (Spitzer, 1987):

$$r_c(t) = r_c(0)\left(1 - \frac{t}{t_{cc}}\right)^{0.53}, \tag{4}$$

$$M_c(t) = M_c(0)\left(1 - \frac{t}{t_{cc}}\right)^{0.42}, \tag{5}$$

$$t_{cc} \sim 3\left(\frac{r_c(0)}{10\,\mathrm{pc}}\right)^{3/2}\left(\frac{M_c(0)}{10^4\,M_\odot}\right)^{1/2}\left(\frac{10\,M_\odot}{m_{PBH}}\right)\ \mathrm{Gyr}, \tag{6}$$

after collapse: selfsimilar expansion $r \propto t^{2/3} \rightarrow$ evolution of the merger rate:

$$\mathcal{R}(t) \propto \left(1 - \frac{t}{t_{cc}}\right)^{-0.66}, \quad t < t_{cc} \tag{7}$$

$$\mathcal{R}(t) \propto t^{-1.48}, \quad t > t_{cc} \tag{8}$$

Equations

The kinetic Fokker-Planck equation (Vasiliev, 2017):

$$\frac{\partial N}{\partial t} = -\frac{\partial}{\partial E}\big(N\langle \Delta E\rangle\big) + \frac{1}{2}\frac{\partial^2}{\partial E^2}\Big(N\big\langle(\Delta E)^2\big\rangle\Big), \tag{9}$$

$N = 4\pi^2 p(E) f(E,t)$
coefficient $\langle \Delta E\rangle$ also includes heating term due to binaries formation in three-body interaction (Cohn et al., 1989) $\rightarrow$ core collapse is terminated

Gravitational potential:

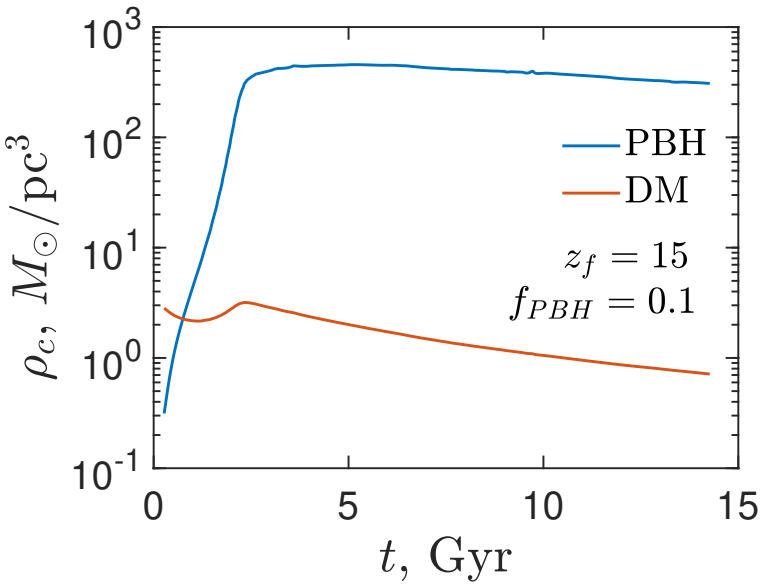
$$\phi(r) = -4\pi G\left[\frac{1}{r}\int_0^r dr' r'^2 \rho(r') + \int_r^\infty dr' r' \rho(r')\right], \tag{10}$$

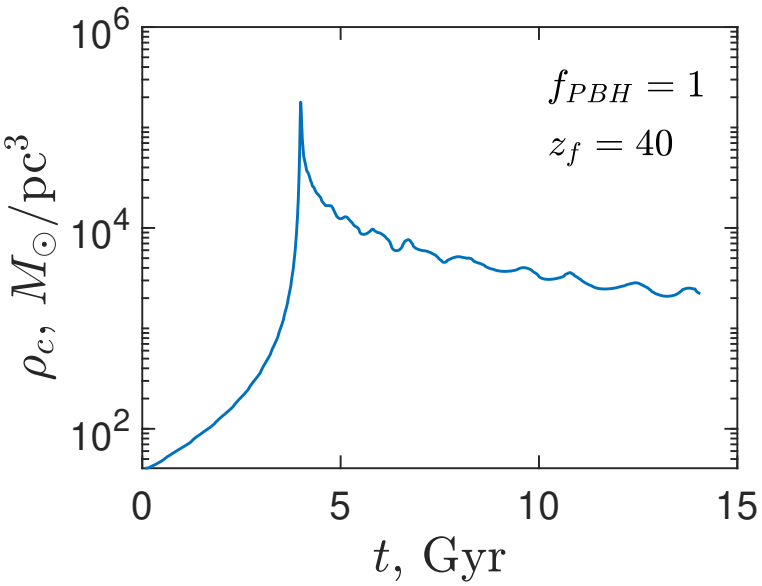
density profile:

$$\rho(r) = 4\pi m \int_{\phi(r)}^0 dE \sqrt{2(E-\phi(r))}\, f(E). \tag{11}$$

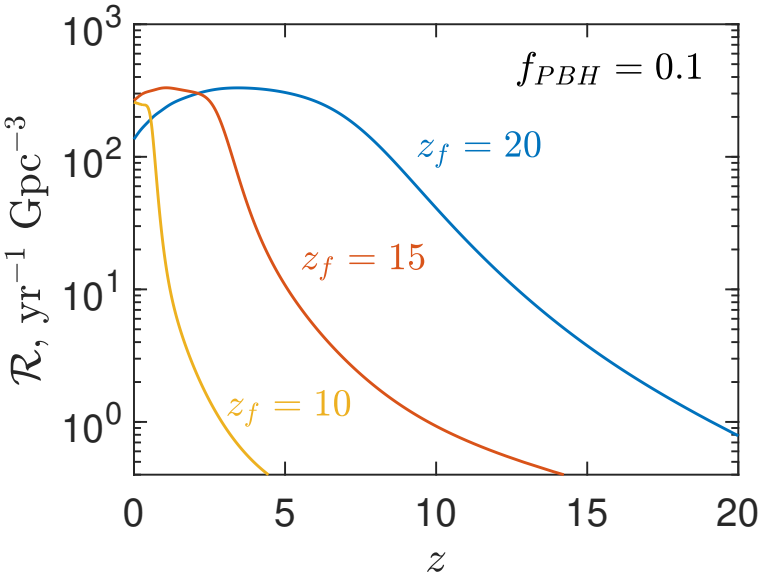
Central density

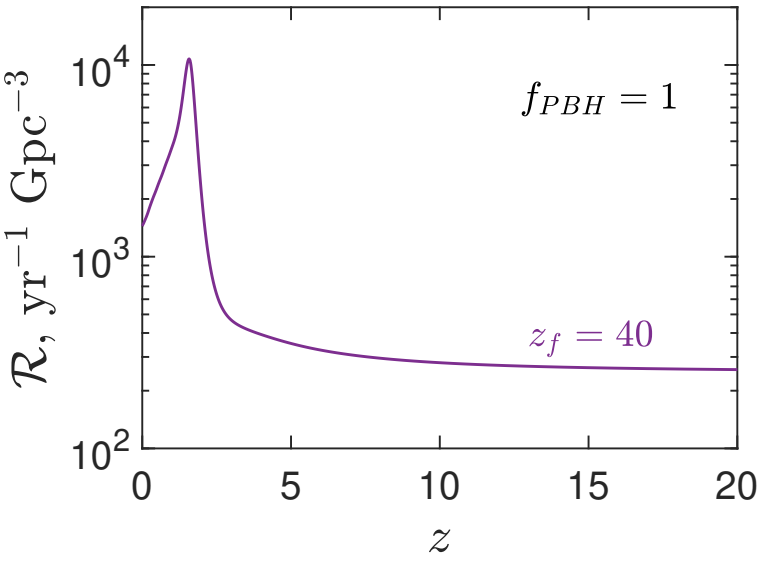
Initial density profile: $\rho \propto \left[1 + (r/r_0)^2\right]^{-3/2}$, $r_0 = R_{vir}/10$, $M_h = 10^5\, M_\odot$, $M_{PBH} = 10\, M_\odot$





Merger rate





The PBHs merger rate, provided that all the DM in the 1σ halo formed at different redshifts z_f

The destruction of dark halos are not taken into account during structure formation $\rightarrow$ the presented merger rate at low z is overestimated

Suppression factor for PBHs binaries forming in the early universe

The merger rate is suppressed due to the destruction of binaries in early halos: $\mathcal{R} = S \times \mathcal{R}_{np}$ (Vaskonen and Veermäe, 2019)

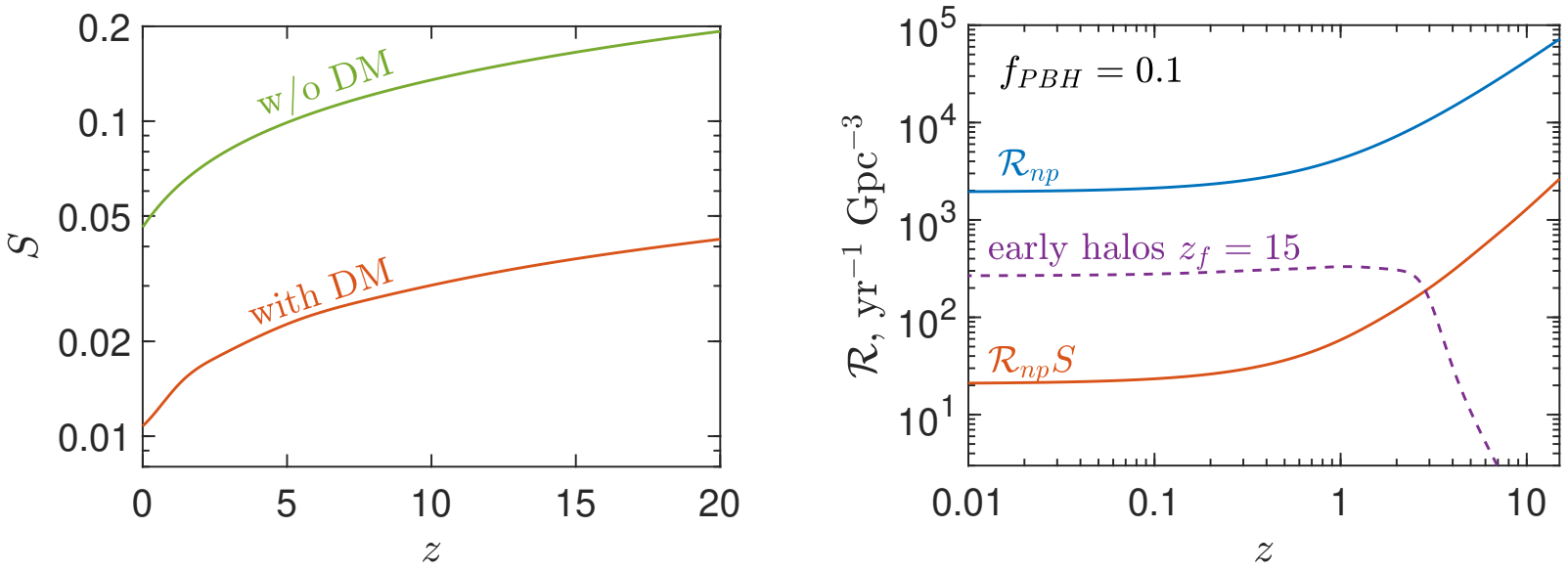
$$S(z) = 1 - \sum_{N=3}^{N_c(z)} \overline{p}_N(z_c) - \sum_{N'>N_c} \left(\sum_{N=3}^{N_c(z)} \tilde{p}_N(z_c) \right) \overline{p}_{N'}(z_c), \quad (12)$$

distribution of haloes containing N PBHs at redshift z_c:

$$p_N \propto N^{-1/2} e^{-N/N^*(z_c)}, \quad \sum_{N\geq 2} \overline{p}_N = 1, \quad \sum_{N=2}^{N'} \tilde{p}_N = 1, \quad (13)$$

simplified model (w/o DM): $t_{cc}(N_c) \approx 3.6 N_c^{7/4} / f_{PBH}^{5/2}$ kyr $< t(z)$
this work: $N_c = 2N^*(z_c)$

Merger rate of early binaries



left: suppression factor, the green line is in a simple model w/o DM particles (Vaskonen and Veermäe, 2019), the red line is this work.
right: the merger rate of early (solid) and late (dotted) PBHs binaries

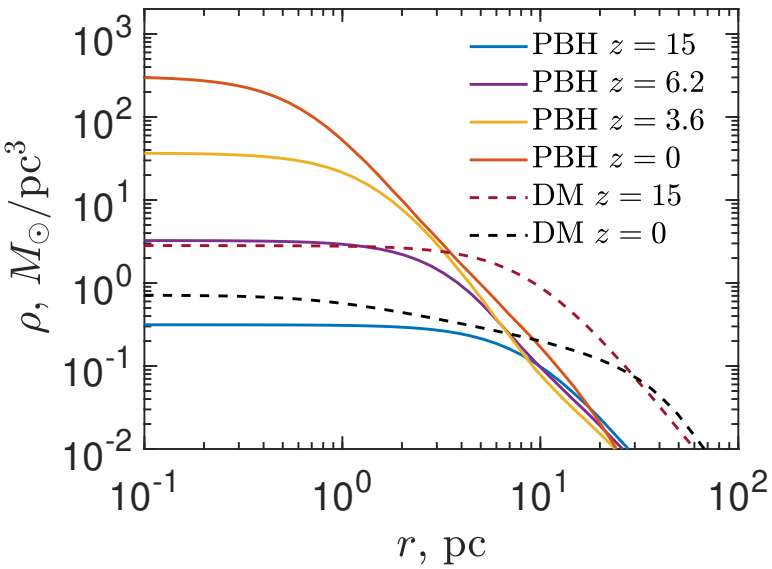
LIGO/Virgo (Abbott et al., 2010.14533): $\mathcal{R} \lesssim 40$ yr^{-1} Gpc^{-3}

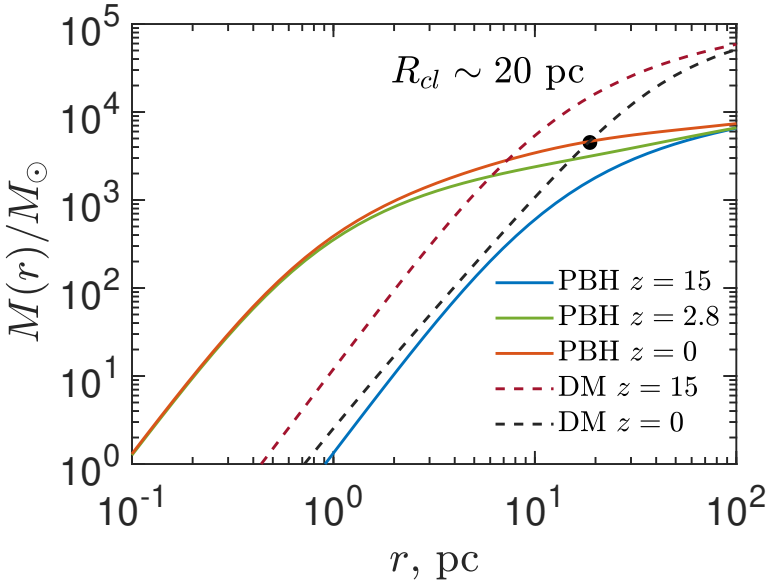
Only at high $z \gtrsim 10$, the merger rate of early binaries exceeds late ones

Conclusion

- PBHs lead to the early formation of DM halos, in the central parts of which the formation of PBH clusters is possible
- Suppression of the merger rate of early PBHs binaries weakens the gravitational waves constraints $f_{PBH} \lesssim 0.1$
- At high z, the merger rate from early binaries dominates, and (probably) at low z from late binaries

Mass distribution





$R_{cl} \sim 20$ pc, $M_{cl} \sim 10^4\, M_\odot$ including DM

Hil sphere: $R_{cl} = R_g (M_{cl}/3M_g)^{1/3} \rightarrow$ such substructures are located $R_g \gtrsim 13$ kpc