

Generating peaks in primordial perturbation spectra

Alexei A. Starobinsky

Landau Institute for Theoretical Physics RAS,
Moscow - Chernogolovka, Russia

The 6th International Conference on Particle Physics and
Astrophysics ICPPA 2022

Moscow, 30.11.2022

Inflation and two new fundamental observational parameters

Beyond the slow-roll approximation

Multiple inflation with variable number of slow-roll inflatons

Conclusions

Inflation

The inflationary hypothesis :

Some part of the world which includes all its presently observable part was as much symmetric as possible during some period in the past - both with respect to the geometrical background and to the state of all quantum fields (no particles).

Non-universal (due to the specific initial condition) explanation of the cosmological arrow to time - chaos, entropy (in some not well defined sense) can only grow after inflation.

Still this state is an intermediate attractor for a set of pre-inflationary initial conditions with a non-zero measure. Also it is not a unique one, there exists a class of such states leading to the same observable predictions.

Successive realization of this idea is based on the two more detailed and independent assumptions.

1. Existence of a metastable quasi-de Sitter stage in our remote past which preceded the hot Big Bang. During it, the expansion of the Universe was accelerated and close to the exponential one, $|\dot{H}| \ll H^2$ where H is the Hubble function.
2. The origin of all inhomogeneities in the present Universe is the effect of **gravitational creation of pairs of particles - antiparticles and field fluctuations** during inflation from the adiabatic vacuum (no-particle) state for Fourier modes covering all observable range of scales (and possibly somewhat beyond).

Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation in the synchronous gauge with some additional conditions fixing it completely:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\xi(\mathbf{r})\delta_{lm} + \sum_{a=1}^2 g^{(a)}(\mathbf{r}) e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \quad g^{(a)}_{,l} e_m^{l(a)} = 0, \quad e_{lm}^{(a)} e^{lm(a)} = 1$$

$\xi = -\mathcal{R}$ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)). The most important quantities:

$$P_\xi(k), \quad \frac{d \ln P_\xi(k)}{d \ln k} \equiv n_s(k) - 1, \quad r(k) \equiv \frac{P_g}{P_\xi}$$

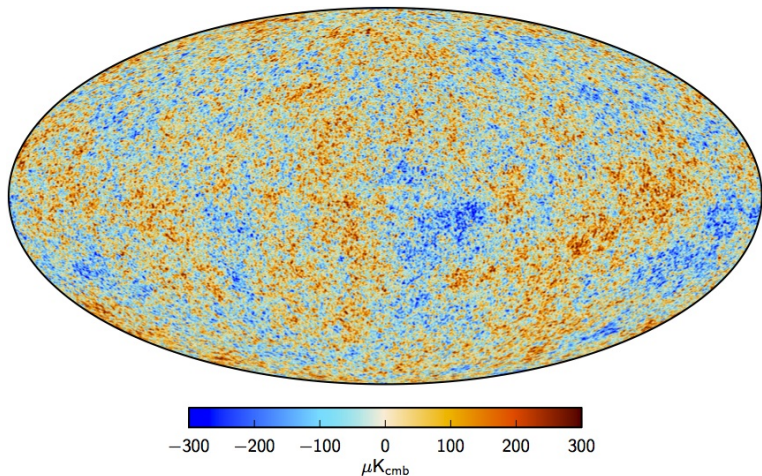
Both $|n_s - 1|$ and r are small during slow-roll inflation.

Existence of constant (quasi-isotropic) modes

For FLRW models filled by ideal fluids, it was known already to Lifshitz (1946). For a wide class of modified scalar-tensor gravity theories, it was proved in A. A. Starobinsky, S. Tsujikawa and J. Yokoyama, Nucl. Phys. B 610, 383 (2001). However, their existence is much more general. From the mathematical point of view, constant modes appear simply due to the existence of non-degenerate solutions of the same gravity models in the isotropic and spatially flat FLRW space-time. By construction, these solutions always have 3 non-physical (gauge) arbitrary constants of integration due to the possibility of arbitrary and independent rescaling of all spatial coordinates. Making these constants slightly inhomogeneous converts them to the leading terms of physical constant modes (one scalar and two tensor ones). Moreover, these constants (now functions of spatial coordinates) need not be small, they can be arbitrarily large: $a^2(t)\delta_\alpha^\beta \rightarrow a^2(t)c_\alpha^\beta(\mathbf{r})$.

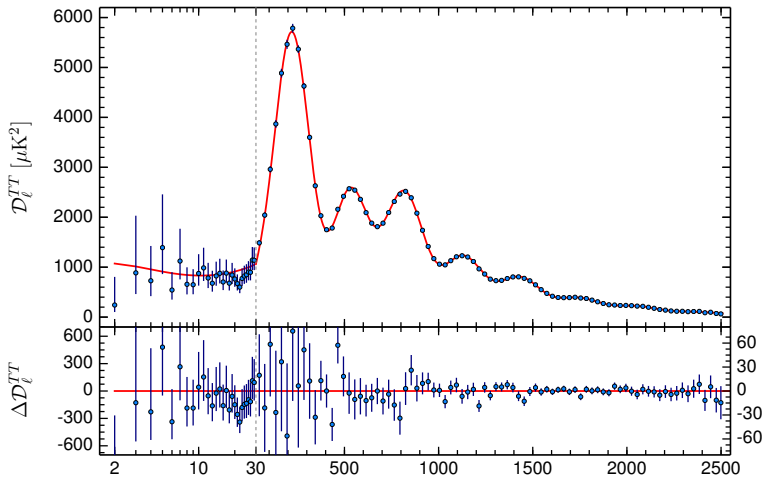
CMB temperature anisotropy

Planck-2015: P. A. R. Ade et al., arXiv:1502.01589



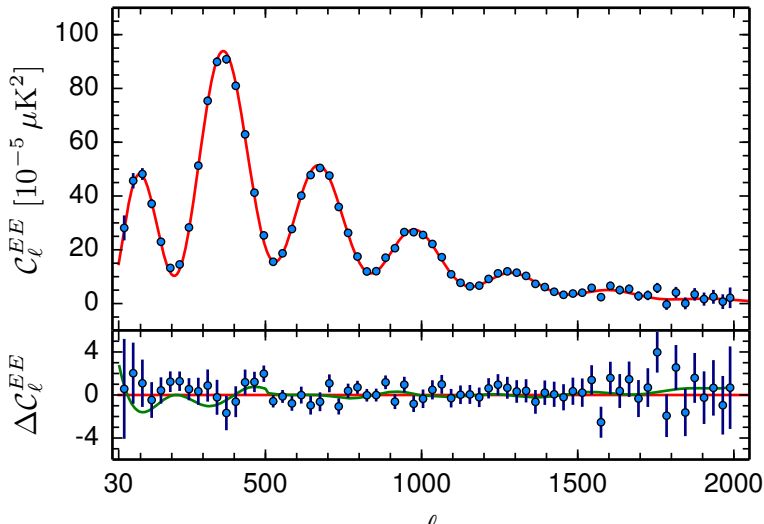
CMB temperature anisotropy multipoles

$$\Delta T(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi), \quad D_l^{TT} = \frac{l(l+1)}{2\pi(2l+1)} \sum_m |a_{lm}|^2.$$



CMB E-mode polarization multipoles

Only the E-mode has been found. The expected correlation between CMB temperature anisotropy and E-mode polarization has been discovered, too.



New cosmological parameters relevant to inflation

Now we have numbers: N. Agranim et al., arXiv:1807.06209

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N_H^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$\langle \xi^2(\mathbf{r}) \rangle = \int \frac{P_\xi(k)}{k} dk, \quad P_\xi(k) = (2.10 \pm 0.03) \cdot 10^{-9} \left(\frac{k}{k_0} \right)^{n_s-1}$$

$$k_0 = 0.05 \text{ Mpc}^{-1}, \quad n_s - 1 = -0.035 \pm 0.004$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$. (note that $(1 - n_s)N_H \sim 2$).

The simplest models producing the observed scalar slope

Arithmetical classification of inflationary models - by the number of free parameters of these models fixed by observations only.

The 3 simplest models having

$n_s - 1 = -\frac{2}{N}$, $r = \frac{12}{N^2} = 3(n_s - 1)^2$ are one-parametric.

1. The $R + R^2$ inflationary model.
2. The Higgs inflationary model.
3. The combined Higgs- R^2 model.

Perspectives of future discoveries

- ▶ Primordial gravitational waves from inflation: r .
 $r \lesssim 8(1 - n_s) \approx 0.3$ (confirmed!) but may be much less.
However, under reasonable assumptions one may expect that $r \gtrsim (n_s - 1)^2 \approx 10^{-3}$. The target prediction in the simplest (one-parametric) models is
 $r = 3(n_s - 1)^2 \approx 0.004$.
- ▶ A more precise measurement of $n_s - 1 \implies$ duration of transition from inflation to the radiation dominated stage \implies information on inflaton (scalaron) couplings to known elementary particles at super-high energies $E \lesssim 10^{13}$ GeV.
- ▶ Local non-smooth features in the scalar power spectrum at cosmological scales (?).
- ▶ Local enhancement of the power spectrum at small (non-cosmological) scales leading to a significant amount of primordial black holes and related peaks in primordial GW background (?).

Generating peaks and depressions in the primordial scalar spectrum

To obtain large peaks and depressions in P_ξ , temporal breaking of the slow-roll approximation during inflation is needed. The simplest way: fast break in the first derivative of the inflaton potential $V(\phi)$ (A. A. Starobinsky, JETP Lett. 55, 489 (1992)). Leads to a step in P_ξ with superimposed oscillations. To obtain a peak, two such features with opposite signs, or a fast break in the $V(\phi)$ itself are needed (so that an inflection point appears in between). However, it is not sufficient to have an inflection point only, it should be combined with a strong breaking of the slow-roll conditions.

Let $V(\phi) = V_0 + A_+ \phi \theta(\phi - \phi_0) + A_- \phi \theta(\phi_0 - \phi)$ for ϕ close to ϕ_0 . Then

$$\dot{\phi} = -\frac{A_+}{3H_0} \theta(-t) - \frac{A_- + (A_+ - A_-)e^{-3H_0 t}}{3H_0} \theta(t)$$

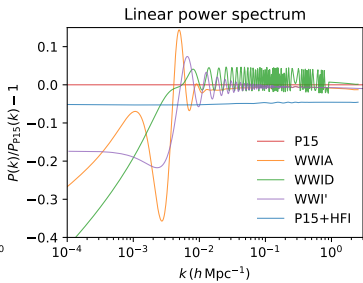
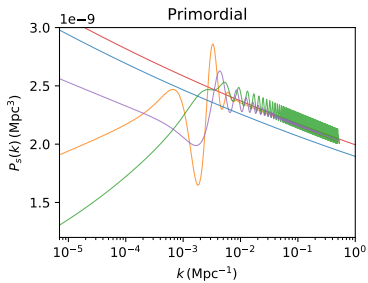
The slow-roll spectrum P_ξ is modulated by the multiplier

$$D^2 = 1 - 3 \left(\frac{A_-}{A_+} - 1 \right) \left[\left(1 - \frac{1}{y^2} \right) \sin 2y + \frac{2}{y} \cos 2y \right] +$$

$$\frac{9}{2} \left(\frac{A_-}{A_+} - 1 \right)^2 \frac{1}{y^2} \left(1 + \frac{1}{y^2} \right) \times$$

$$\left[1 + \frac{1}{y^2} + \left(1 - \frac{1}{y^2} \right) \cos 2y - \frac{2}{y} \sin 2y \right],$$

$$y = \frac{k}{k_0}, \quad D(0) = \frac{A_-}{A_+}, \quad D(\infty) = 1$$



Non-scale-free features at cosmological scales

The most recent analysis of this type of spectra with power suppression at large scales (D. K. Hazra, D. Paoletti, I. Debono, A. Shafieloo, G. F. Smoot, A. A. Starobinsky, JCAP 2112 (2021) 038; [arXiv:2107.09460](#)) using the CMB temperature and polarization data from the Planck 2018 data release shows marginal (68% C.L.) preference of suppression from the large scale temperature angular power spectrum. However, the large-scale E-mode likelihood does not support this suppression and in the combined data the preference towards the suppression becomes negligible. For models with oscillatory features along with the suppression, unbinned data from the recently released CamSpec 12.5 likelihood was used which updates Planck 2018 results. Comparison of the Bayesian evidences of the feature models with their baseline slow-roll inflaton potentials showed that the latter are moderately preferred against potentials with features.

Perturbations in multiple slow-roll inflation

The constant mode of ξ always exists in the super-Hubble regime, but it is not tangent to the trajectory of a background solution in the field space generically.

The simplest case: minimally coupled to gravity scalar fields with $V = \sum_n V_n(\phi_n)$.

A. A. Starobinsky, JETP Lett. 42, 152 (1985).

The number of e-folds:

$$N = -8\pi G \sum_n \int^{\phi_n} \frac{V_n(\tilde{\phi}_n)}{dV_n/d\tilde{\phi}_n} d\tilde{\phi}_n,$$

where only **slow-roll** inflatons are taken into account in the sum.

The scalar perturbation spectrum (δN formalism):

$$\xi(\mathbf{r}) = \sum_n \left(\frac{\delta N}{\delta \phi_n} \right)_b \delta \phi_n(\mathbf{r}),$$

where the subscript b means the background quantity.
In the Fourier space:

$$\xi(\mathbf{k}) = 4G \sum_n \left(\frac{H V_n}{dV_n/d\phi_n} \right)_k c_n(\mathbf{k}),$$

where $c_n(\mathbf{k})$ are independent Gaussian variables with zero average and unit dispersion, and the subscript k means that the background quantity is taken at the moment of the mode Hubble radius crossing $k = aH$ during inflation.

Generalization to an arbitrary $V(\phi_n)$: [M. Sasaki, E. D. Stewart, Prog. Theor. Phys. 95 \(1996\) 71; astro-ph/9507001](#).

PBHs and small-scale GWs in two-field models of inflation

The simplest one-parameter inflationary models do not predict PBHs, at least whose existing at present with $M > 10^{15}$ g. Previously known ways to obtain a large peak in the primordial power spectrum of scalar adiabatic perturbations at small scales:

1. A local feature in the inflaton potential $V(\phi)$ (a rapid change of its slope or its amplitude, an inflection point with a large $\frac{|V'''V'|}{\kappa^2 V^2}$ if $\epsilon \equiv -\frac{\dot{H}}{H^2} \lesssim 1$).
2. A rapid turn of the inflaton trajectory in the field space in the case of many-field models of inflation.
3. Phase transitions leading to large isocurvature perturbations which transform to adiabatic ones afterwards.
4. Generically, the **dimensionality** of the internal space of slow-roll inflatons changes with time during multiple inflation.

Two-field inflation with large kinetic coupling

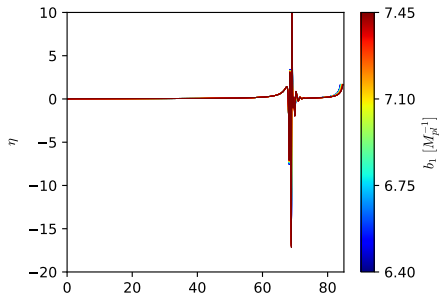
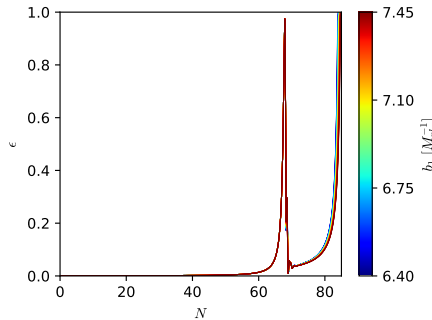
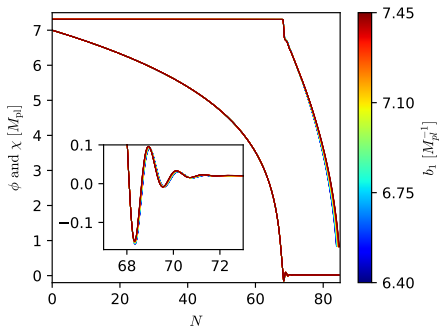
A novel mechanism: a two-field inflation with different inflaton effective masses (that leads to two stages of inflation) and a **large** non-standard kinetic coupling of the heavier field.

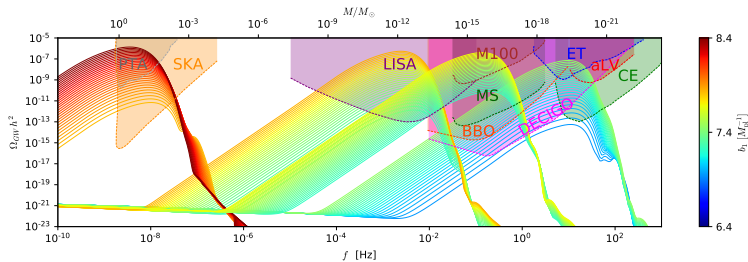
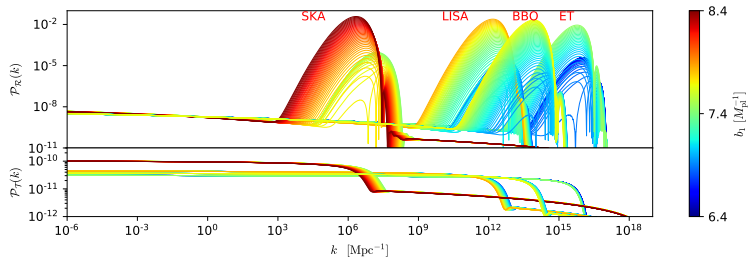
M. Braglia, D. K. Hazra, F. Fimelli, G. F. Smoot,
L. Sriramkumar, A. A. Starobinsky, JCAP 2008 (2020) 001.

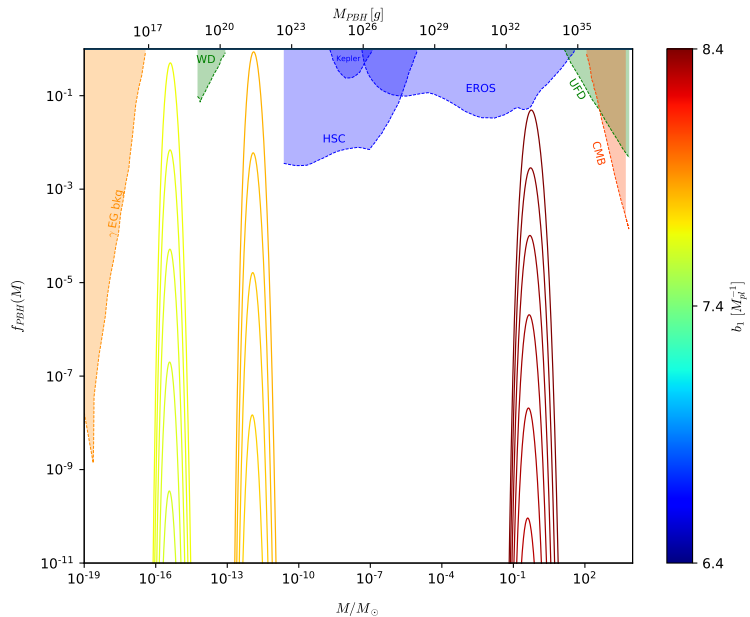
$$S(\phi, \chi) = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial\phi)^2 - \frac{f(\phi)}{2}(\partial\chi)^2 - V(\phi, \chi) \right]$$

$$V(\phi, \chi) = V_0 \frac{\phi^2}{\phi_0^2 + \phi^2} + \frac{m_\chi^2}{2} \chi^2$$

Large kinetic coupling: $f(\phi) = \exp(b\kappa\phi)$, $b \gg 1$. The peak in the spectrum arises when the heavier field goes out of the slow-roll regime. It can lead to the formation of PBHs with a wide range of masses and to the generation of stochastic background of primordial gravitational waves produced by second order scalar perturbations.







Conclusions

- ▶ In two-field inflationary models with two stages of inflation having different number of slow-roll inflatons and a large non-standard kinetic coupling of a heavier inflaton field, it is possible to produce a large peak at small scales in the primordial power spectrum of scalar adiabatic perturbations leading to the formation of PBHs and to the peak in the stochastic background of primordial gravitational waves produced by second order scalar perturbations. However, large peaks in the spectra require a large value of some (at least one) of the model parameters.
- ▶ As for local non-scale-free features at cosmological scales, the present CMB temperature anisotropy and polarization data do not favor them, but are not able to exclude them completely.