Radiation of a twisted neutron in the presence of a dispersion medium

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Various representations of particles

Wave, beam and packet

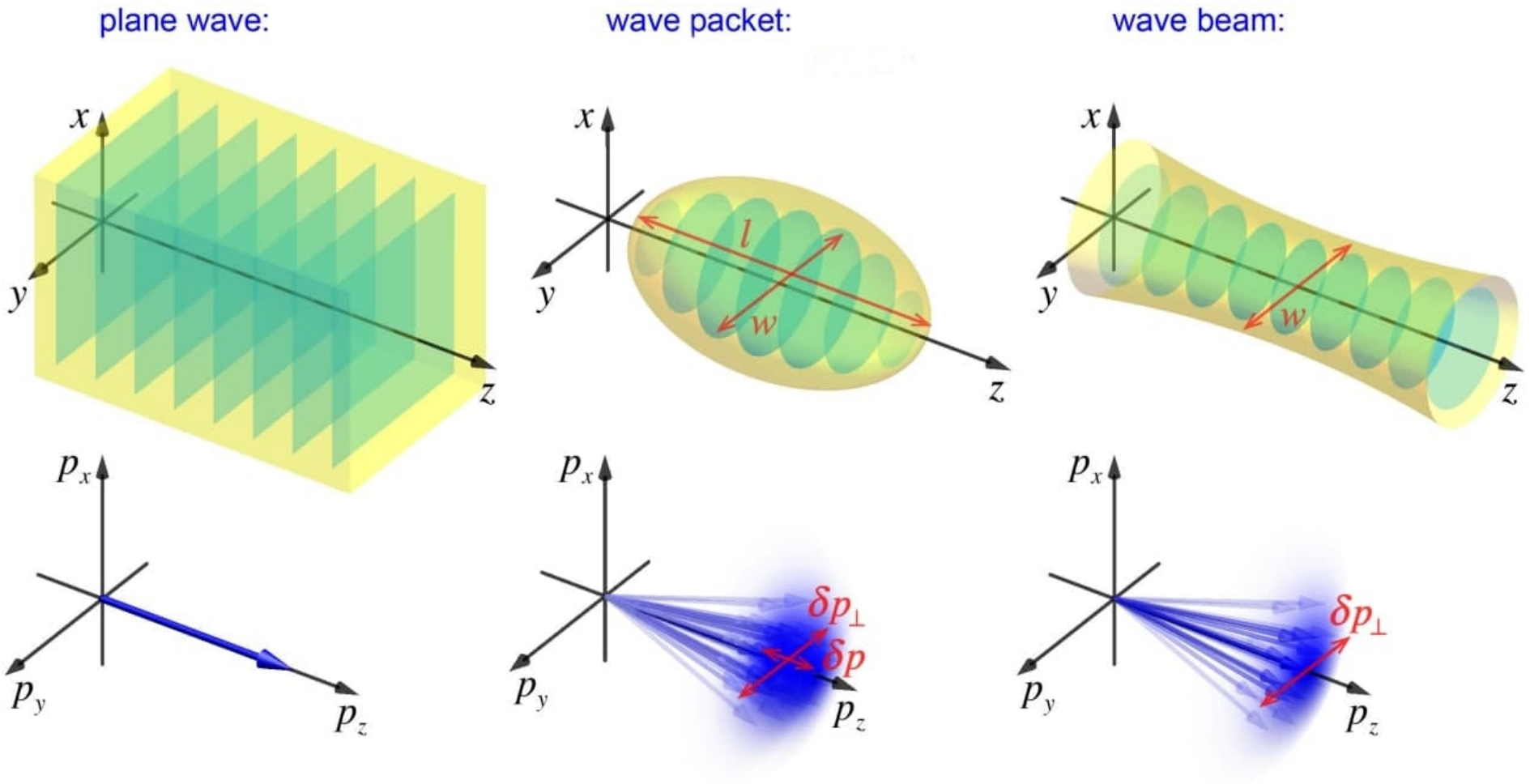
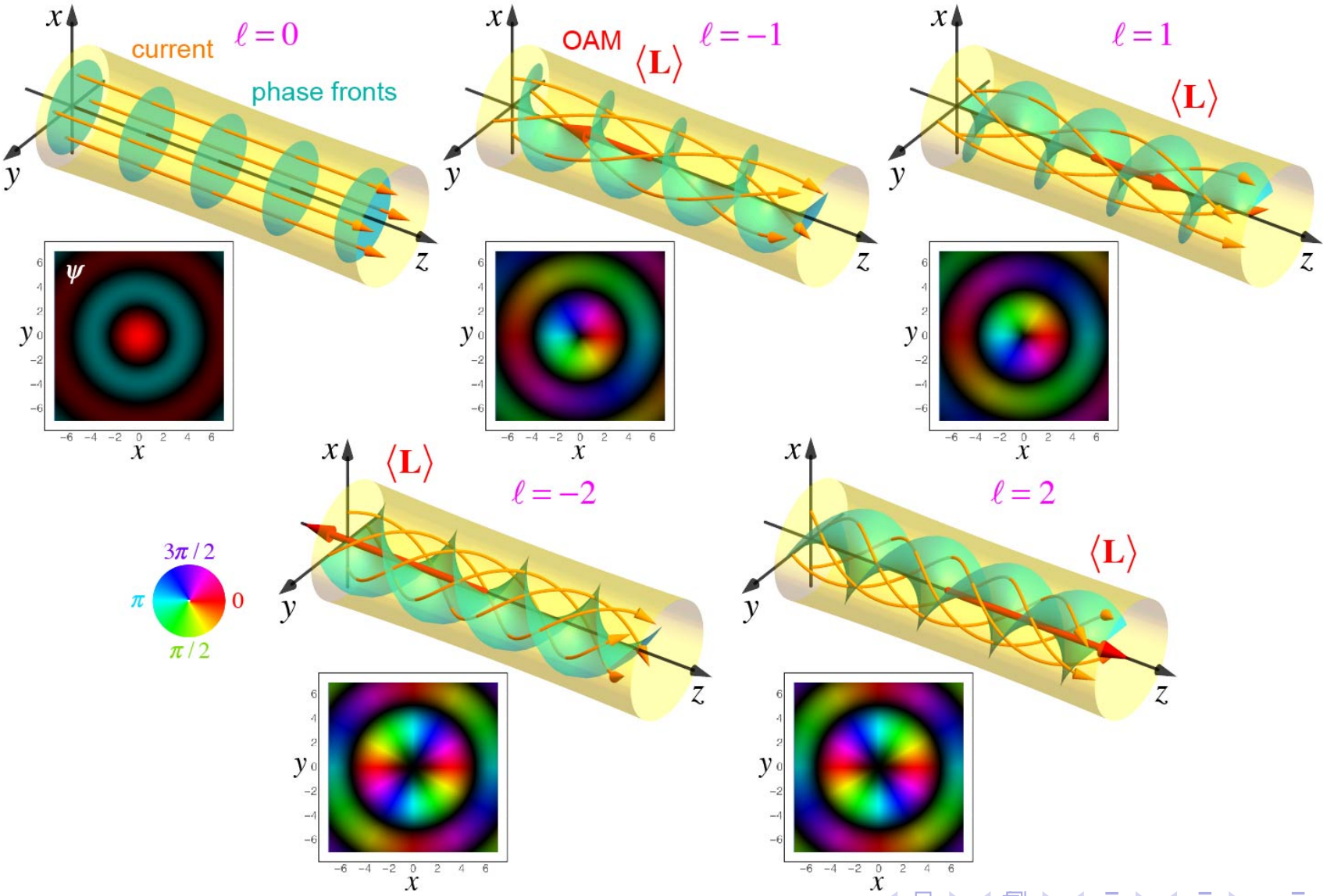


Figure: Assuming the paraxial approximation $p_z \simeq p$, $p_\perp \ll p$, the characteristic dimensions of the real- and momentum-space distributions satisfy the uncertainty relations $l \sim \hbar/\delta p$ and $w \sim \hbar/\delta p_\perp$ [K. Y. Bliokh et al., Phys. Rep. 690, 1 (2017)].

Twisted wave packets

See, for details, in K. Y. Bliokh et al., Phys. Rep. 690, 1 (2017).



Methods for the generation of vortex beams

See, for details, in C. W. Clark et al., Nature **525**, 504 (2015); Sarenac et al., Sci. Adv.**8**, eadd2002 (2022).

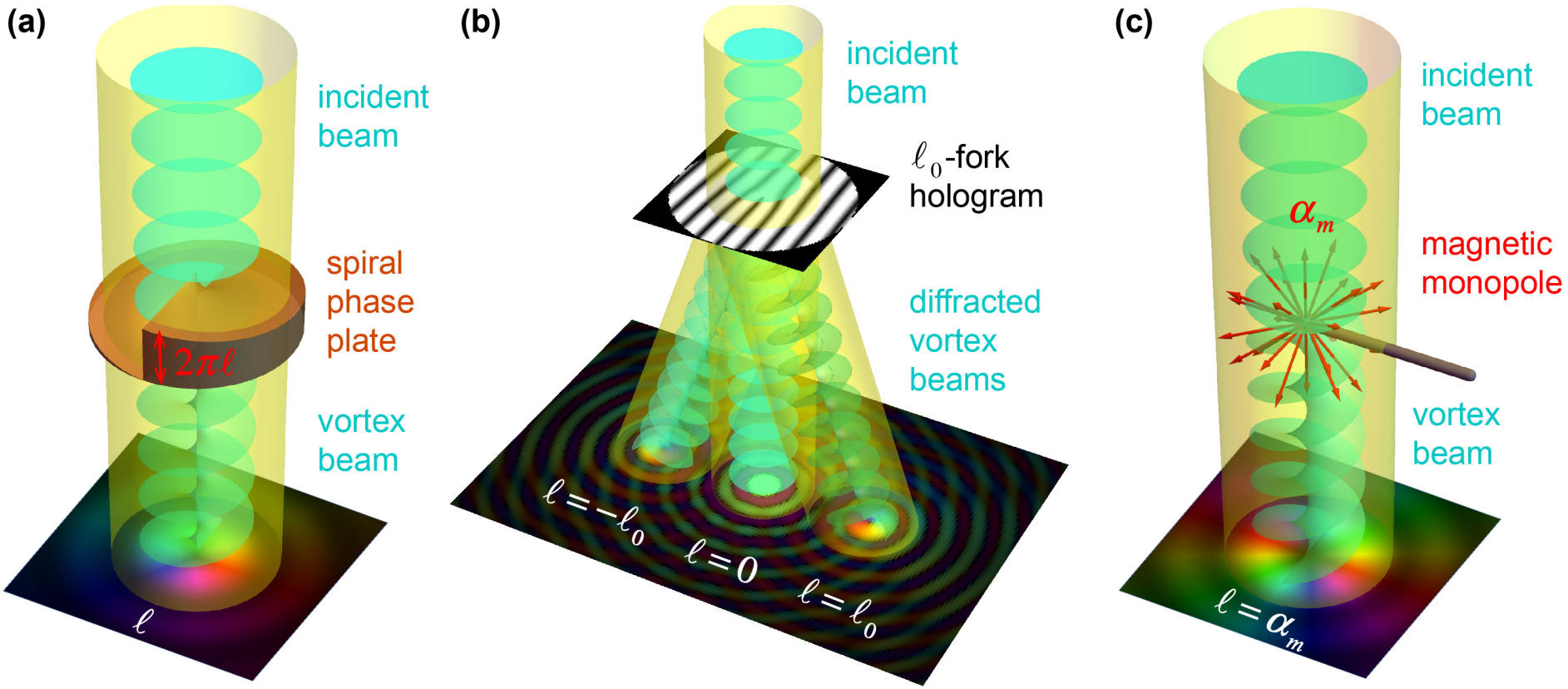


Figure: Schematics of basic methods for the generation of vortex beams.

Twisted Neutrons

Profile of the wave packet

$$\varphi_s(\mathrm{p}) = \delta_{s\sigma} k p_\perp^{|l|} \exp\Big(-\frac{(p_3 - p)^2}{4\sigma_3^2} - \frac{\mathrm{p}_\perp^2}{4\sigma_\perp^2} + il \arg(p^1 + ip^2) \Big), \qquad (1)$$

Normalization constant

$$k^{-2} = (2\pi)^{3/2} \sigma_3 \sigma_\perp^2 (2\sigma_\perp^2)^{|l|} |l|!, \qquad l \in \mathbb{Z}, \qquad (2)$$

$\sigma = \pm 1$ characterizes the spin projection of the state onto the z axis.

- The state (1) is the eigenvector of the projection of the total angular momentum operator on the z axis with the eigenvalue $l + \sigma/2$.
- The case $l = 0$ describes a cylindrically symmetric Gaussian wave packet.

$$c(\mathrm{p}) = k^2 p_\perp^{2|l|} \exp\Big(-\frac{(p_3 - p)^2}{2\sigma_3^2} - \frac{\mathrm{p}_\perp^2}{2\sigma_\perp^2} \Big). \qquad (3)$$

Problem statement

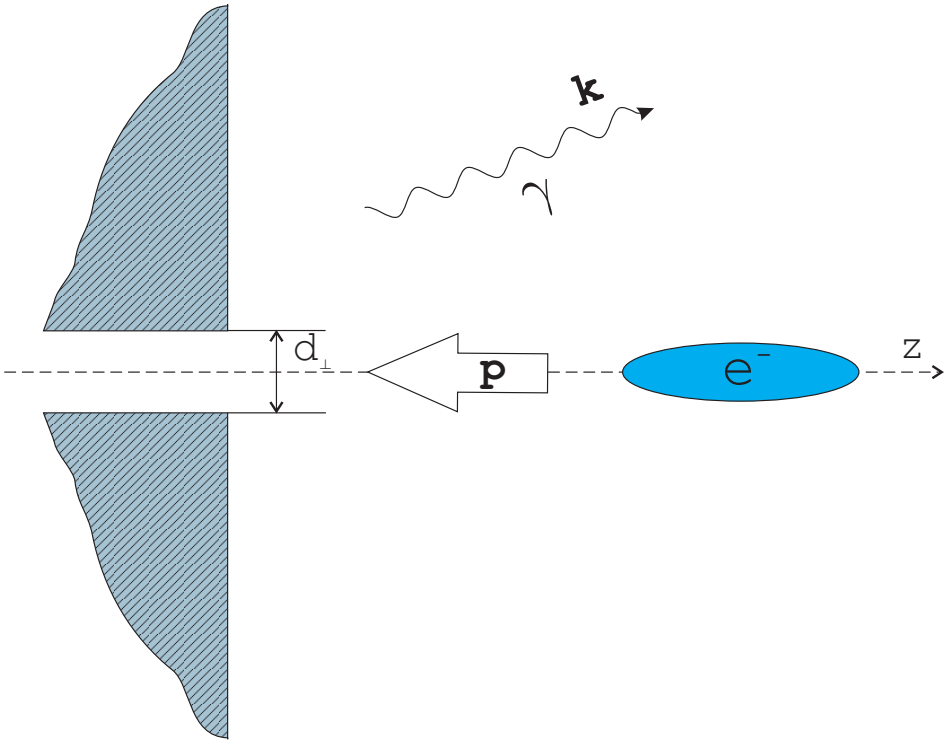


Figure: The schematic representation of the experimental installation for observation of transition radiation.

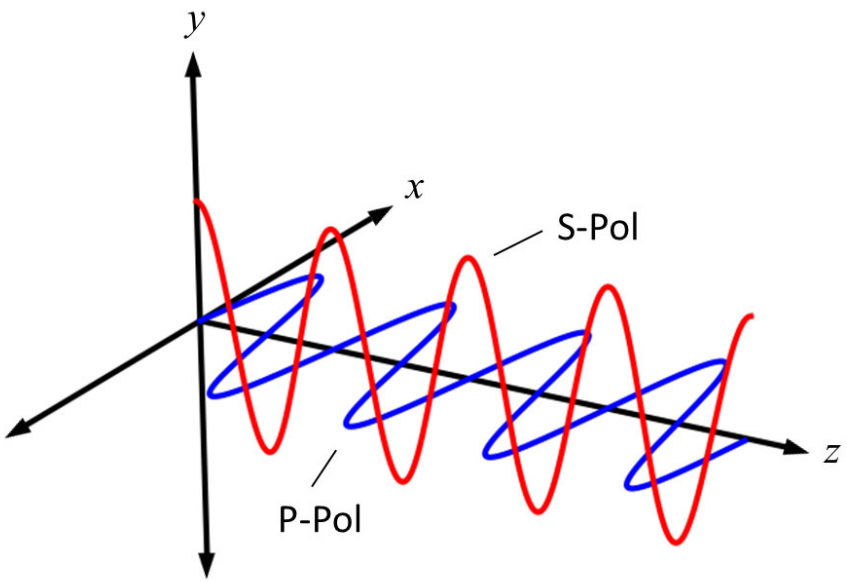


Figure: Diagram showing s-polarized and p-polarized light relative.

QED with wave packet

Lagrangian density of the system

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[\gamma^{\mu}(i\partial_{\mu} - eA_{\mu}) - m]\psi - \frac{\mu_a}{2}F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi, \qquad (4)$$

μ_a is the anomalous magnetic moment, m is the particle mass.
Quantum fields in the interaction picture

$$\hat{\mathrm{A}}_i(x) = \sum_{\lambda}\int_{+}\frac{V d\mathrm{k}}{(2\pi)^3}\frac{1}{\sqrt{2k_0 V}}\left[\mathrm{a}_i^{(\lambda)}(k_3;z)e^{-ik_0x^0+i\mathrm{k}_{\perp}\mathrm{x}_{\perp}}\hat{c}_{\lambda}(\mathrm{k}) + \mathrm{H.c.}\right], \qquad (5)$$

$k_0 = |\mathrm{k}|$, V characterizes the normalization volume.

$$\mathrm{k} = k_0\mathrm{n} = k_0(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \qquad (6)$$

The integration region is restricted by the inequality $k^3 \geqslant 0$.

QED with wave packet

State of the Dirac particle

$$|\varphi\rangle := \sum_{\alpha} \sqrt{\frac{(2\pi)^3}{V}} \varphi_\alpha |\alpha\rangle, \qquad \sum_{\alpha} \frac{(2\pi)^3}{V} |\varphi_\alpha|^2 = \sum_{s} \int d\mathrm{p} |\varphi_s(\mathrm{p})|^2 = 1. \quad (7)$$

Density matrix

$$R_{s,\bar{s}}(\mathrm{p},\bar{\mathrm{p}}) := \varphi_s(\mathrm{p})\varphi^*_{\bar{s}}(\bar{\mathrm{p}}). \quad (8)$$

Spin density matrix

$$\rho_{s,\bar{s}}(\mathrm{p},\bar{\mathrm{p}}) := R_{s,\bar{s}}(\mathrm{p},\bar{\mathrm{p}})/c(\mathrm{p}), \quad c(\mathrm{p}) := \sum_{s} R_{s,s}(\mathrm{p},\mathrm{p}), \quad \int d\mathrm{p} c(\mathrm{p}) = 1. \quad (9)$$

Reference

P.O. Kazinski, G.Yu. Lazarenko, Phys. Rev. A. **103**, 012216 (2021).

General result

Inclusive radiation probability

$$dP(\lambda,\mathrm{k};\varphi) = \mu_a^2 \int \frac{d\mathrm{p}c(\mathrm{p})}{|p_3' p_3|} F_i^{(\lambda)}(\mathrm{k},q_3)\left(F_j^{(\lambda)}(\mathrm{k},q_3)\right)^* \times \\ \left\{q^2[2m^2\eta^{ij} - (p^i+p'^i)(p^j+p'^j)/2] - 2m^2 q^i q^j + \right. \\ \left. + 2im[(qs)p_\mu q_\nu + q^2 q_\mu s_\nu/2]\varepsilon^{\mu\nu ij}\right\} \frac{d\mathrm{k}}{32\pi^3 k_0} \tag{10}$$

Describes the properties of matter

$$F_i^{(\lambda)}(\mathrm{k},q_3) := \int_{-\infty}^{+\infty} dx_3\, e^{iq_3x_3}\left(a_i^{(\lambda)}(k_3;x_3)\right)^*, \tag{11}$$

where

$$\varepsilon^{0123} = 1, \quad \eta^{ij} = -\delta_{ij}, \quad q_\mu := p_\mu - p'_\mu, \\ q^2 = 2(m^2 - pp') = k_3^2 - (p_3 - p_3')^2 < 0. \tag{12}$$

Polarization of radiation

s-polarization vector

$$\mathrm{f}^{(2)}(\mathrm{k}) \sim [\mathrm{e}_3, \mathrm{k}] \tag{13}$$

Inclusive radiation probability

$$dP(2,\mathrm{k};\varphi) = -\mu_a^2 q^2 \int \frac{d\mathrm{p}c(\mathrm{p})}{|p_3' p_3|} \left((\mathrm{p}, \mathrm{f}^{(2)})^2 + m^2 \right) \left| F_i^{(2)} \right|^2 \frac{d\mathrm{k}}{16\pi^3 k_0} \tag{14}$$

In the limit of a small quantum recoil $k_0 \ll p_3 \beta_3$

$$q_3 \approx k_0 (1 - \beta_\perp \mathrm{n}_\perp)/\beta_3, \quad \text{where} \quad \beta = \mathrm{p}/(m\gamma). \tag{15}$$

The amplitude semiclassical radiation*

$$\left(\beta, \mathrm{F}^{(\lambda)}(\mathrm{k}, q_3)\right)^* / \beta_3 \approx \int_{-\infty}^{+\infty} dt e^{-ik_0 t} \left(\beta, \mathrm{A}^{(\lambda)}(k_3; x_3)\right) \tag{16}$$

*L.D. Landau, E.M. Lifshitz, The Classical Theory of Fields, Pergamon, Oxford, 1962.

Ideally conducting plate

If $p_3 \gg \sigma_3$, where σ_3^2 is the dispersion of p_3 in the packet and

$$\mathrm{p} = (0, 0, p), \qquad p < 0. \tag{17}$$

Normal falling

$$dP(\lambda, \mathrm{k}; \varphi) = -\mu_a^2 m^2 \int \frac{d\mathrm{p}c(\mathrm{p})}{|p_3' p_3| q^2} \frac{k_3^2 d\mathrm{k}}{4\pi^3 k_0} \tag{18}$$

In the nonrelativistic limit, $\gamma^2 \approx 1$

$$dP(\lambda, \mathrm{k}; \varphi) = \mu_a^2 \frac{n_3^2 d\mathrm{k}}{4\pi^3 k_0}. \tag{19}$$

Ideally conducting plate

Probability radiation p-polarization

$$dP(1,\mathrm{k};\varphi) = \mu_a^2\left(1 - \frac{(|l|+1)\sigma_\perp^2 n_3^2}{m^2 + n_\perp^2 p^2}\right)\frac{d\mathrm{k}}{4\pi^3 k_0} \tag{20}$$

Probability radiation s-polarization

$$\begin{aligned} dP(2,\mathrm{k};\varphi) = \frac{\mu_a^2}{m^2 + n_\perp^2 p^2}\Big\{ m^2 - \frac{(|l|+1)\sigma_\perp^2}{(m^2 + n_\perp^2 p^2)^2}\big[m^4(1 - 3n_\perp^2) - \\ - m^2 p^2 n_\perp^2 (4 - n_\perp^2) - p^4 n_\perp^4 \big]\Big\}\frac{n_3^2 d\mathrm{k}}{4\pi^3 k_0} \end{aligned} \tag{21}$$

- The anomalous magnetic moment behaves as a "true" magnetic moment [V. A. Bordovitsyn, I. M. Ternov, and V. G. Bagrov, Spin light, Phys. Usp. 38, 1037 (1995)].
- In the ultrarelativistic limit, the expressions for the leading contributions to (20) and (21) turn into classical formulas of [V. L. Ginzburg, V. N. Tsytovich, Transition Radiation and Transition Scattering (Hilger, Bristol, 1990)].

Plasma permittivity $\varepsilon(k_0) = 1 - \frac{\omega_p^2}{k_0^2}$

Approximations

$$\frac{\omega_p}{k_3} \ll 1, \qquad \text{and} \qquad \frac{L\omega_p^2}{k_3} \ll 1, \tag{22}$$

where L - thickness plate.

Probability radiation s-polarization

$$dP(2,\mathrm{k};\varphi) = \mu_a^2 \left(\frac{\omega_p}{2k_3}\right)^4 \int d\mathrm{p}c(\mathrm{p}) \left(\frac{\beta_3 - 2n_3}{\gamma}\right)^2 (1 + (\gamma\beta_2)^2) \frac{n_3^2 d\mathrm{k}}{(2\pi k_0)^3} \tag{23}$$

Normal falling

$$dP(2,\mathrm{k};\varphi) = \mu_a^2 \left(\frac{\omega_p}{2k_3}\right)^4 \left(\frac{\beta_3 - 2n_3}{\gamma}\right)^2 \left(1 + \frac{(|l|+1)\sigma_\perp^2}{m^2}\right) \frac{n_3^2 d\mathrm{k}}{(2\pi k_0)^3} \tag{24}$$

Conclusion

1. A formula was obtained for the inclusive probability transition radiation from a neutron wave packet falling on a dispersive medium. It generalizes the results of the work Phys. Rev. A 103, 012216 (2021).
2. The semiclassical limit of the formula is found.
3. In transition radiation the probability to detect a photon with polarization vector lying in the reaction plane does not depend on the observation angle and the energy of the incident particle.
4. To date, corrections to the probability for the Gaussian profile of a wave packet of the order $\max\left[(1+|l|)(\overline{\lambda}_c/\sigma_\rho)^2\right] \approx 10^{-17}$ Nature **525**, 504 (2015); Sci. Adv.**8**, eadd2002 (2022).
5. The transition radiation probability for high-energy photons is obtained.