

# An Effective Field Theory for Large Oscillons

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based on: Levkov, VM, Nugaev, Panin, [arXiv:2208.04334](https://arxiv.org/abs/2208.04334)

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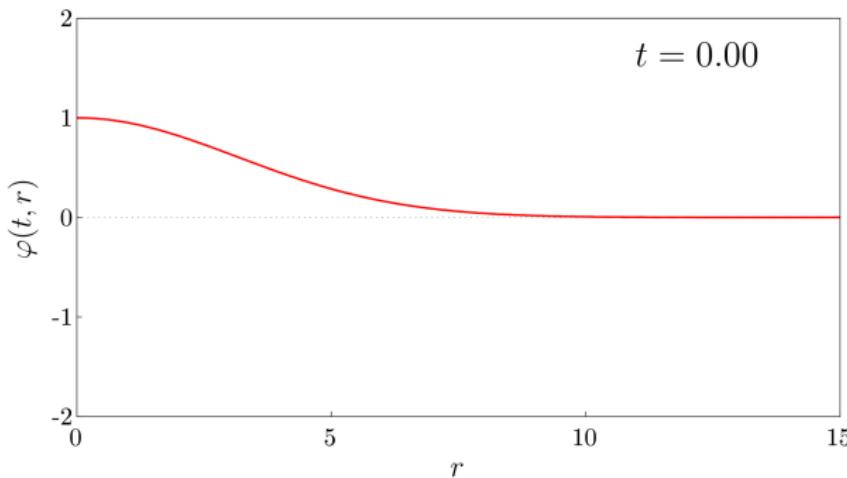
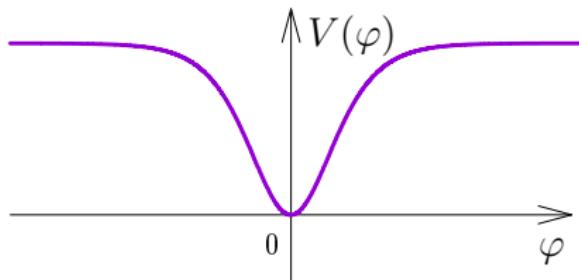
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# Oscillons: introduction

Example: scalar field theory

$$\partial_t^2 \varphi - \Delta \varphi = -V'(\varphi)$$

$$V(\varphi) = \frac{1}{2} \tanh^2 \varphi$$



$$d = 3$$

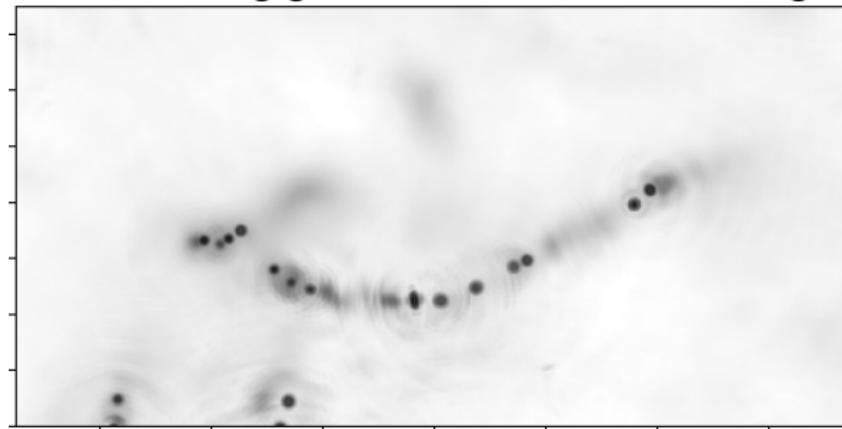
$$\varphi(0, r) = \varphi_0 e^{-r^2/\sigma^2}$$
$$\varphi_0 = 1, \quad \sigma = 20$$

Lifetime:

$\gtrsim 10^5$  periods

# Introduction: oscillons in cosmology

- nucleate during generation of axion or ultra-light DM



*Kolb, Tkachev '94*

Vaquero, Redondo,  
Stadler '19

*Buschmann, Foster,  
Safdi '20*

- accompany cosmological phase transitions

*Dymnikova, Kozel, Khlopov, Rubin '00  
Gleiser, Graham, Stamatopoulos '10*

- formed by inflaton field during preheating

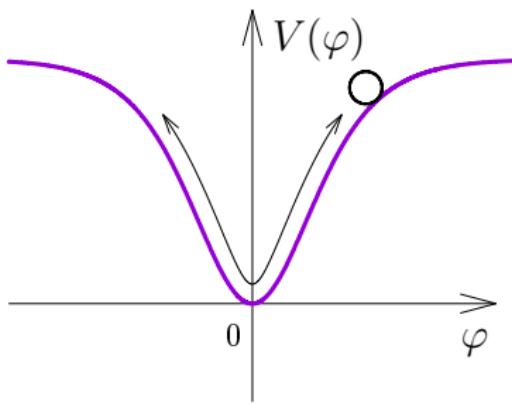
*Amin, Easter, Finkel, '10  
Hong, Kawasaki, Yamazaki '18*

Why are oscillons so long-lived?

How to describe them?

# Effective Field Theory: action-angle variables

- Main idea: consider large-size oscillons
- Zero order approx.:  $\partial_t^2 \varphi - \cancel{\Delta} \varphi = -V'(\varphi) \implies$  Nonlinear oscillator



$$V(\varphi) = \frac{1}{2} \tanh^2 \varphi$$

$$\varphi = \operatorname{arcsinh} \left( \frac{\sqrt{I(2-I)}}{1-I} \cos \theta \right)$$

$$h(I) = I - I^2/2.$$

- Exactly solvable: action-angle variables

$$(\pi_\varphi \equiv \dot{\varphi}) \quad (\varphi, \dot{\varphi}) \rightarrow (I, \theta)$$

- Hamiltonian:  $h = \frac{\dot{\varphi}^2}{2} + V(\varphi) \equiv h(I)$

- Explicitly:  $I(\varphi, \dot{\varphi}) \propto \oint \sqrt{h - V} d\varphi$

$$\theta(\varphi, \dot{\varphi}) \propto \frac{\partial}{\partial I} \int \sqrt{h - V} d\varphi'$$

- Classical solution:

$$I = \text{const}, \theta = \Omega t + \text{const}, \quad \boxed{\Omega = \frac{\partial h}{\partial I}}$$

- General case:  $\varphi = \Phi(I, \theta), \dot{\varphi} = \Pi(I, \theta)$

# Effective Field Theory: leading-order effective action

- **BUT** oscillon depends on  $\mathbf{x} \implies I = I(\mathbf{x})$ ,  $\theta = \theta(t, \mathbf{x})$ , but **slowly**
- Let us derive effective action:

$$\mathcal{S} = \int dt d^d \mathbf{x} \left( \underbrace{\frac{1}{2} \dot{\varphi}^2 - V(\varphi)}_{I \partial_t \theta - h} - \underbrace{\frac{1}{2} (\partial_i \varphi)^2}_{\text{subleading}} \right)$$

- Averaging over period

$$(\partial_i \varphi)^2 \longrightarrow \langle (\partial_i \varphi)^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} (\partial_i \Phi(I, \theta))^2 d\theta$$

- Slow-varying  $\partial_i I$ ,  $\partial_i \theta$  are moved *out* of the average

$$\langle (\partial_i \varphi)^2 \rangle \approx \frac{(\partial_i I)^2}{\mu_I(I)} + \frac{(\partial_i \theta)^2}{\mu_\theta(I)} + \cancel{\langle \partial_I \Phi \partial_\theta \Phi \rangle \partial_i I \partial_i \theta}$$

$$\mu_I \equiv \langle (\partial_I \Phi)^2 \rangle^{-1}, \quad \mu_\theta \equiv \langle (\partial_\theta \Phi)^2 \rangle^{-1}$$

# Effective Field Theory: properties

## Effective action in the leading order

$$\mathcal{S}_{\text{eff}} = \int dt d^d x \left( I \partial_t \theta - h(I) - \frac{(\partial_i I)^2}{2\mu_I(I)} - \frac{(\partial_i \theta)^2}{2\mu_\theta(I)} \right)$$

- Introducing complex field  $\psi(t, \mathbf{x}) = \sqrt{I} \cdot e^{-i\theta}$  gives

Example:  $V(\varphi) = \frac{1}{2} \tanh^2 \varphi$

$$\mu_I = I(2-I)^2(1-I)^2, \quad \mu_\theta = I^{-1} - 1$$

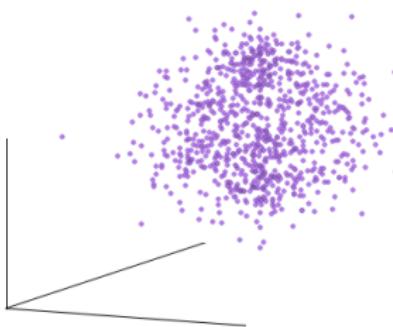
$$\mathcal{S}_{\text{eff}} = \int dt d^d x \left( i\psi^* \partial_t \psi - h - \frac{|\partial_i \psi|^2}{2\mu_1} - \frac{1}{2\mu_2} [\psi^{*2} (\partial_i \psi)^2 + \text{h.c.}] \right)$$

nonlinear Schrödinger model

Form factors:  $\mu_i = \mu_i(|\psi|^2)$ ,  $h = h(|\psi|^2)$

- Global symmetry:  $\theta \rightarrow \theta + \alpha \iff \psi \rightarrow e^{-i\alpha} \psi$   
 $\Downarrow$
- Conserved charge:  $N = \int d^d x I = \int d^d x |\psi|^2$

Attraction + charge conservation = solitons!



- Stationary ansatz:

$$\psi = \psi(r) e^{-i\omega t}, \quad \text{or} \quad \theta = \omega t$$

- Alternatively:

minimize the energy  $E$  at fixed charge  $N$ .

- Oscillon profile equation

$$\Omega = \partial h / \partial I$$

$$-\frac{2\psi^2}{\mu_I} \Delta\psi - (\partial_i\psi)^2 \frac{d}{d\psi} \left(\psi^2/\mu_I\right) + \Omega\psi = \omega\psi$$

- Physical interpretation:  
 $N$  — „number of particles”  
 $\omega$  — „energy of a particle”

Example:  $V(\varphi) = \frac{1}{2} \tanh^2 \varphi$

$$d = 1 : (t, x)$$

- ODE is integrable

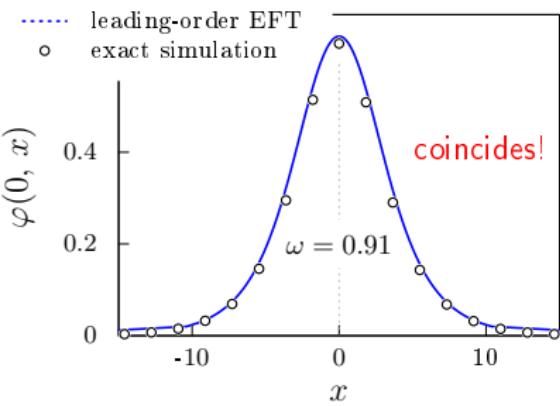
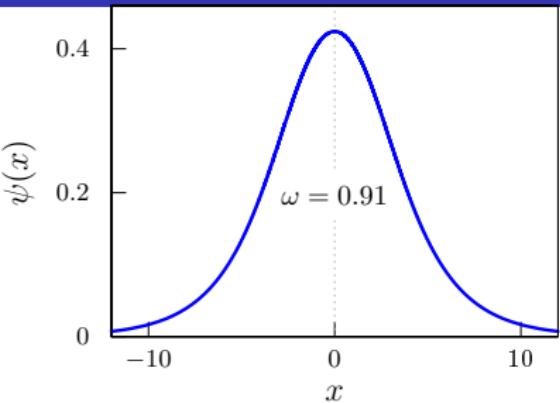
$$\begin{aligned} x &= \frac{2}{\sqrt{2\omega - 1}} \arctan \frac{\zeta(\psi)}{\sqrt{2\omega - 1}} \\ &- \frac{1}{\sqrt{2\omega}} \arctan \frac{\zeta(\psi)}{\sqrt{2\omega}} \\ &+ \frac{1}{\sqrt{2 - 2\omega}} \operatorname{arctanh} \frac{\zeta(\psi)}{\sqrt{2 - 2\omega}}, \end{aligned}$$

where  $\zeta(\psi) = \sqrt{2 - 2\omega - \psi^2}$

- Oscillon profile:

$$\varphi(t, x) = \Phi(\psi^2, \omega t)$$

- $\omega \rightarrow 1$ :  $R \propto (1 - \omega)^{-1/2} \rightarrow \infty$



# Corrections

- Goal: Develop asymptotic expansion in  $R^{-2}$ :

$$S_{\text{eff}} = \underbrace{S_{\text{eff}}^{(1)}}_{R^0 + R^{-2}} + \underbrace{\overbrace{S_{\text{eff}}^{(2)} + S_{\text{eff}}^{(3)} + \dots}^{\text{corrections}}}_{R^{-4} + R^{-6}}$$

- Field corrections:

$$I = \underbrace{I}_{\text{slow}} + \underbrace{\delta I}_{\text{fast}}, \quad \theta = \underbrace{\bar{\theta}}_{\text{slow}} + \underbrace{\delta\theta}_{\text{fast}}$$
$$\langle \delta I \rangle = \langle \delta\theta \rangle = 0, \quad \delta I \ll I, \quad \delta\theta \ll \theta$$

- Solve eqs. for  $\delta I, \delta\theta \Rightarrow$  plug the result into action +  $\boxed{\bar{\theta} = \omega t}$

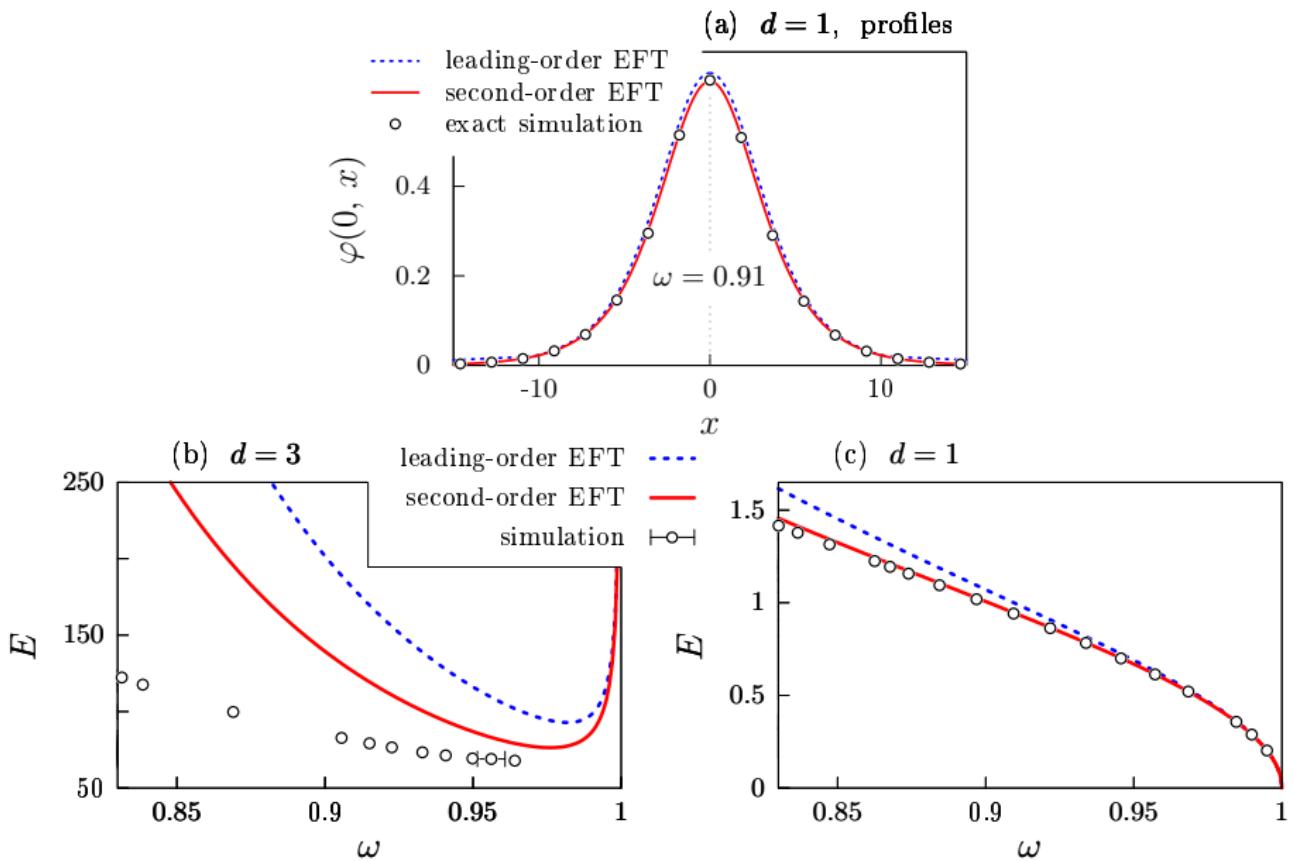
$$S_{\text{eff}} = S_{\text{eff}}^{(1)} + S_{\text{eff}}^{(2)}$$

$$S_{\text{eff}}^{(2)} = \int dt d^d x [d_1 (\partial_i \psi)^4 + d_2 \psi \Delta \psi (\partial_i \psi)^2 + d_3 (\Delta \psi)^2]$$

Note. Four spatial derivatives

$d_i(\psi^2)$  — form factors

# Second-order EFT: comparison for $V(\varphi) = \frac{1}{2} \tanh^2 \varphi$



## EFT.

- Sole parameter of the expansion:  $(mR)^{-2}$
- Action-angle variables  $\Rightarrow$  smooth fields  $I(x), \theta(t, x)$ .
- Fast-oscillating subleading terms — averaged over period
- Global  $U(1)$ -symmetry  $\Rightarrow$  oscillons
- Conditions for existence of long-lived oscillons

$$V(\varphi) \quad \left\{ \begin{array}{l} \text{attractive} \\ \text{nearly quadratic potential} \end{array} \right.$$

- Best predictions — at  $\omega \rightarrow m$  and smaller  $d$ .

## Perspective.

- Decay of oscillons — nonperturbative in EFT?

# Thank you for your attention!