Catalytic effect on pion decay caused by monopole and instanton creations in QCD

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References

I will talk about the contents in the following papers:

[1] **M. Hasegawa**, *Monopole and instanton effects in QCD*, JHEP 09 (2020) 113, [arXiv: 1807.04808], (2020).

[2] **M. Hasegawa**, *Color confinement, chiral symmetry breaking, and catalytic effect induced by monopole and instanton creations*, Eur. Phys. J. C 82 (2022) 1040, [arXiv: 2203.11357], (2022).

[3] **M. Hasegawa**, open access data tables for this research are available in "figshare" with DOI: [https://doi.org/10.6084/m9.figshare.20942866].

Monopoles and instantons

- What **magnetic monopole** we are studying is closely related to color confinement.
- 'tHooft [Proceedings of EPS international (1976)] and Mandelstam [Phys. Rep 23, 245 (1976)] provided a convincing description that a magnetic monopole that condenses in the QCD vacuum causes the dual Meissner effect and that color-charged particles are confined.
- Instantons induce chiral symmetry breaking.
- **The phenomena are well explained by models of the instantons [Dyakonov and Shuryak].**

Magnetic monopole search

- In condensed matter physics, a research group has created Dirac monopoles in a Bose-Einstein condensate and detected the monopoles through their experiments [Nature 505 (2014) 657, Science 348 (2015) 544].
- In high-energy physics, the “Monopole and Exotics Detector at the LHC (MoEDAL)’’ has already operated [https://videos.cern.ch/record/1360998].
- This experiment aims to explore **magnetic monopoles**, **dyons**, and other highly ionizing particles in proton-proton collisions at the LHC.
- This detector has been upgraded to the "MoEDAL-Apparatus for Penetrating Particles (MoEDAL-APP) [https://home.cern/science/experiments/moedal-mapp].

Purpose of this research

- **Our project aims to reveal the effects of magnetic monopoles and instantons on observables, which experiments can detect.**
- **Our final goal is to find a clue to observe magnetic monopoles and instantons in QCD in the real world.**
- **For these purposes, we perform simulations of lattice gauge theory.**

Simulation parameters

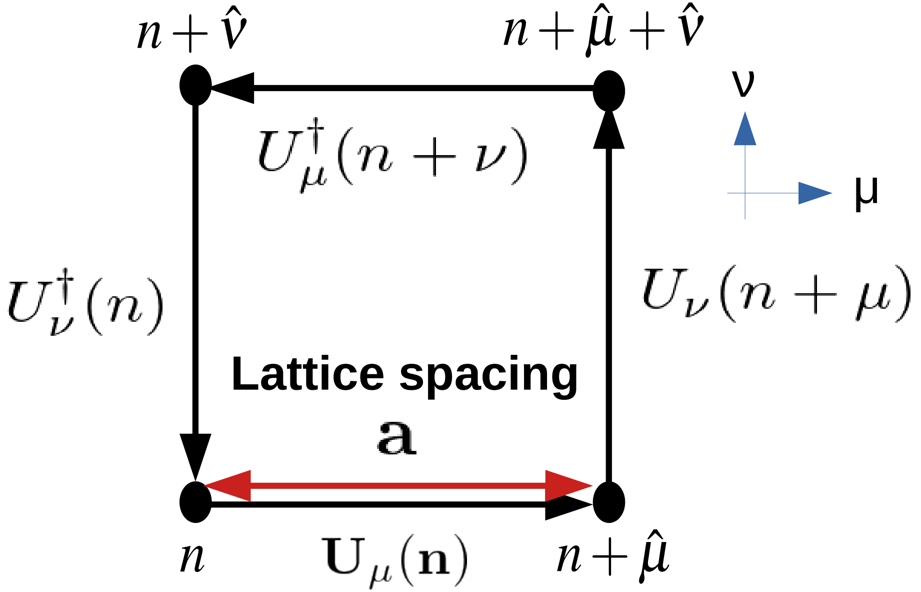
- We add a monopole and anti-monopole pair to gauge field configurations of quenched approximation in SU(3).
- We generate the standard configurations and the configurations with the additional monopoles and anti-monopoles.
- To change the number of monopoles and anti-monopoles, we vary the magnetic charges.
- The magnetic charges of the monopole m_c (positive) range from **0** to **5**.
- The magnetic charges of the anti-monopoles $-m_c$ (negative) range from **0** to **-5**. Hereafter, the magnetic charge m_c indicates that both charges are added.
- The electric charge **g** is added.

Numerical calculations

First, we generate the SU(3) normal configurations as follows:

- The action is the plaquette (Wilson) gauge action ***S***.
- The lattice spacing is calculated [NPB 622 (2002) 328].
- Sommer scale $r_0 = 0.5$ [fm].
- Standard Monte Carlo simulations in which the gauge links ***U*** are updated using the heat bath and overrelaxation methods.

- **Plaquette gauge action**



$$S = \sum_{n,\ \mu\neq\nu} \beta \mathrm{tr}[U_\mu(n)U_\nu(n+\mu)U^\dagger_\mu(n+\nu)U^\dagger_\nu(n)]$$

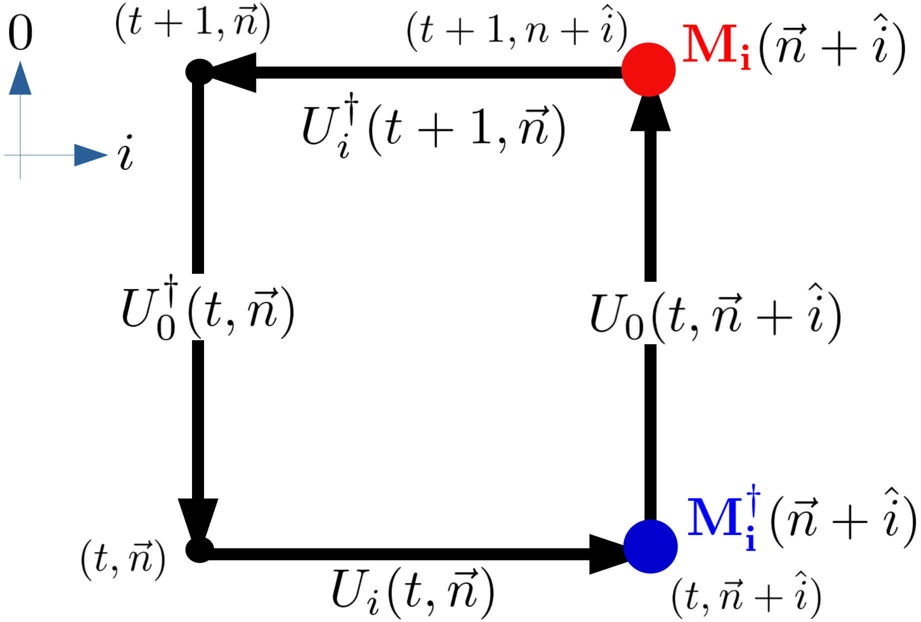
$$= 2\sum_{n,\ \mu<\nu} \beta \mathrm{Retr}[U_{\mu\nu}(n)]$$

Monopole creation operator

Next, we create a pair of monopole and anti-monopole in the QCD vacuum as follows:

- The monopole creation operator $\bar{\mu} = \exp\left(-\beta \overline{\Delta S}\right)$ acts on the vacuum.
- The plaquette action ***S*** is shifted as follows: $\mathbf{S} \to \mathbf{S} + \overline{\mathbf{\Delta S}}$
- A static Abelian monopole and anti-monopole **(Wu-Yang form)** pair is created [PRD 85 (2012) 065001].

- **The shifted plaquette action**



$$\mathbf{M_i}(\vec{n}) = \exp(iA_i^0(\vec{n}-\vec{x}_1))$$

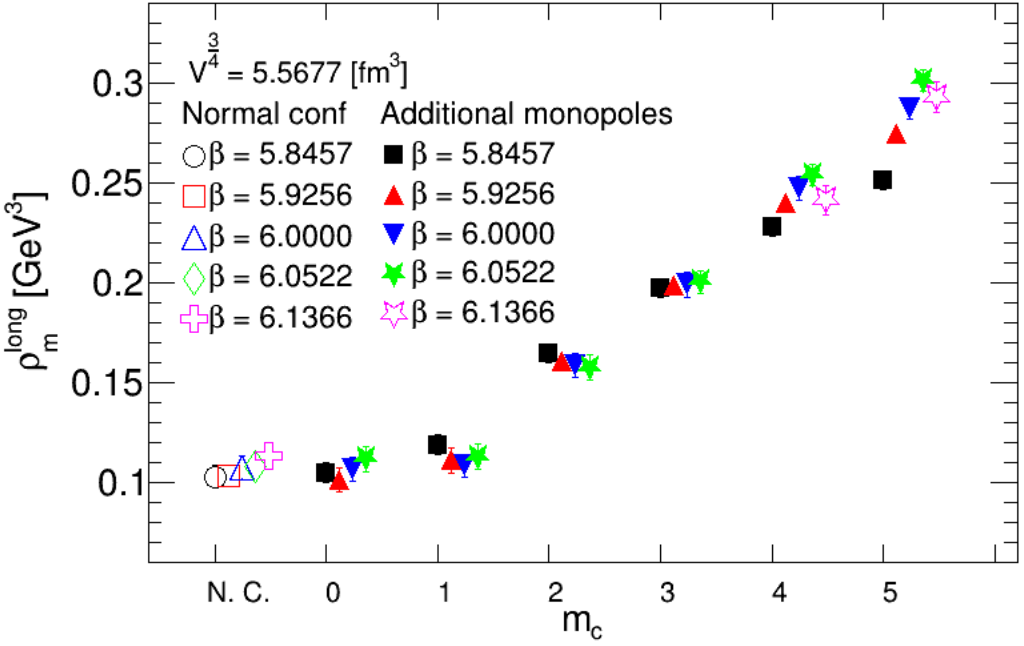
$$\mathbf{M_i^\dagger}(\vec{n}) = \exp(-iA_i^0(\vec{n}-\vec{x}_2))$$

Number density of long monopole loops

- The monopole current satisfies the current conservation law. Therefore, the monopole currents form closed loops [NPB PS 34 (1994) 549].
- The monopole loops are classified into two clusters; long loops and short loops.
- The definition of the number density of the long monopole loops is as follows:

$$\rho_m^{long} \equiv \left(\frac{1}{12V} \sum_{i,\mu} \sum_{^*n \in C} |k_\mu^i(^*n)| \right)^{\frac{3}{4}} \ [\mathrm{GeV}^3]$$

[EPJC 82 (2022) 1040]



Overlap Dirac operator

- Chiral symmetry: $\mathcal{L} = \bar{\psi} D \psi, \ \gamma_5 D + D\gamma_5 = 0.$
- The Dirac operator ***D***, which preserves the exact chiral symmetry in lattice gauge theory satisfies the following Ginsparg-Wilson relation [P. H. Ginsparg and K. G. Wilson PRD 25 (1982) 2649]: $\gamma_\mathbf{5}\mathbf{D} + \mathbf{D}\gamma_\mathbf{5} = \mathbf{aDR}\gamma_\mathbf{5}\mathbf{D}.$
- We calculate **overlap Dirac operator** ***D*** that satisfies the Ginsparg-Wilson relation [H. Neuberger, PLB 427 (1998) 353]:

$$\mathbf{D}(\rho) = \frac{\rho}{\mathbf{a}}\{\mathbf{1} + \gamma_\mathbf{5}\epsilon(\mathbf{H_W}(\rho))\}$$

- The sign function ***ϵ*** is approximated by the Chebyshev polynomials [Com. Phys. Comm. 153 (2003) 31].
- We solve the eigenvalue problems using ARPACK: $\mathbf{D}(\rho)|\psi_\mathbf{i}\rangle = \lambda_\mathbf{i}|\psi_\mathbf{i}\rangle$
- We compute pairs of eigenvalues $\boldsymbol{\lambda}_i$ and eigenvectors $|\psi_\mathbf{i}\rangle$ from the lowest energy level to approximately 100.

Operators and correlation functions

- **Quark propagator:**

$$G(\vec{y}, y^0; \vec{x}, x^0) \equiv \sum_i \frac{\psi_i(\vec{x}, x^0)\psi_i^\dagger(\vec{y}, y^0)}{\lambda_i^{mass}}$$

- λ_i^{mass} **of massive Dirac operator:**

$$\lambda_i^{mass} = \left(1 - \frac{a\bar{m}_q}{2\rho}\right)\lambda_i + \bar{m}_q$$

Light quark masses

- **Pion:** $\bar{m}_{ud} \equiv \dfrac{m_u + m_d}{2}$

- **Kaon:** $\bar{m}_{sud} \equiv \dfrac{m_s + \bar{m}_{ud}}{2}$

- **Scalar:** $\mathcal{O}_S = \bar{\psi}_1 \left(1 - \frac{a}{2\rho} D\right) \psi_2$

- **Pseudoscalar:** $\mathcal{O}_{PS} = \bar{\psi}_1 \gamma_5 \left(1 - \frac{a}{2\rho} D\right) \psi_2$

- **Scalar density:**

$$C_{SS}(\Delta t) = \frac{a^3}{V} \sum_{\vec{x}_1} \sum_{\vec{x}_2,\ t} \langle \mathcal{O}_S^C(\vec{x}_2, t) \mathcal{O}_S(\vec{x}_1, t + \Delta t) \rangle$$

- **Pseudoscalar density:**

$$C_{PS}(\Delta t) = \frac{a^3}{V} \sum_{\vec{x}_1} \sum_{\vec{x}_2,\ t} \langle \mathcal{O}_{PS}^C(\vec{x}_2, t) \mathcal{O}_{PS}(\vec{x}_1, t + \Delta t) \rangle$$

- **Correlation function** [PRD 69 (2004) 074502]:

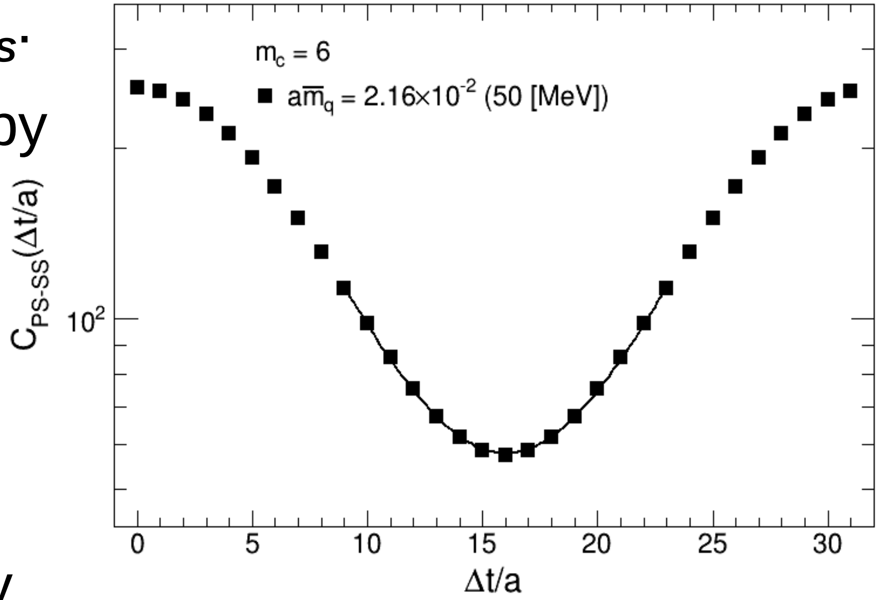
$$C_{PS-SS}(\Delta t) \equiv C_{PS}(\Delta t) - C_{SS}(\Delta t)$$

Fitting to the correlations

- The bar quark mass range is $\bar{\mathbf{m}}_{\mathbf{q}} = 30 - 150\ [\mathrm{MeV}]$.
- We compute the correlation function $\boldsymbol{C_{PS\text{-}SS}}$.
- We obtain parameters $\boldsymbol{a^4G_{PS\text{-}SS}}$ and $\boldsymbol{am_{PS}}$ by fitting the following function:

$$C_{PS-SS}(t) = \frac{a^4 G_{PS-SS}}{am_{PS}} \exp\left(-\frac{m_{PS}}{2}T\right) \cosh\left[m_{PS}\left(\frac{T}{2} - t\right)\right].$$

- We then evaluate chiral condensate, light quark masses, meson masses, and decay constants using the fitting results $\boldsymbol{a^4G_{PS\text{-}SS}}$ and $\boldsymbol{am_{PS}}$.

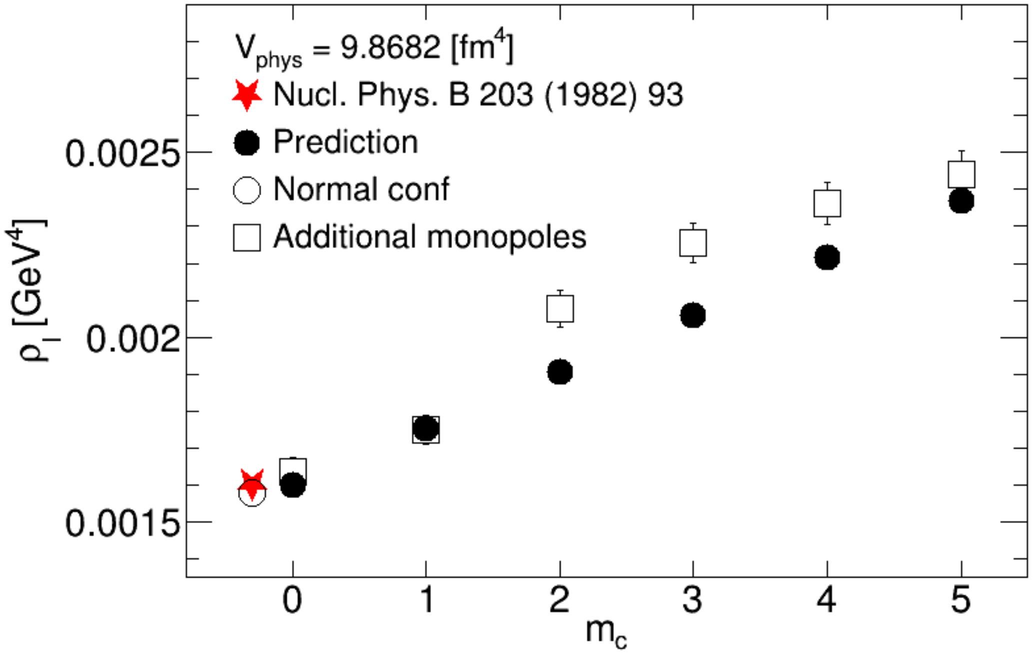


Fitting result:

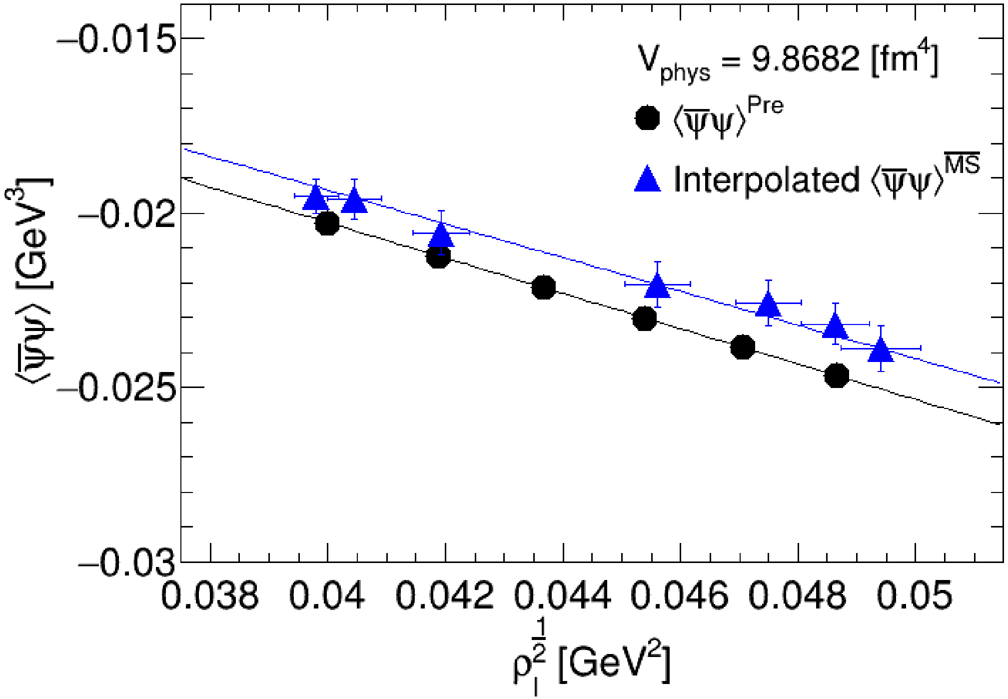
χ^2/d.o.f. ≈ 0.6 (FR Δt/a = 9-23).

Instantons and chiral symmetry breaking

- **Instanton density**



- **Chiral condensate (GMOR)**



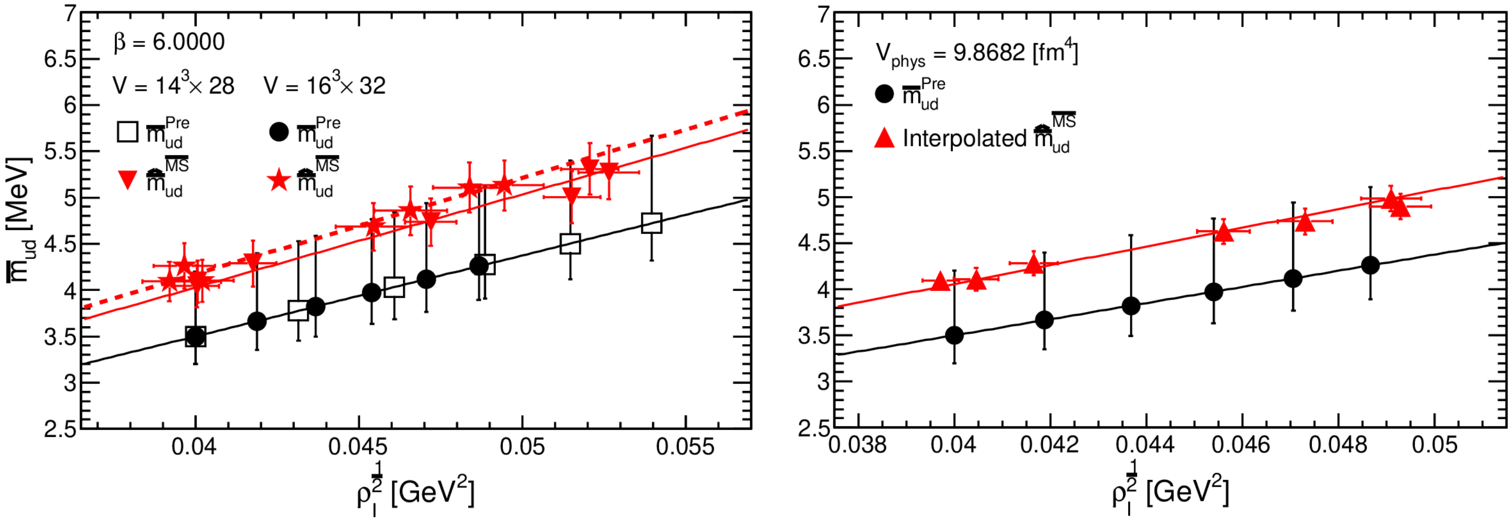
[EPJC 82 (2022) 1040]

Instanton effects on quark mass

- The prediction of the light quark masses is as follows:

$$\bar{m}_{ud}^{\mathrm{Pre}}(m_c) = \bar{m}_{ud}^{\mathrm{Exp}} \left[\frac{\rho_I^{\mathrm{Pre}}(m_c)}{\rho_I^{\mathrm{sta}}} \right]^{\frac{1}{2}}$$

[EPJC 82 (2022) 1040]



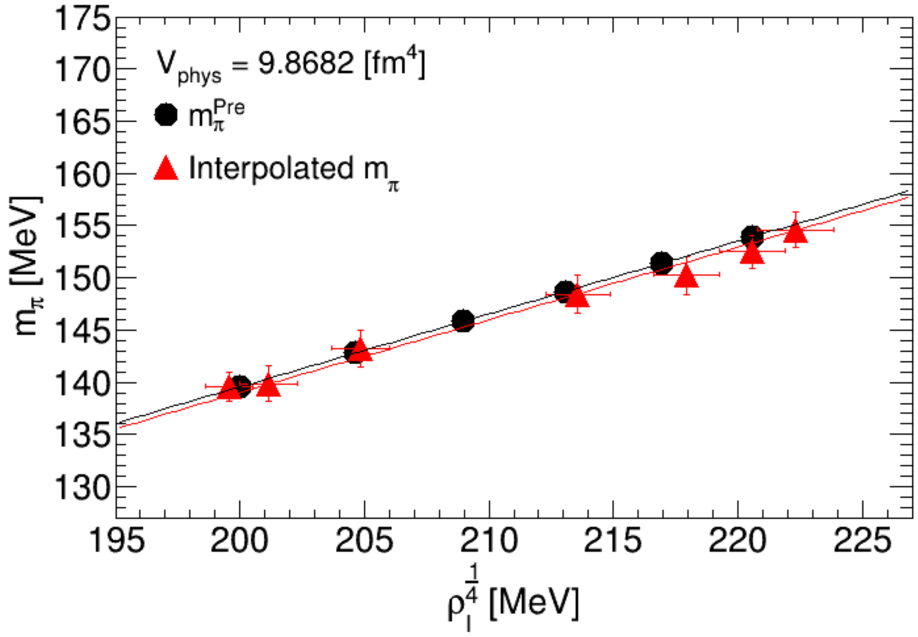
- The light quark masses increase in **direct proportion** to the **square root** of the number density of the instantons and anti-instantons.

Instanton effects on pion mass

- From the PCAC relation, suppose that the pion mass increases in direct proportion to the one-fourth root of the number density of the instantons and anti-instantons. $(am_{PS})^2 = Aa\bar{m}_q$
- The prediction of the pion mass:

$$m_{\pi}^{\rm Pre}(m_c) = m_{\pi^{\pm}}^{\rm Exp}\left[\frac{\rho_I^{\rm Pre}(m_c)}{\rho_I^{\rm sta}}\right]^{\frac{1}{4}}$$

- The pion mass increases in **direct proportion** to the **one-fourth root** of the number density of the instantons and anti-instantons.



[EPJC 82 (2022) 1040]

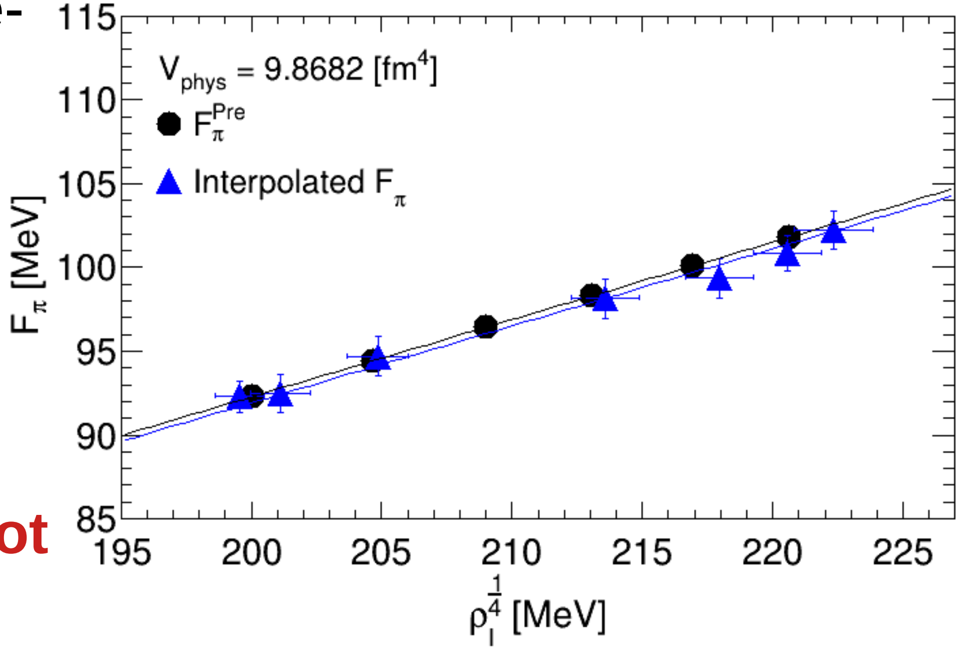
Instanton effects on pion decay constant

- The formula in the quenched chiral perturbation theory in SU(2) holds. Suppose that the pion decay constant increases in direct proportion to the one-fourth root of the number density of the instantons and anti-instantons.

$$aF_{PS} = \frac{2a\bar{m}_q\sqrt{a^4 G_{PS-SS}}}{(am_{PS})^2}, \quad (F_\pi = 93 \text{ [MeV]}).$$

- Prediction of the pion decay constant:

$$F_\pi^{\mathrm{Pre}}(m_c) = \frac{F_{\pi^-}^{\mathrm{Exp}}}{\sqrt{2}} \left[\frac{\rho_I^{\mathrm{Pre}}(m_c)}{\rho_I^{\mathrm{sta}}}\right]^{\frac{1}{4}}$$



- The pion decay constant increases in **direct proportion** to the **one-fourth root** of the number density of the instantons and anti-instantons.

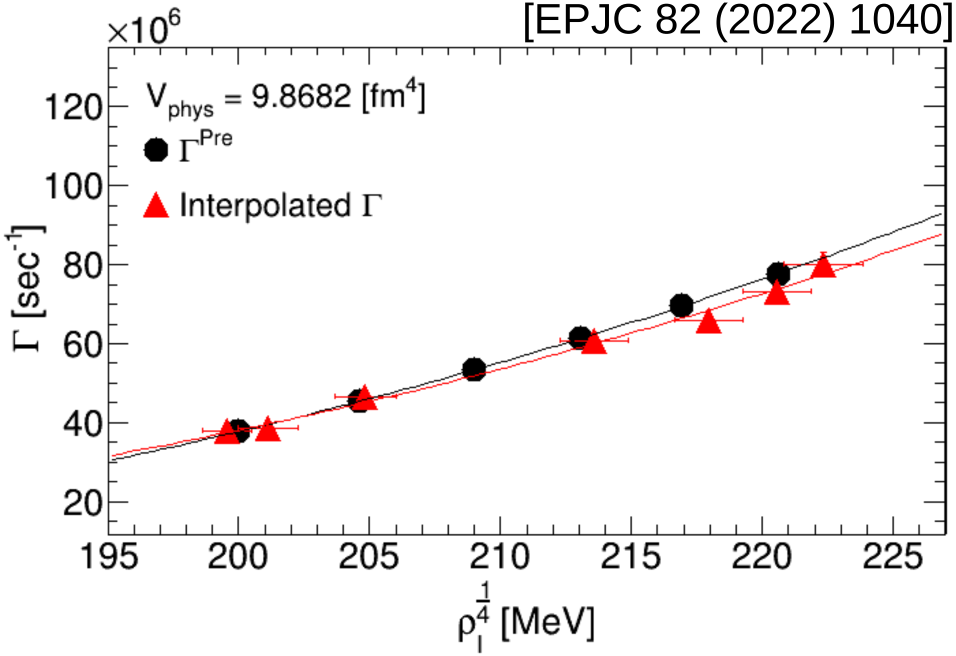
[EPJC 82 (2022) 1040]

Catalytic effect on the pion decay

- One charged pion decays to an electron or a **muon** and a neutrino as follows: $\pi^+ \to l^+ + \nu_l,\ \pi^- \to l^- + \bar{\nu}_l$
- The decay width of the charged pion is derived [textbook, T. Kugo] as follows:
$$\Gamma(\pi^- \to l + \bar{\nu}_l) = \frac{(G_F F_\pi \cos\theta_c)^2}{4\pi m_\pi^3} m_l^2 (m_\pi^2 - m_l^2)^2.$$
- We approximate the total decay width of the charged pion from the partial decay width, where the charged pion decays to the **muon**.
- The lifetime of charged pion is estimated as follows: $\tau = \frac{\mathbf{1}}{\mathbf{\Gamma}}.$
- We substitute the numerical results of the pion mass and pion decay constant and estimate the catalytic effect on the charged pion.
- To evaluate the catalytic effect, we fit these curves:
$$y(x) = p_1 x^3 - p_2 x + \frac{p_3}{x},\ y(x)^{-1},\ x = \left(\frac{N_I}{V}\right)^{\frac{1}{4}}$$

Catalytic effect on the charged pion

[EPJC 82 (2022) 1040]



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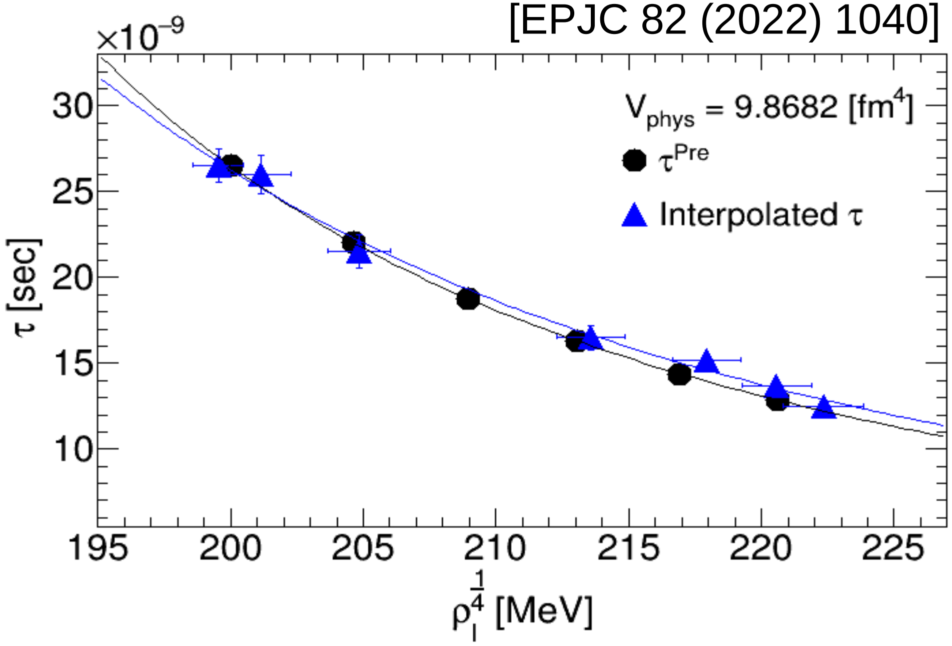


- The decay width of the charged pion becomes wider than the experimental outcome.
- The lifetime of the charged pion becomes shorter than the experimental outcome.

Conclusions

- The monopole creation operator makes only the long monopole loops, which are the crucial elements for the mechanism of color confinement.
- The added monopoles and anti-monopoles create instantons and anti-instantons.
- Chiral condensate decreases in direct proportion to the **square root** of the number density of the instantons and anti-instantons.
- The mass and decay constant of the pion increase in direct proportion to the **one-fourth root** of the number density of the instantons and anti-instantons.
- The decay width of the charged pion becomes wider and the lifetime of the charged pion becomes shorter by increasing the number density of the instantons and anti-instantons.

Acknowledgments

- I performed simulations using supercomputers (SX-series, OCTOPUS, SQUID, SR, XC40) and PC-clusters at the Research Center for Nuclear Physics (RCNP) and Cybermedia Center (CMC) at Osaka University and the Yukawa Institute for Theoretical Physics at Kyoto University.
- I use the storage elements of the Japan Lattice Data Grid at the RCNP.
- I appreciate the computer resources and technical support that these facilities provided for this project.

Monopole creation operator

- **Abelian monopole (Wu-Yang form)** [PRD 91 (2015) 054512]

$$\textbf{Monopole}: \mathbf{M_i}(\vec{n}) = \exp(iA_i^0(\vec{n} - \vec{x}_1))$$

$$\textbf{Anti-monopole}: \mathbf{M_i^\dagger}(\vec{n}) = \exp(-iA_i^0(\vec{n} - \vec{x}_2))$$

$(i)\ n_z - z \geqq 0$

$$\begin{pmatrix} A_x^0 \\ A_y^0 \\ A_z^0 \end{pmatrix} = \begin{pmatrix} \frac{\mathbf{m_c}}{2\mathbf{g}r}\frac{\sin\phi(1+\cos\theta)}{\sin\theta}\lambda_3 \\ -\frac{m_c}{2gr}\frac{\cos\phi(1+\cos\theta)}{\sin\theta}\lambda_3 \\ 0 \end{pmatrix}$$

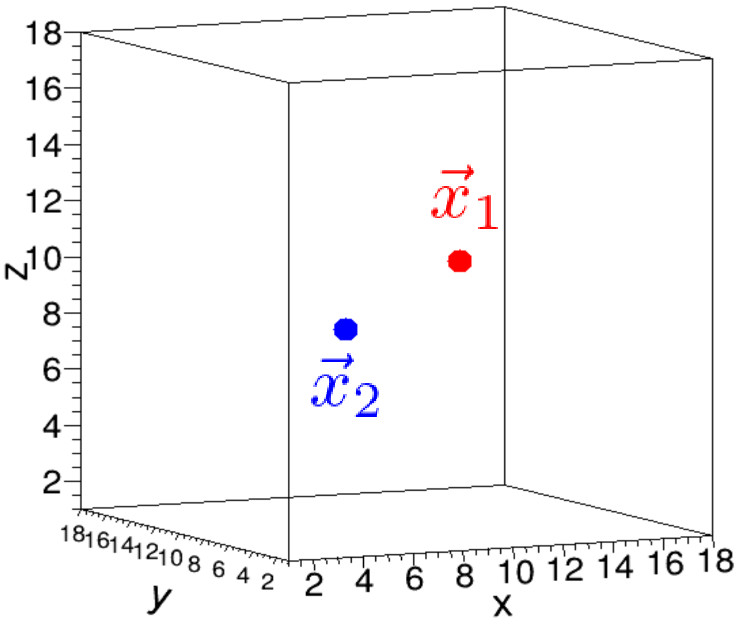
$(ii)\ n_z - z < 0$

$$\begin{pmatrix} A_x^0 \\ A_y^0 \\ A_z^0 \end{pmatrix} = \begin{pmatrix} -\frac{m_c}{2gr}\frac{\sin\phi(1-\cos\theta)}{\sin\theta}\lambda_3 \\ \frac{m_c}{2gr}\frac{\cos\phi(1-\cos\theta)}{\sin\theta}\lambda_3 \\ 0 \end{pmatrix}$$

$$\mathbf{g} = \sqrt{\frac{6}{\beta}} : \text{(gauge coupling constant)}$$

- **Location of the monopole and anti-monopole**

$$D = |\vec{x}_1 - \vec{x}_2| \approx 1.1\ [\text{fm}]$$



Instanton effects

- **To compare the numerical results with the experimental results, we improve the determination method of the lattice spacing** [NPB 489 (1997) 427, PRD 64 (2001) 114508]**.**
- We set the scale of the lattice using analytical function [NPB 622 (2002) 328].
- We match the numerical results of the decay constants and the square of the masses with the experimental results of the pion and kaon and determine the **normalization factors**.
- We evaluate the instanton effects on chiral condensate, the light quark masses, mesons masses, and decay constants of mesons.
- Finally, we estimate the catalytic effect on the charged pion.