Nonperturbative corrections and checking of the hypothesis of vacuum dominance

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Plan of the presentation

- **Introduction**: existing studies of non-perturbative corrections and hypothesis of vacuum dominance
- **Theoretical framework:** R-ratio, Adler function, Borel transform and processing of the experimental data
- **Exploring of the condensates in OPE**: C_2, C_4, C_6. Checking of the possibility of other intermediate states, contributing to four-quark condensate. How other nonperturbative corrections (C_2 and C_4 or gluon condensate) change when the additional contribution of intermediate states is taken into account?
- **Summary**

Introduction

- Studies of quark and gluon condensates in the framework of operator product expansion (OPE) were initiated by M.A. Shifman, A.I. Vainstein and V.I. Zakharov and reflected in the seminal review (Nucl. Phys. B 147 (1979), 385-447). Nonperturbative effects are related to non-vanishing vacuum average values for the local O_n operators in OPE. Also these topics are discussed in work of B.L.Ioffe and A.V.Smilga (Nucl. Phys. B 216 (1983), 373-407).
- Our previous works (EPJ Web Conf. 138 (2017), 02006; EPJ Web Conf. 204 (2019), 02005; Nonlin. Phenom. Complex Syst. 22 (2019) no.2, 151-163) were mostly devoted to the operator C_2 (short string), while the quark condensate or C_6 was taken as a constant value. As a result, it is shown that C_2 values. are strongly (anti)correlated with C_4 and compared with zero.
- How this picture changes when the quark condensate is not fixed? In this work, we estimate the range in which the value of C_6 can vary, comparing C_4 to the available estimates (SVZ, Geshkenbein, Ioffe).
- As a reference point we use the vacuum dominance approach, when C_6 is related to the square of the quark condensate estimated of using the Gell-Mann-Oakes-Renner (GMOR) formula (Phys. Rev. 175 (1968), 2195-2199) for the two lightest flavors:

$$(m_u + m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle = -m_\pi^2 f_\pi^2$$

  where m_π is pion mass and f_π is the decay constant. We use the data from FLAG Review 2021, estimate quark condensate $\langle 0|\bar{q}q|0\rangle \approx -(0.290 \pm 1.1\ \mathrm{MeV})^3$ and calculate the coefficient C_6(through 4-quark operator) which we fix:

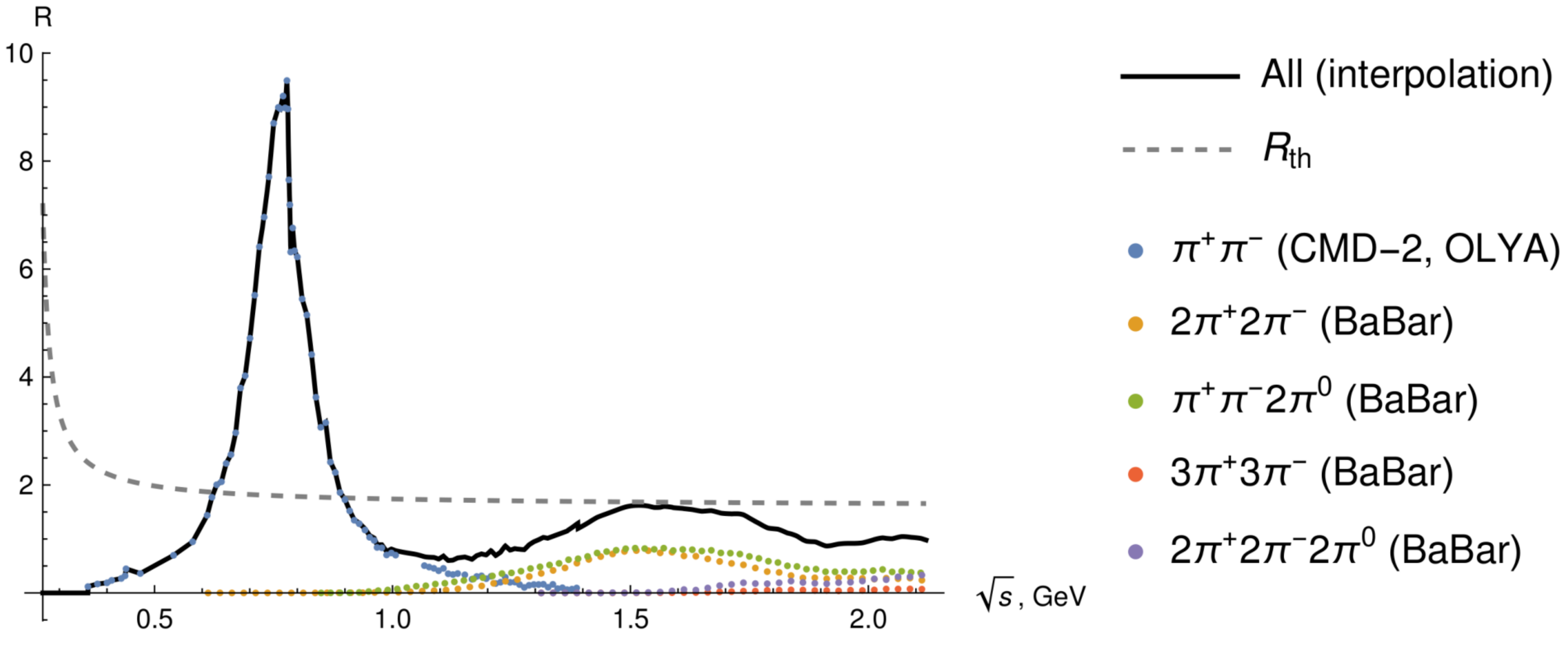
$$C_6 = -\frac{448\pi^3}{27}\alpha_s\langle 0|\bar{q}q|0\rangle^2 \approx -0.104 \pm 0.002\ \mathrm{GeV}^6 \text{ at 2 GeV}$$

- The purpose of the work is to check how vacuum dominance works: the possibility of other intermediate states, contributing to four-quark condensate. How other nonperturbative corrections (C_2 and C_4 or gluon condensate) change?

Theoretical framework: R-ratio

- R-ratio by definition: $R(s) = \frac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}}$
- Total ratio is $R(s) = \sum_k R_k(s)$, where k is the channel number.
- experimental data on e^+e^--annihilation into even number of pions: **CMD-2, OLYA and BaBar**.

Fig. 1. The experimental data for different channels, the total R -ratio as sum of all interpolated data and theoretical prediction. The decline of the total R -ratio at values $\sqrt{s} \geq 1.5$ GeV is explained by the fact that there are no data on e^+e^--annihilation into 8 pions and more.



Theoretical framework: Adler function

- Adler function can be constructed by two ways, by dispersional approach:

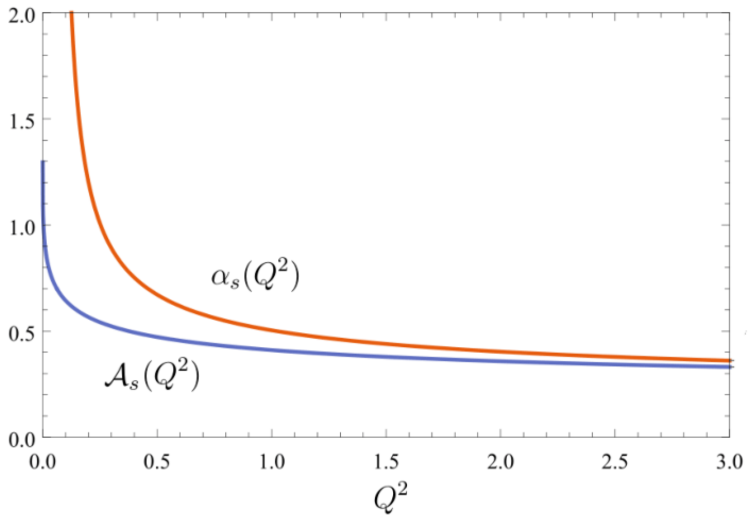
$$D_{\rm exp}(Q^2) = Q^2 \int_{4\,m_\pi^2}^{\infty} \frac{R_{\text{exp-th}}(s)\, ds}{(s+Q^2)^2} \qquad R_{\rm th}^{\rm PT/APT}(s) = N_c \sum_q e_q^2 \left(1 + \frac{\alpha_s^{\rm PT/APT}(s)}{\pi}\right)$$

where $R_{\text{exp-th}}(s) = R_{\rm exp}(s)\theta(s-s_0) + R_{\rm th}(s)\theta(s_0 - s)$

and s_0 is continuum threshold.

And by OPE in **PT** and **APT** (D. V. Shirkov, I. L. Solovtsov, Phys. Rev. Lett. 79, 1209 (1997); K. A. Milton andI. L. Solovtsov, Phys. Rev. D 55, 5295 (1997)):

$$D_{\rm PT(APT)+OPE} = N_c \sum_q e_q^2 \left[1 + \frac{\alpha_s^{PT(APT)}(Q^2)}{\pi} + \sum_{n\geq 1} \Gamma(n) \frac{c_{2n}}{Q^{2n}}\right],$$



$$\textbf{PT} \Rightarrow \alpha_s(Q^2) = \frac{4\pi}{b_0}\left[\frac{1}{\ln(Q^2/\Lambda^2)}\right], \quad \textbf{APT} \Rightarrow \mathcal{A}_s(Q^2) = \frac{4\pi}{b_0}\left[\frac{1}{\ln(Q^2/\Lambda_{\rm APT}^2)} - \frac{\Lambda_{\rm APT}^2}{Q^2 - \Lambda_{\rm APT}^2}\right].$$

where $N_c = 3$, and for $I = 1$ we get $\sum_q e_q^2 = (1/2)^2 + (1/2)^2$, $n = 1, 2, 3$, $b_0 = 11 - 2N_f/3$ is the first coefficient of the β-function expansion and Λ is the QCD scale parameter, which we choose as 0.25 GeV.

Defining $C_n = \Gamma(n) c_n$, we get $D_{\rm PT(APT)+OPE} = \frac{3}{2}\left[1 + \frac{\alpha_s^{PT(APT)}(Q^2)}{\pi} + \frac{C_2}{Q^2} + \frac{C_4}{Q^4} + \frac{C_6}{Q^6}\right]$

Theoretical framework

- The continuum threshold s_0, we fixed from the local quark-hadron duality. The corresponding duality relation reads

$$I_{\rm dual}(s_0) \equiv \int\limits_{4\,m_\pi^2}^{s_0} ds\, R_{\rm exp}(s)) = \int\limits_{4\,m_\pi^2}^{s_0} ds\, R_{\rm th}(s)$$

- Using the experimental data and the perturbative ansatz in the leading order (LO) and in the next to leading order (NLO), we find that the best value is $s_0 \simeq 1.51^2\mathrm{GeV}^2$: we show integrals as functions of the upper limit of the integration in **Fig. 2**.

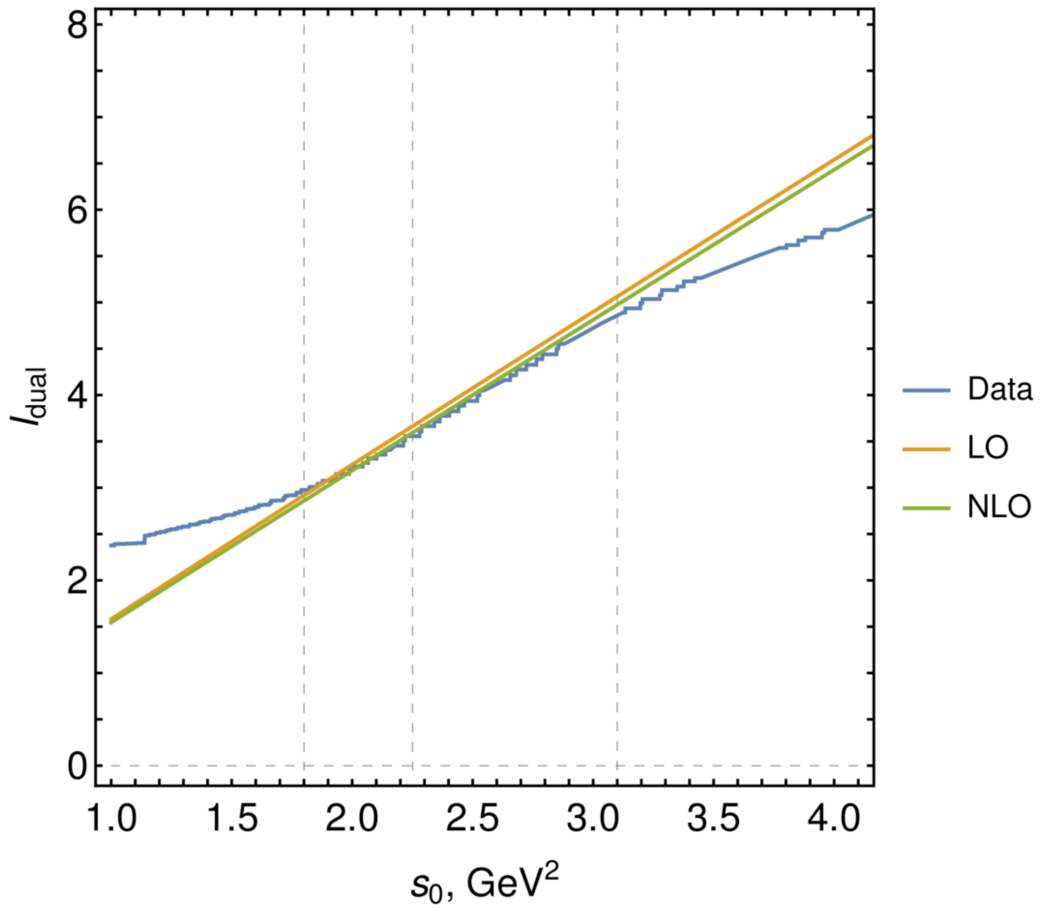


Fig.2. The integral $I_{\rm dual}$ versus the upper integration limit, s_0 . The integral for the experimental data corresponds to the blue line (Data). The theoretical curves are the orange (LO) and green (NLO) lines.

Theoretical framework: Borel transform

- In order to smooth the function and reduce errors we apply the Borel transform.
- The Borel transform of the function f by definition is:

$$\hat{B}_{x\to y}[f(x)] = \lim_{n\to\infty} \frac{(-x)^n}{\Gamma(n)} \left[\frac{d^n}{dx^n} f(x)\right]_{x=ny}.$$

- The Borel transformed forms in PT and APT of the D-function are

$$\hat{B}_{Q^2\to M^2}[D_{\text{exp}}(Q^2)] = \Phi_{\text{exp}}(M^2) = \int_0^\infty R_{\text{exp-th}}(s) \left(1 - \frac{s}{M^2}\right) e^{-s/M^2} \frac{ds}{M^2},$$

$$\hat{B}_{Q^2\to M^2}[D_{\text{PT+OPE}}(Q^2)] = \Phi_{\text{PT+OPE}}(M^2) = \frac{3}{2}\frac{\hat{B}_{Q^2\to M^2}[\alpha_s(Q^2)]}{\pi} + \frac{3}{2}\left(\frac{C_2}{M^2} + \frac{C_4}{M^4} + \frac{C_6}{M^6}\right).$$

- The Borel transform of the running coupling give the following results:

$$\hat{B}_{Q^2\to M^2}\left[\alpha_s(Q^2)\right] = \frac{4\pi}{b_0}\left[\frac{1}{M^2}\int_0^\infty \frac{e^{-s/M^2} ds}{\ln^2(s/\Lambda^2) + \pi^2} + \frac{\Lambda^2}{M^2} e^{\Lambda^2/M^2}\right] \qquad \hat{B}_{Q^2\to M^2}\left[\mathcal{A}_s(Q^2)\right] = \frac{4\pi}{b_0}\left[\frac{1}{M^2}\int_0^\infty \frac{e^{-s/M^2} ds}{\ln^2(s/\Lambda^2) + \pi^2}\right].$$

- Equating the two forms of the Adler function, we get the sum rule: $\Phi_{\text{exp}}(M^2) = \Phi_{\text{PT(APT)+OPE}}(M^2)$.
- Using the method of Monte-Carlo generating of events and simulating measurements with their statistic and systematic errors, we extract coefficients C_2, C_4 and C_6. In this work usually - by fixing one of them and calculating others. We control the errors of the experimental data more precisely.

Exploring of the condensates I

- Let us start the analysis from the consideration of quark condensate in vacuum dominance approach defined by GMOR formula and data from FLAG Review 2021 with $\langle 0|\bar{q}q|0\rangle \approx -(0.290 \pm 1.1\ \mathrm{MeV})^3$, corresponding to $C_6 = -\frac{448\pi^3}{27}\alpha_s\langle 0|\bar{q}q|0\rangle^2 \approx -0.104 \pm 0.002\ \mathrm{GeV}^6$ at 2 GeV.
- The value of M = 2 GeV is outside the selected region on Borel mass, [0.75;3] GeV^2, however, the $C_6 \sim \alpha_s\langle 0|\bar{q}q|0\rangle^2$ is the renorm-invariant value with good precision, therefore, we can use it at values in the selected region.
- When the found $C_6 \approx -0.104\ \mathrm{GeV}^6$ is fixed, the corresponding regions of C_2 and gluon condensate at 1σ, 2σ and 3σ are calculated and shown on **Fig. 3**, first panel.
- We explore the deviations from vacuum dominance approach by taking into account of the intermediate states different from the vacuum one. This effect is parameterized by multiplication factors for value of C_6 .

Exploring: fixing of C6 at different values

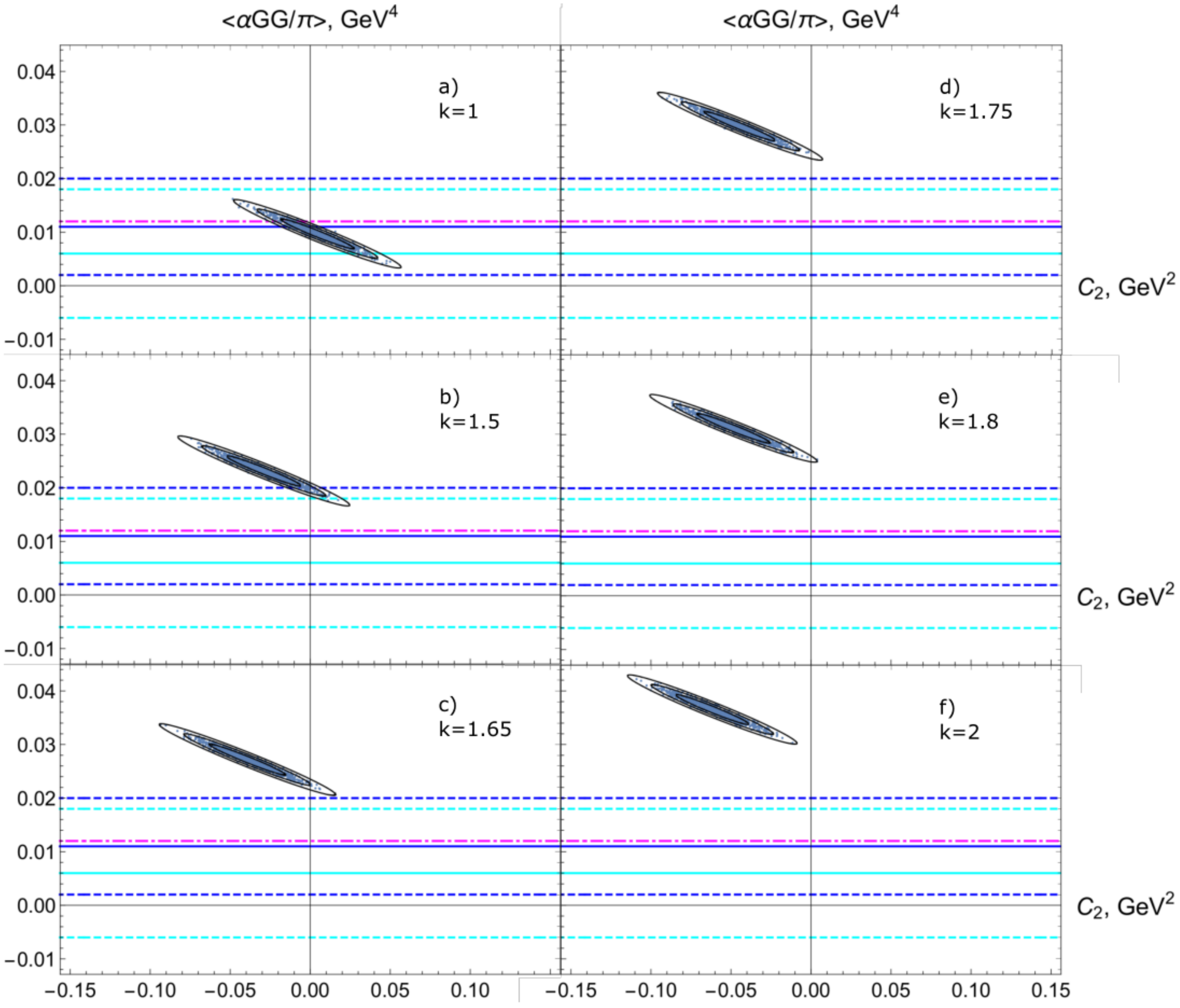


FIG. 3. Allowed regions of C_2 and gluon condensate with various multiplication factors for quark condensate (or C_6) within 1, 2 and 3 standard deviations. The values for gluon condensate from the literature together with corresponding limits are also shown.

TABLE I: The extracted values for the interval of M^2 : $[0.75, 3]$ GeV^2 **in PT**. The obtained values of condensates with errors, $\Lambda = 0.25$ GeV, $s_0 = 1.51^2$ GeV^2 and using $C_6^{\text{GMOR, FLAG}} = -0.104$ GeV^6 fixed. In the last column the estimate of anticorrelation between gluon condensate and C_2 is given.

C_6	C_2, GeV^2	C_4, GeV^4	$\langle\frac{\alpha_s GG}{\pi}\rangle$, GeV^4	χ^2	$\langle\alpha_s GG/\pi\rangle(C_2)$, GeV^4
C_6^{GMOR}	0.004 ± 0.015	0.064 ± 0.012	0.010 ± 0.002	0.936	$-0.122C_2 + 0.010$
$C_6^{\text{GMOR}} \times 1.5$	-0.029 ∓ 0.016	0.152 ± 0.013	0.023 ± 0.002	0.428	$-0.122C_2 + 0.020$
$C_6^{\text{GMOR}} \times 1.65$	-0.039 ∓ 0.016	0.179 ± 0.013	0.027 ± 0.002	0.415	$-0.121C_2 + 0.022$
$C_6^{\text{GMOR}} \times 1.75$	-0.045 ∓ 0.015	0.196 ± 0.012	0.030 ± 0.002	0.443	$-0.123C_2 + 0.024$
$C_6^{\text{GMOR}} \times 1.8$	-0.048 ∓ 0.015	0.205 ± 0.012	0.031 ± 0.002	0.464	$-0.120C_2 + 0.025$
$C_6^{\text{GMOR}} \times 2.0$	-0.062 ∓ 0.015	0.240 ± 0.012	0.037 ± 0.002	0.631	$-0.122C_2 + 0.029$

Exploring of the condensates II

- Let us now find the allowed domains for C_2 and C_6 calculated at various fixed values of the gluon condensate, see **Fig. 4**, ranging from the value $\left\langle 0 \left| \frac{\alpha_s}{\pi} GG \right| 0 \right\rangle = 0.006\ \mathrm{GeV}^4$ (Ioffe, Zyabluk) to $\left\langle 0 \left| \frac{\alpha_s}{\pi} GG \right| 0 \right\rangle = 0.012\ \mathrm{GeV}^4$ (SVZ). We also show the available estimates of the quark condensate converted to C_6 (green lines) for comparison — our estimate using the GMOR formula, the value given in review SVZ and the value of Geshkenbein, Ioffe, Zyablyuk.
- It can be seen that C_2 is compatible to zero within one standard deviation whereas our calculation of the value of C_6 gives a reasonable value using the GMOR formula. Furthermore, it is possible to increase C_6 by 20%, especially in the case of $\left\langle 0 \left| \frac{\alpha_s}{\pi} GG \right| 0 \right\rangle = 0.012\ \mathrm{GeV}^4$ (SVZ). As soon as the gluon condensate increases to large values, C_2 becomes essentially negative.

Exploring: fixing of C4 at different values

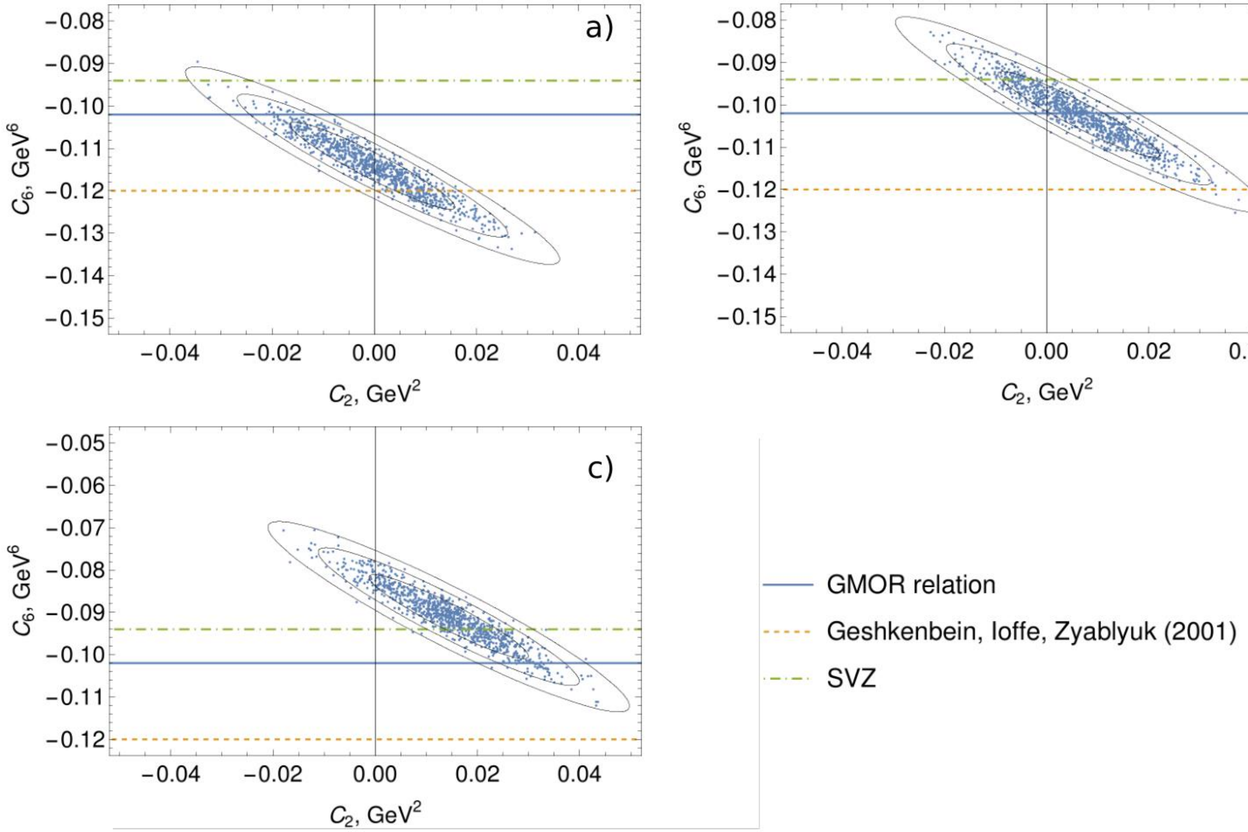


FIG. 4. The regions of existence of condensates C_2 and C_6 for different fixed C_4 - from SVZ values to Ioffe-Zyabluk. The known values of quark condensate recalculated into C_6 are shown.

TABLE III: The extracted values for the interval of M^2 : $[0.75, 3]$ GeV^2 **in PT**. The obtained values of condensates with errors, $\Lambda = 0.25$ GeV, $s_0 = 1.51^2$ GeV^2 and different C_4 fixed. In the last column the estimate of anticorrelation between C_6 and C_2 is given.

C_4	$\langle\frac{\alpha_s GG}{\pi}\rangle$, GeV^4	C_2, GeV^2	C_6, GeV^6	χ^2	$C_6(C_2)$, GeV^6
C_4^{SVZ}	0.012	-0.0004 ∓ 0.011	-0.114 ∓ 0.007	0.786	$-0.637C_2 - 0.114$
C_4^{mid}	0.009	0.006 ± 0.010	-0.102 ∓ 0.007	0.969	$-0.642C_2 - 0.098$
C_4^{IZ}	0.006	0.014 ± 0.010	-0.091 ∓ 0.007	1.181	$-0.636C_2 - 0.082$

Exploring of the condensates III

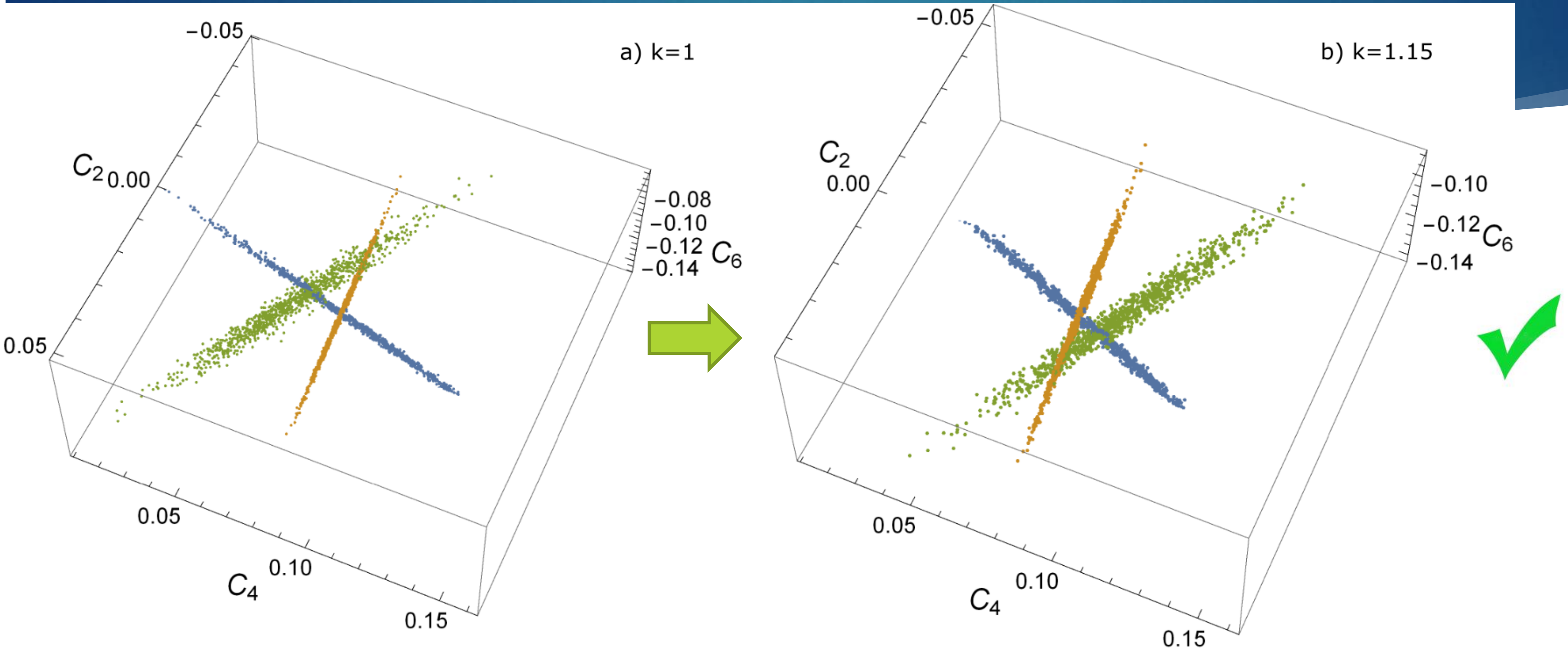


FIG. 5. The allowed regions for condensates $C_2 - C_4$, $C_2 - C_6$ and $C_4 - C_6$ for fixed values C_6 , C_4 and C_2 respectively. On the left panel we take the value 1.15 × C_6 - all three regions intersect at one point. Hypothesis of vacuum dominance performed with an accuracy of 15% (comparing SVZ).

Exploring: fixing of C6 at different values in APT

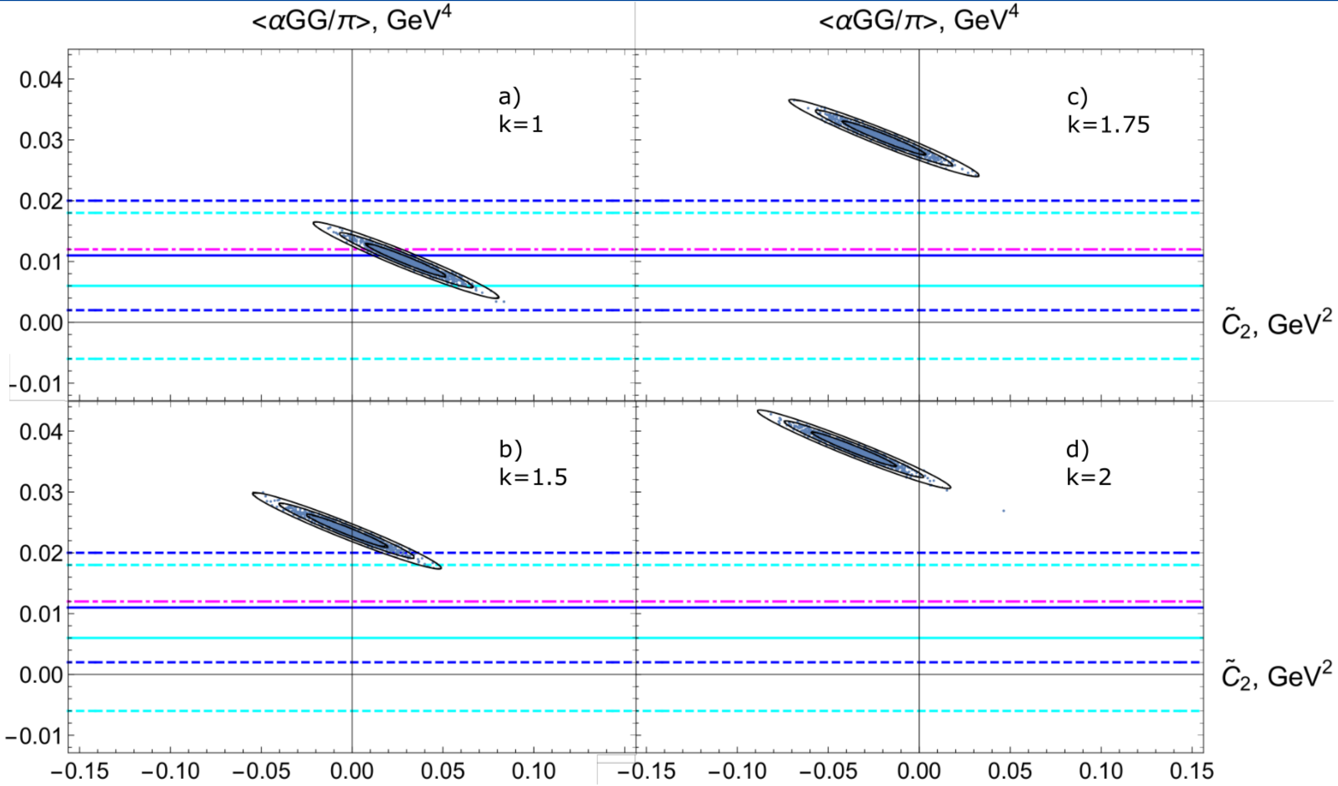


FIG. 3. Allowed regions of C_2 and gluon condensate **in APT** with various multiplication factors for quark condensate (or C_6) within 1, 2 and 3 standard deviations. The values for gluon condensate from the literature together with corresponding limits are also shown.

In APT the effect of non-vacuum intermediate states seems to be about 50%.

TABLE II: The extracted values for the interval of M^2 : $[0.75, 3]$ GeV^2 **in APT**. The obtained values of condensates with errors, $\Lambda = 0.277$ GeV, $s_0 = 1.51^2$ GeV^2 and using $\widetilde{C}_6^{\text{GMOR, FLAG}} = -0.104$ GeV^6 fixed. In the last column the estimate of anticorrelation between gluon condensate and $\widetilde{C}_2$ is given.

$\widetilde{C}_6$	$\widetilde{C}_2$, GeV^2	$\widetilde{C}_4$, GeV^4	$\langle\frac{\alpha_s GG}{\pi}\rangle$, GeV^4	χ^2	$\langle\alpha_s GG/\pi\rangle(\widetilde{C}_2)$, GeV^4
$\widetilde{C}_6^{\text{GMOR}}$	0.030 ± 0.015	0.067 ± 0.012	0.010 ± 0.002	1.071	$-0.123C_2 + 0.014$
$\widetilde{C}_6^{\text{GMOR}} \times 1.5$	-0.003 ∓ 0.015	0.155 ± 0.012	0.024 ± 0.002	0.491	$-0.121\widetilde{C}_2 + 0.023$
$\widetilde{C}_6^{\text{GMOR}} \times 1.65$	-0.013 ∓ 0.016	0.182 ± 0.013	0.028 ± 0.002	0.454	$-0.122\widetilde{C}_2 + 0.026$
$\widetilde{C}_6^{\text{GMOR}} \times 1.75$	-0.019 ∓ 0.015	0.199 ± 0.012	0.030 ± 0.002	0.467	$-0.121\widetilde{C}_2 + 0.028$
$\widetilde{C}_6^{\text{GMOR}} \times 1.8$	-0.022 ∓ 0.015	0.208 ± 0.012	0.032 ± 0.002	0.484	$-0.122\widetilde{C}_2 + 0.029$
$\widetilde{C}_6^{\text{GMOR}} \times 2.0$	-0.036 ∓ 0.015	0.244 ± 0.012	0.037 ± 0.002	0.621	$-0.121\widetilde{C}_2 + 0.033$

Summary

- We have presented the checking of hypothesis of vacuum dominance based on processing of the data of e^+e^--annihilation into pions.
- The method: construction of the Adler function for the modern data on e^+e^--annihilation into hadrons (an even number of pions), the Borel transform, sum rule construction, extraction the nonperturbative corrections in OPE, is developed. The Monte-Carlo simulating of events with taking into account errors of the data gives good accuracy. The interval on Borel mass is selected in analysis, not one point - that gives stability of the result.
- The found anticorrelation between OPE corrections remains strong.
- We explored the possible deviation from vacuum dominance and its interplay with short strings contribution. **We checked that it is possible to increase the coefficient C_6 in OPE, and hence the 4-quark condensate, but no more than 20%, and it is better by 15% -** that may be due to the correction to vacuum dominance of intermediate states. In this case the value of C_2 is compatible with zero, and C_4 is within reasonable values.
- **In APT** this effect is about **50%** - much more larger than in PT.

Thank you for your attention!