Little String Theory, Holography and the Non-Abelian Vortex String in $\mathcal{N} = 2$ supersymmetric QCD

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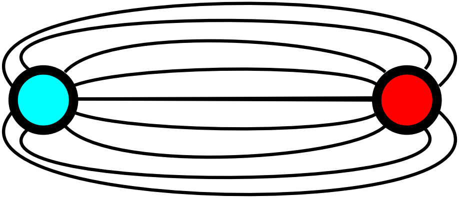
Confinement in QCD

QCD: quarks, gluons $\rightarrow$ mesons, baryons

The question: How does confinement work?

Idea: some kind of flux tube



What is the underlying mechanism?

Theorist's bread and butter: start from a simpler (maybe even solvable) theory

4D $\mathcal{N} = 2$ supersymmetric QCD

Quark vacuum

Bulk theory: $\mathcal{N} = 2$ supersymmetric QCD
Scalar quarks condense $\Rightarrow$ broken symmetry.

$$m_G \sim g\sqrt{\xi}$$

In the limit of equal quark masses, a *global* subgroup survives:

$$\mathrm{U}(N_c)_{\text{gauge}} \times \mathrm{SU}(N_f)_{\text{flavor}} \to \boxed{\mathrm{SU}(N_c)_{C+F}} \times \mathrm{SU}(\widetilde{N})_F \times \mathrm{U}(1)$$

$\widetilde{N} = N_f - N$

Abrikosov-Nielsen-Olesen strings emerge, which then are promoted to non-Abelian strings.

World sheet

2D $\mathcal{N} = (2,2)$ $\mathbb{WCP}(2,2)$ sigma model

Bosonic action:

$$S_{\text{bos}} = \int d^2x \left\{ \left|\nabla_\alpha n^P\right|^2 + \left|\tilde{\nabla}_\alpha \rho^K\right|^2 + 2\left|\sigma + \frac{m_P}{\sqrt{2}}\right|^2 \left|n^P\right|^2 \right.$$

$$\left. + 2\left|\sigma + \frac{m_K}{\sqrt{2}}\right|^2 \left|\rho^K\right|^2 + \frac{e^2}{2}\left(|n^P|^2 - |\rho^K|^2 - \beta\right)^2 \right\}$$

When does the string become critical

The theory is superconformal at $\widetilde{N} = N_f - N_c = N_c$.
At strong coupling $\to$ thin string regime '15 [Shifman, Yung]
DOF: 4 translational, $2N$ from n^P, $2N$ from ρ^K, minus one constraint and $U(1)$ gauge inv. $\to$

$$\mathrm{DOF} = 4 + 2 \cdot 2N - 2 \stackrel{?}{=} 10 \implies \text{critical at } N = 2$$

We see a 10d space required for a critical superstring (Type IIA).

b-baryon

Gauge invariant variables:

$$w^{PK} = n^P \rho^K$$

Condition:

$$\det w^{PK} = 0.$$

$\rightarrow$ singular CY space Y_6 $\rightarrow$ conifold

At $\beta = 0$ a " deformed conifold " is possible

$$\det w^{PK} = b.$$

b becomes a complex modulus — VEV of a massless 4D field (lowest component of a BPS hypermultiplet)

Non-critical $c = 1$ string

String theory on a non-compact CY is dual to a non-critical $\boldsymbol{c} = 1$ string [Giveon Kutasov '99] .
Relevant non-critical $\boldsymbol{c} = 1$ string theory is formulated on the target space $\mathbb{R}^4 \times \mathbb{R}_\phi \times \boldsymbol{S}^1$, where $\mathbb{R}_\phi$ is a real line associated with the Liouville field ϕ and the theory has a linear in ϕ dilaton, such that string coupling is given by $\boldsymbol{g_s} = \boldsymbol{e}^{-\frac{1}{\sqrt{2}}\phi}$.

Non-critical $c = 1$ string

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Deformation of the conifold (complex structure deformation) mentioned above translates into adding the Liouville interaction to the world-sheet sigma model

$$\delta L = b \int d^2\theta \, e^{-\frac{\phi + iY}{Q}}$$

Non-critical $c = 1$ string

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The mirror description of the Liouville $c = 1$ non-critical string theory is in terms of a two-dimensional Witten black hole, specifically $\mathrm{SL}(2, \mathbb{R})/\mathrm{U}(1)$ coset WZNW theory at level $k = 1$.

This is an exactly solvable model with known spectrum $\longrightarrow$ we are able to get the masses of 4d states.

$$\frac{(M_m^S)^2}{8\pi T} = -\frac{p_\mu p^\mu}{8\pi T} = m^2 - \frac{1}{4}, \quad m = \pm(1/2, 3/2, 5/2, \ldots)$$

Solitonic string-gauge duality versus holography

4D $\mathcal{N} = 2$ SQCD with $N_f = 2N = 4$:

- weak coupling — Higgs phase, (s)quarks and Higgsed gauge bosons
- strong coupling — closed string states formed by the non-Abelian vortex string

Solitonic string-gauge duality

Solitonic string-gauge duality versus holography

Solitonic string-gauge duality	AdS/CFT
10d space is artificial: 4d "real" + 6d internal moduli space	10d fundamental string to begin with
String tension at 4d SQCD FI parameter	String tension at Plank scale
No branes, large b (weak gravity)	D-branes, large N (weak gravity)
4d states $\leftrightarrow$ normalizable vertex op. near singularity	Off-shell QFT corr. $\leftrightarrow$ On-shell string corr. on the boundary, e.g. far from branes

Vertex operators

$$V_{j,m_L,m_R} = e^{i\sqrt{2}(m_L Y_L + m_R Y_R)} \left[e^{\sqrt{2}j\phi} + R(j, m_L, m_R; k) e^{-\sqrt{2}(j+1)\phi} + \cdots \right]$$

Generally this v.o. is non-normalizable. However, it becomes normalizable at special values of j, m_L, m_R, when R vanishes or has a pole.

2pt correlation functions

LSZ relation:

$$V^{\mathrm{non-norm}} \sim \frac{1}{p_\mu^2 + M^2}\, V^{\mathrm{norm}},$$

2pt correlation function:

$$\left\langle V_{j_1;m_L,m_R} V_{j_2;-m_L,-m_R} \right\rangle = R(j, m_L, m_R; k)\, \delta(j_1 - j_2)$$

LSZ in our case:

$$\langle V_{\tilde{j},m_L,m_R}, O_1, ..., O_n \rangle \sim \frac{1}{j - j_0} \langle V_{j,m_L,m_R}, O_1, ..., O_n \rangle$$

3pt correlation functions

$j = (-1, -1/2, -1/2)$ gives a scattering amplitude

$$\left\langle V_{j_1=-1;m_1,-m_1} V_{j_2=-1/2;m_2,-m_2} V_{j_3=-1/2;m_3,-m_3} \right\rangle$$

$$= \frac{\operatorname*{Res}_{\tilde{j}_1=0} \left\langle V_{\tilde{j}_1;m_1,-m_1} V_{j_2=-1/2;m_2,-m_2} V_{j_3=-1/2;m_3,-m_3} \right\rangle}{\operatorname*{Res}_{\tilde{j}_1=0} R(\tilde{j}_1, m_1, -m_1; k)}$$

$$= \frac{1}{m_1^2}$$

Conclusions and outlook

- Studied 4d SQCD hadron correlation functions in terms of string theory on $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ WZNW coset and studied their analytic structure.
- LSZ and holography works for $j = -1$ (normalizable) but not for $j = -1/2$ (log-normalizable)
- What is more direct relation between critical string theory on the conifold and non-critical $c = 1$ string theory with the Liouville field?
- Construct low energy effective theory of hadron interactions