Coherent radiation of photons by particle wave packets

Timofey V. Solovyev,
Peter O. Kazinski

TOMSK STATE UNIVERSITY
Faculty of Physics, Quantum Field Theory Department

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Research object

Framework

The radiation of photons by electrons is investigated in the framework of quantum electrodynamics up to the second order in the coupling constant e.

States

We consider the N-particle, coherent, and thermal initial states and the forms of the electron wave packets are taken into account.

Tasks

- the explicit expressions for the intensity of radiation
- the inclusive probability to record a photon

Initial and final states

The density matrix of the initial state is

$$\hat{R} = \hat{R}_{ph} \otimes \hat{R}_e \otimes |0\rangle_{e^+} \langle 0|_{e^+}. \tag{1}$$

The projector to the states in the Fock space containing at least one particle with quantum numbers specified by the projector D

$$\hat{P}_{ph} = \hat{1} - :\exp(-\hat{c}^\dagger D\hat{c}): = \hat{c}^\dagger D\hat{c} + \cdots, \quad D^\dagger = D. \tag{2}$$

The measurement in the final state is specified by the projector

$$\hat{P} = \hat{P}_{ph} \otimes \hat{1}_e \otimes \hat{1}_{e^+} \tag{3}$$

We suppose that either D is diagonal in the energy basis or the operator D is taken at the instant of time $t = 0$

$$D_{\alpha\bar{\alpha}}(out) = D_{\alpha\bar{\alpha}} e^{-i(k_{0\alpha} - k_{0\bar{\alpha}})t_{out}}. \tag{4}$$

Contributions to the S-matrix

The radiation of a photon by an electron in the external field or in the presence of the dispersive medium is described by the operator

$$\hat{V} = V^{\bar{\gamma}}_{\bar{\alpha}\alpha}\hat{a}^{\dagger}_{\bar{\alpha}}\hat{a}_{\alpha}\hat{c}^{\dagger}_{\bar{\gamma}} - V^{\dagger\gamma}_{\bar{\alpha}\alpha}\hat{a}^{\dagger}_{\bar{\alpha}}\hat{a}_{\alpha}\hat{c}_{\gamma}, \tag{5}$$

of the first and higher orders in the coupling constant e.
The Compton process is specified by the operators,

$$\begin{aligned} \hat{W} &= \hat{W}_1 + \hat{W}_2 + \hat{W}_3, & W^{\bar{\gamma}\gamma}_{1\bar{\alpha}\alpha} &= 2W^{AB}_{\bar{\alpha}\alpha}\bar{e}^{\bar{\gamma}}_{A}e^{\gamma}_{B}, \\ \hat{W}_1 &= W^{\bar{\gamma}\gamma}_{1\bar{\alpha}\alpha}\hat{a}^{\dagger}_{\bar{\alpha}}\hat{a}_{\alpha}\hat{c}^{\dagger}_{\bar{\gamma}}\hat{c}_{\gamma}, & W^{\gamma_1\gamma_2}_{2\bar{\alpha}\alpha} &= W^{AB}_{\bar{\alpha}\alpha}e^{\gamma_1}_{A}e^{\gamma_2}_{B}, \\ \hat{W}_2 &= W^{\gamma_1\gamma_2}_{2\bar{\alpha}\alpha}\hat{a}^{\dagger}_{\bar{\alpha}}\hat{a}_{\alpha}\hat{c}_{\gamma_1}\hat{c}_{\gamma_2}, & W^{\bar{\gamma}_1\bar{\gamma}_2}_{3\bar{\alpha}\alpha} &= W^{AB}_{\bar{\alpha}\alpha}\bar{e}^{\bar{\gamma}_1}_{A}\bar{e}^{\bar{\gamma}_2}_{B}, \\ \hat{W}_3 &= W^{\bar{\gamma}_1\bar{\gamma}_2}_{3\bar{\alpha}\alpha}\hat{a}^{\dagger}_{\bar{\alpha}}\hat{a}_{\alpha}\hat{c}^{\dagger}_{\bar{\gamma}_1}\hat{c}^{\dagger}_{\bar{\gamma}_2}, \end{aligned} \tag{6}$$

of the second order in the coupling constant.

Contributions to the S-matrix

Of the same order are the contributions of the mass operator,

$$\hat{M} = M_{\bar{\alpha}\alpha}\hat{a}^\dagger_{\bar{\alpha}}\hat{a}_\alpha, \tag{7}$$

and the polarization operator,

$$\begin{aligned}
\hat{\Pi} &= \hat{\Pi}_1 + \hat{\Pi}_2 + \hat{\Pi}_3, & \Pi_1^{\bar{\gamma}\gamma} &= 2\Pi^{AB}\bar{e}_A^{\bar{\gamma}} e_B^{\gamma}, \\
\hat{\Pi}_1 &= \Pi_1^{\bar{\gamma}\gamma}\hat{c}^\dagger_{\bar{\gamma}}\hat{c}_\gamma, & \Pi_2^{\gamma_1\gamma_2} &= \Pi^{AB} e_A^{\gamma_1} e_B^{\gamma_2}, \\
\hat{\Pi}_2 &= \Pi_2^{\gamma_1\gamma_2}\hat{c}_{\gamma_1}\hat{c}_{\gamma_2}, & \Pi_3^{\bar{\gamma}_1\bar{\gamma}_2} &= \Pi^{AB}\bar{e}_A^{\bar{\gamma}_1}\bar{e}_B^{\bar{\gamma}_2}, \\
\hat{\Pi}_3 &= \Pi_3^{\bar{\gamma}_1\bar{\gamma}_2}\hat{c}^\dagger_{\bar{\gamma}_1}\hat{c}^\dagger_{\bar{\gamma}_2}, & &
\end{aligned} \tag{8}$$

Moreover, the process of scattering of an electron by an electron,

$$\hat{C} = C_{\bar{\alpha}\bar{\beta}\beta\alpha}\hat{a}^\dagger_{\bar{\alpha}}\hat{a}^\dagger_{\bar{\beta}}\hat{a}_\beta\hat{a}_\alpha \tag{9}$$

Then the expansion of the S-matrix with respect to the coupling constant is given by

$$\hat{S} = \hat{1} + \hat{V} + \hat{E} + \hat{W} + \hat{\Pi} + \hat{M} + \hat{C} + \cdots . \tag{10}$$

Contributions to the inclusive probability

The inclusive probability to record a photon in the states specified by the projector D becomes

$$P_D = \operatorname{Sp}(\hat{P}\hat{S}\hat{R}\hat{S}^\dagger). \tag{11}$$

The intensity of radiation, I_D, is a linear part of (11) with respect to D.

$$D_{\alpha\bar{\alpha}} = k_{0\gamma} \operatorname{pr}^{\gamma}_{\alpha\bar{\alpha}} \tag{12}$$

The leading contributions to the process are determined by traces[1]

$$\begin{gathered} \operatorname{Sp}(\hat{P}\hat{R}), \quad \operatorname{Sp}(\hat{P}\hat{V}\hat{R}), \quad \operatorname{Sp}(\hat{P}\hat{V}\hat{R}\hat{V}^\dagger), \quad \operatorname{Sp}(\hat{P}\hat{E}\hat{R}\hat{E}^\dagger), \\ \operatorname{Sp}(\hat{P}\hat{W}\hat{R}), \quad \operatorname{Sp}(\hat{P}\hat{\Pi}\hat{R}), \quad \operatorname{Sp}(\hat{P}\hat{M}\hat{R}), \quad \operatorname{Sp}(\hat{P}\hat{C}\hat{R}). \end{gathered} \tag{13}$$

[1] P.O. Kazinski, T.V. Solovyev, Eur. Phys. J. C 82, 790 (2022)

Stimulated radiation from a wave packet

As the example of stimulated radiation produced by a wave packet, we consider transition radiation from the Dirac fermion traversing a mirror in the field of a laser wave[2]

$$
\begin{aligned}
V^{\bar{\gamma}}_{\bar{\alpha}\alpha} &= -i\frac{m}{V}\int dx a^*_{i\lambda}(x_3)\bar{u}_{\bar{\alpha}}\big[e\gamma^i - i\mu_a(k_0\sigma^{i0} \\
&\qquad + (p_j - p'_j)\sigma^{ij})\big]u_\alpha \frac{e^{ik_0x^0 - i\mathbf{k}_\perp\mathbf{x}_\perp + i(p'_\mu - p_\mu)x^\mu}}{\sqrt{2Vk_0p_0p'_0}} \\
&= -i\frac{m}{V}\int dx a^*_{i\lambda}(x_3)\bar{u}_{\bar{\alpha}}\big[e\gamma^i - i\mu_a(p'_\nu - p_\nu)\sigma^{\nu i}\big] \\
&\qquad \times u_\alpha \frac{e^{ik_0x^0 - i\mathbf{k}_\perp\mathbf{x}_\perp + i(p'_\mu - p_\mu)x^\mu}}{\sqrt{2Vk_0p_0p'_0}},
\end{aligned}
$$

It was assumed that the mirror is ideally conducting and it is placed at $z \leq 0$.

[2] P.O. Kazinski, G.Yu. Lazarenko, Phys. Rev. A 103, 012216 (2021)

Stimulated radiation from a wave packet

Explicit expression for $dF_\lambda(k)$ neglecting the recoil due to photon radiation and setting $\mu_a = 0$

$$\bar{u}^{s'}(\mathbf{p}')\gamma^\mu u^s(\mathbf{p}) \approx \delta_{s's} p^\mu / m, \tag{14}$$

we derive

$$dF_\lambda(\mathbf{k}) = \sum_s \int d\mathbf{p} \sum_{N=1}^{\infty} N\rho_{ss}^{(N,1)}(\mathbf{p},\mathbf{p}') \times \frac{2e|\beta_3|\beta^i\left[k_3 f_i^{*(\lambda)} + (q_3 - k_3)\delta_i^3 f_3^{*(\lambda)}\right]}{k_0\left[(1-\boldsymbol{\beta}_\perp\mathbf{n}_\perp)^2 - \beta_3^2 n_3^2\right]} \frac{d_\lambda^*(\mathbf{k})}{\sqrt{2(2\pi)^3 k_0}} d\mathbf{k},$$

where

$$q_\mu := p_\mu - p'_\mu, \qquad \mathbf{p}'_\perp = \mathbf{p}_\perp - \mathbf{k}_\perp,$$
$$p'_3 = p_3 - k_0(1-\boldsymbol{\beta}_\perp\mathbf{n}_\perp)/\beta_3.$$

Susceptibility of an electron wave packet

The effective susceptibility tensor is equal to

$$\Pi_{eff}^{AB} = \sum_{N=1}^{\infty} N\rho_{\alpha\bar{\alpha}}^{(N,1)} W_{\bar{\alpha}\alpha}^{AB} = \sum_{N=1}^{\infty} N\rho_{\alpha\bar{\alpha}}^{(N,1)} W_{1\bar{\alpha}\alpha}^{AB}, \tag{15}$$

Susceptibility of the wave packet of a single electron in the small recoil limit.

$$\chi_{ij}(k_0;\mathbf{x}) = -\frac{e^2\rho(\mathbf{x})}{k_0^2 m}\delta_{ij}, \tag{16}$$

this susceptibility has the standard plasma form.

Susceptibility of an electron wave packet

The intensity of the radiation has the form

$$dI_{\lambda'}(\mathbf{k}') = \Big| d_{\lambda'}(\mathbf{k}')\sqrt{1+\mathcal{N}} - \frac{ie^2}{m} \times \sum_{\lambda} \int \frac{d\mathbf{k}}{2(2\pi)^2 k_0} \delta(k_0' - k_0) F(\mathbf{k}' - \mathbf{k}) e_i^{*(\lambda')}(\mathbf{k}') e_i^{(\lambda)}(\mathbf{k}) d_\lambda(\mathbf{k}) \Big|^2 \times k_0' d\mathbf{k}'.$$

As we see, the contribution to the radiation intensity we consider describes the interference of the incident and reradiated photons and it is of the order e^2.

Proposals for experiments

- To use stimulated transition and diffraction radiation for mapping the profiles of radiating particle wave packets. To this end, one can employ the same methods as for noninvasive diagnostics of beams of charged particles[3][4][5].
- To use the interference of incident photons with scattered ones in the Compton process in order to restore the form of the wave packet of the electron participating in this process.

[3] A. P. Potylitsyn, M. I. Ryazanov, M. N. Strikhanov, A. A. Tishchenko, Springer Tracts in Modern Physics, Vol. 239

[4] L.G. Sukhikh, G. Kube, A.P. Potylitsyn, Phys. Rev. Accel. Beams 20, 032802 (2017)

[5] L. G. Sukhikh, Doctor thesis, Tomsk Polytechnic University, (2018)

Conclusion

- We obtain the general formulas for radiation intensity and inclusive probability to record a photon in QED up the second order in the coupling constant e. These formulas differ from those presented in papers[6,7].
- A special attention was paid to the processes where the wave packets of electrons radiate coherently.
  We found the three such processes in the second order of perturbation theory: stimulated radiation produced by an electron wave packet[8,9,10,11]; coherent radiation from N wave packets of particles arranged symmetrically; reradiation by an electron wave packet in the Compton process.
- For the process of reradiation by an electron wave packet, the susceptibility tensor of a single electron wave packet in a vacuum was found.

[6] G.L. Kotkin, V.G. Serbo, Phys. Rev. STAB 7 (2004)
[7] G.L. Kotkin, V.G. Serbo and A. Schiller, Int. J. Mod. Phys. A 7, 4707 (1992)
[8] N. Talebi, New J. Phys. 18, 123006 (2016)
[9] Y. Pan, A. Gover, J. Phys. Commun. 2, 115026 (2018)
[10] Y. Pan, A. Gover, Phys. Rev. A 99, 052107 (2019)
[11] Y. Pan, A. Gover, New J. Phys. 23, 063070 (2021)