6th International Conference on Particle Physics and Astrophysics, MEPhi, 29 November - 2 December 2022

Relativistic theory of paired heavy meson and baryon production

A.P. Martynenko (Samara U.),
In collaboration with R.N. Faustov (FRC CSC RAS, Moscow),
D. Ebert (Humboldt U., Berlin),
V.O. Galkin (FRC CSC RAS, Moscow),
F.A. Martynenko (Samara U., Samara)

Dedicated to the memory of Rudolf Nikolaevich Faustov

Contents

The current stage of research on the processes of pair production of heavy mesons and baryons began in 2004 after Belle and BaBar experiments

- K. Abe, et al., Phys. Rev. D **70**, 071102 (2004).
- B. Aubert, et al., Phys. Rev. D **72**, 031101 (2005).

During a short period of research it has been shown that the theoretical results for the pair production of charmonium can be reconciled with experimental data taking into account

- 1. Radiative corrections of order $O(\alpha_s)$
- 2. Relativistic corrections due to relative motion of heavy quarks and transformation law of the bound state wave functions

A small list of publications on the issue

- E. Braaten, J. Lee, Phys. Rev. D **67**, 054007 (2003); Phys. Rev. D **72**, 099901(E) (2005).
- K.-Y. Liu, Z.-G. He, K.-T. Chao, Phys. Lett. B **557**, 45 (2003).
- K. Hagiwara, E. Kou, C.-F. Qiao, Phys. Lett. B **570**, 39 (2003).
- K.-Y. Liu, Z.-G. He, K.-T. Chao, Phys. Rev. D **77**, 014002 (2008).
- A.E. Bondar, V.L. Chernyak, Phys. Lett. B **612**, 215 (2005).
- V.V.Braguta, A.K. Likhoded, A.V. Luchinsky, Phys. Rev. D **72**, 074019 (2005).
- Y.-J. Zhang, Y.-J. Gao, K.-T. Chao, Phys. Rev. Lett. **96**, 092001 (2006).
- G.T. Bodwin, D. Kang, J. Lee, Phys.Rev. D **74**, 114028 (2006).
- D. Ebert, A.P. Martynenko, Phys. Rev. D **74**, 054008 (2006).
- H.-M.Choi, Ch.-R. Ji, Phys. Rev. D **76**, 094010, (2007).
- Z.-G. He, Y.Fan, K.-T. Chao, Phys. Rev. D **75**, 074011 (2007).
- A.V. Berezhnoy, Phys. Atom. Nucl. **71**, 1803 (2007).
- G.T. Bodwin, J. Lee, Ch.Yu, Phys. Rev. **D77**, 094018 (2008).
- D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, Phys. Lett. **B672**, 264 (2009).

The study of exclusive processes of pair production of heavy mesons and baryons is of interest both from the point of view of testing the Standard Model and the theory of bound states of quarks. Experimental studies of such reactions are currently being carried out both at electron-positron colliders and in proton-proton interaction at the LHC.

- R. Aaij et al., Observation of J/Ψ pair production in pp collisions at $\sqrt{s} = 7$ TeV , Phys. Lett. B707 (2012) 5259, arXiv:1109.0963 [hep-ex].
- V. Khachatryan et al., Measurement of prompt J/Ψ pair production in pp collisions at $\sqrt{s} = 7$ Tev, JHEP no. 9, (2014) 94, arXiv:1406.0484 [hep-ex]
- CMS Collaboration, Search for Higgs boson decays into Z and J/Ψ and for Higgs and Z boson decays into J/Ψ or Υ pairs in pp collisions at $\sqrt{s} = 13$ TeV, arXiv:2206.03525v1 [hep-ex].

The aim of our work is to give a consistent relativistic description of the processes of pair production within the framework of the Standard Model and the quark model of hadrons.

At the same time, it is necessary:

- To explore various mechanisms of pair hadron production.
- To construct relativistic production amplitudes.
- To construct relativistic cross sections for pair production.
- To take into account relativistic effects in the quark interaction operator.

The main elements of our approach for calculating the observed probabilities for the production of a pair of heavy hadrons are presented below using examples of the production of B_c mesons.

Two production amplitudes of the B_c meson pair in leading order of the QCD coupling constant α_s are presented in Fig. 7. Two other amplitudes can be obtained by corresponding permutations. There are two stages of B_c meson production process.

- At the first perturbative stage the virtual photon γ^* and then a virtual gluon g^* produce two heavy quarks (bc) and two heavy antiquarks $(\bar{b}\bar{c})$.
- At the second nonperturbative stage of the production process, quark-antiquark pairs form double heavy mesons of definite spin.

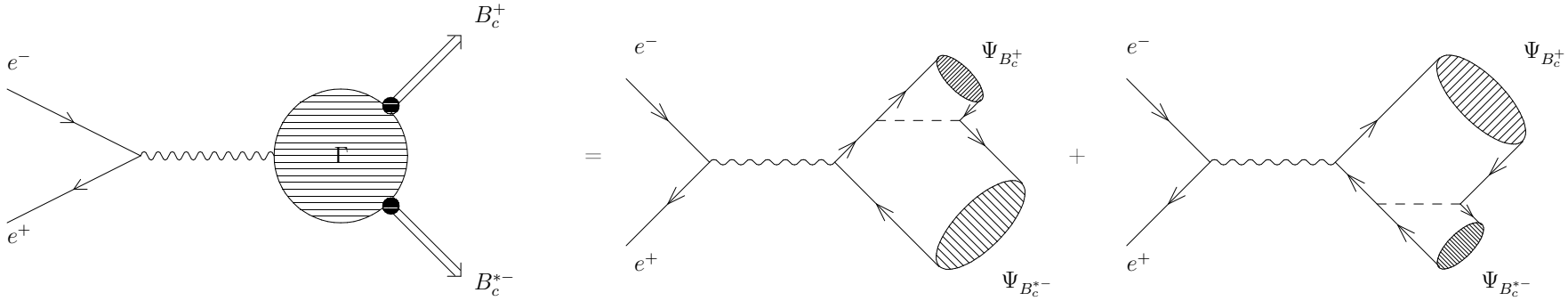

Figure: One-photon mechanism of the pair B_c-meson production in e^+e^- annihilation. B_c^+ and B_c^{*-} denote the B_c-meson states with spin 0 and 1. Γ is the production vertex function.

Another possible two-photon mechanism for the production of a pair of B_c mesons is shown in Fig. 2.

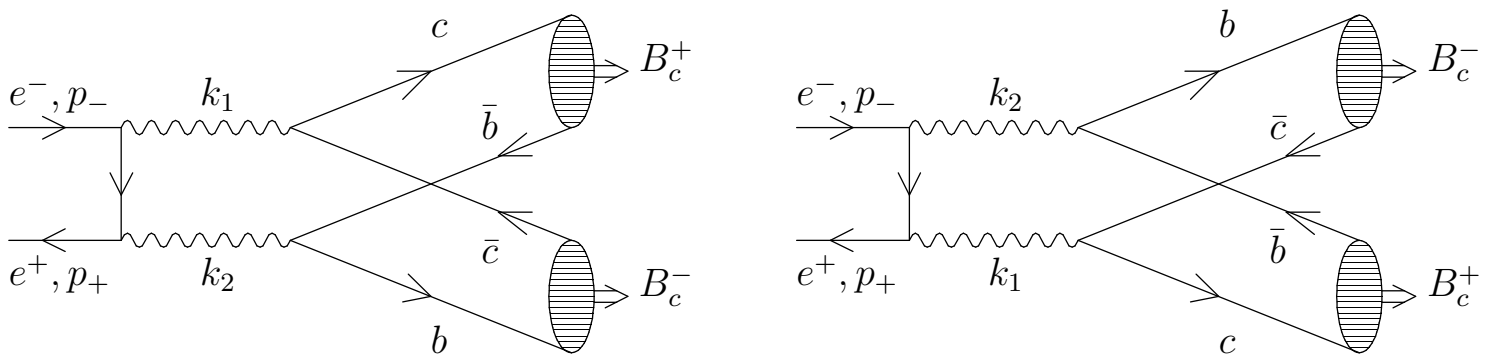

Figure: Two-photon mechanism of the pair B_c-meson production in e^+e^- annihilation. B_c^+ and B_c^- denote the B_c-meson states. Wavy lines show the virtual photons with four momenta k_1 and k_2.

Various meson pair production mechanisms differ from each other in the structure of the main functions and the values of the vertex factors.

Four-momenta of heavy quarks and antiquarks can be expressed in terms of relative and total four momenta as follows:

$$p_1 = \eta_1 P + p,\ p_2 = \eta_2 P - p,\ (p \cdot P) = 0,\ \eta_i = \frac{M_1^2 \pm m_1^2 \mp m_2^2}{2M_1^2},\ (1)$$

$$q_1 = \rho_1 Q + q,\ q_2 = \rho_2 Q - q,\ (q \cdot Q) = 0,\ \rho_i = \frac{M_2^2 \pm m_1^2 \mp m_2^2}{2M_2^2}.$$

In the quasipotential approach the production amplitude can be written as a convolution of perturbative production amplitude of free quarks and antiquarks and the quasipotential wave functions. Using then the transformation law of the bound state wave functions from the rest frame to the moving one with four-momenta P and Q we can present the meson production amplitude in the form:

$$\mathcal{M}(p_-, p_+, P, Q) = \frac{8\pi^2\alpha}{3s^2}\sqrt{M_{B_{\bar{b}c}} M_{B_{b\bar{c}}}}\,\bar{v}(p_+)\gamma^\beta u(p_-)\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\times \qquad (2)$$

$$Tr\left\{\Psi^{\mathcal{P}}_{B_{\bar{b}c}}(p,P)\Gamma^{\beta\nu}(p,q,P,Q)\Psi^{\mathcal{V}}_{B_{b\bar{c}}}(q,Q)\gamma_\nu\right\},$$

where s is the center-of-mass energy.

The relativistic wave functions of the bound quarks accounting for the transformation from the rest frame to the moving one with four momenta P, and Q, are

$$\Psi^{\mathcal{P}}_{B_{\bar{b}c}}(p,P) = \frac{\Psi^0_{B_{\bar{b}c}}(\mathbf{p})}{\sqrt{\frac{\epsilon_1(p)}{m_1}\frac{(\epsilon_1(p)+m_1)}{2m_1}\frac{\epsilon_2(p)}{m_2}\frac{(\epsilon_2(p)+m_2)}{2m_2}}}\left[\frac{\hat{v}_1-1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m_2(\epsilon_2(p)+m_2)}-\frac{\hat{p}}{2m_2}\right] \times\gamma_5(1+\hat{v}_1)\left[\frac{\hat{v}_1+1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m_1(\epsilon_1(p)+m_1)}+\frac{\hat{p}}{2m_1}\right], \quad (3)$$

$$\Psi^{\mathcal{V}}_{B^*_{b\bar{c}}}(q,Q) = \frac{\Psi^0_{B^*_{b\bar{c}}}(\mathbf{q})}{\sqrt{\frac{\epsilon_1(q)}{m_1}\frac{(\epsilon_1(q)+m_1)}{2m_1}\frac{\epsilon_2(q)}{m_2}\frac{(\epsilon_2(q)+m_2)}{2m_2}}}\left[\frac{\hat{v}_2-1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m_1(\epsilon_1(q)+m_1)}+\frac{\hat{q}}{2m_1}\right] \times\hat{\varepsilon}_{\mathcal{V}}(Q,S_z)(1+\hat{v}_2)\left[\frac{\hat{v}_2+1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m_2(\epsilon_2(q)+m_2)}-\frac{\hat{q}}{2m_2}\right], \quad (4)$$

where $v_1 = P/M_{B_{\bar{b}c}}$, $v_2 = Q/M_{B_{b\bar{c}}}$; $\varepsilon_{\mathcal{V}}(Q,S_z)$ is the polarization vector of the $B_c^{*-}(1^-)$ meson, relativistic quark energies $\epsilon_{1,2}(p) = \sqrt{p^2+m_{1,2}^2}$ and $m_{1,2}$ are the masses of c and b quarks.

In our amplitudes we have complicated factor including the bound state wave function in the rest frame. The initial production amplitude contains the integration over the quark relative momenta $\mathbf{p}$ and $\mathbf{q}$. Color part of the meson wave function in the amplitude is taken as $\delta_{ij}/\sqrt{3}$ (color indexes $i, j, k = 1, 2, 3$). Relativistic wave functions are equal to the product of wave functions in the rest frame $\Psi^0_{B_{\bar{b}c}}(\mathbf{p})$ and spin projection operators that are accurate at all orders in $|\mathbf{p}|/m$. Our derivation of production amplitudes accounts for the transformation law of the bound state wave functions from the rest frame to the moving one with four momenta P and Q. This transformation law was discussed in the Bethe-Salpeter approach and in quasipotential method:

- S.J. Brodsky and J.R. Primack, Ann. Phys. **52**, 315 (1969).
- R.N. Faustov, Ann. Phys. **78**, 176 (1973).

When constructing the production amplitudes with the production of a pair of S-wave B_c mesons, we introduce projection operators $\hat{\Pi}^{\mathcal{P},\mathcal{V}}$ for states with total spin $S = 0,\ 1$ of the following form:

$$\hat{\Pi}^{\mathcal{P}} = [v_2(0)\bar{u}_1(0)]_{S=0} = \gamma_5 \frac{1+\gamma^0}{2\sqrt{2}}, \quad \hat{\Pi}^{\mathcal{V}} = [v_2(0)\bar{u}_1(0)]_{S=1} = \hat{\varepsilon}\frac{1+\gamma^0}{2\sqrt{2}}. \quad (5)$$

At leading order in α_s the vertex functions $\Gamma^{\beta\nu}_{1,2}(p,P;q,Q)$ can be written as ($\Gamma^{\beta\nu}_2(p,P;q,Q)$ can be obtained from $\Gamma^{\beta\nu}_1(p,P;q,Q)$ by means of the replacement $p_1 \leftrightarrow p_2$, $q_1 \leftrightarrow q_2$, $\alpha_b \to \alpha_c$, $Q_c \to Q_b$)

$$\Gamma^{\beta\nu}_1(p,P;q,Q) = Q_c\alpha_b \left[\gamma_\mu \frac{(\hat{l}-\hat{q}_1+m_1)}{(l-q_1)^2-m_1^2+i\epsilon}\gamma_\beta + \gamma_\beta \frac{(\hat{p}_1-\hat{l}+m_1)}{(p_1-l)^2-m_1^2+i\epsilon}\gamma_\mu\right] D^{\mu\nu}(k_2), \quad (6)$$

$$\Gamma^{\beta\nu}_2(p,P;q,Q) = Q_b\alpha_c \left[\gamma_\mu \frac{(\hat{l}-\hat{q}_2+m_2)}{(l-q_2)^2-m_2^2+i\epsilon}\gamma_\beta + \gamma_\beta \frac{(\hat{p}_2-\hat{l}+m_2)}{(p_2-l)^2-m_2^2+i\epsilon}\gamma_\mu\right] D^{\mu\nu}(k_1), \quad (7)$$

where the square of total four-momentum $l^2 = s^2 = (P+Q)^2 = (p_- + p_+)^2$.

Accounting for the small ratio of relative quark momenta p and q to the energy s, we use an expansion of inverse denominators of quark and gluon propagators as follows:

$$\frac{1}{(l-q_{1,2})^2-m_{1,2}^2}=\frac{1}{r_{2,1}s^2}\left[Z_{1,2}-\frac{q^2-2qP}{r_{2,1}s^2}+\cdots\right], \tag{8}$$

$$\frac{1}{(l-p_{1,2})^2-m_{1,2}^2}=\frac{1}{r_{2,1}s^2}\left[Z_{3,4}-\frac{p^2-2pQ}{r_{2,1}s^2}+\cdots\right], \tag{9}$$

$$Z_1=\frac{r_2s^2}{\rho_1^2M_P^2+\rho_2^2s^2+\rho_1\rho_2(s^2+M_P^2-M_V^2)-m_1^2}, \tag{10}$$

$$Z_3=\frac{r_1s^2}{\rho_2^2M_V^2+\rho_1^2s^2+\rho_1\rho_2(s^2+M_P^2-M_V^2)-m_2^2}, \tag{11}$$

$$\frac{1}{k_{1,2}^2}=\frac{1}{r_{2,1}^2s^2}\left[Y_{1,2}\pm\frac{2r_{2,1}(pQ+qP)\mp p^2\mp q^2\mp 2pq}{r_{2,1}^2s^2}+\cdots\right], \tag{12}$$

$$Y_{1,2}=\frac{r_{2,1}^2s^2}{\eta_{2,1}^2M_V^2+\rho_{2,1}^2M_P^2+\eta_{2,1}\rho_{2,1}(s^2-M_V^2-M_P^2)}.$$

In a purely non-relativistic approximation factors Z_i and Y_i are equal to 1. They accumulate the bound state effects. The expressions for Z_2 and Z_4 can be obtained from Z_1 and Z_3 after replacement $r_2\to r_1$, $\rho_{1,2}\leftrightarrow\eta_{2,1}$, $m_1\to m_2$.

We obtain relativistic amplitudes of the B_c meson pairs production for 1γ mechanism:

$$\mathcal{M}_{PP} = \frac{256\pi^2\alpha M M_{\mathcal{P}}}{3s^6}(v_1 - v_2)^{\beta}\bar{v}(p_+)\gamma_\beta u(p_-)\Psi^0_{B_{\bar{b}c}}(0)\Psi^0_{B_{b\bar{c}}}(0)\times \tag{13}$$

$$\left[\frac{Q_c\alpha_s(\frac{m_2^2}{M^2}s^2)}{r_2^3}F_1^{\mathcal{P}} - \frac{Q_b\alpha_s(\frac{m_1^2}{M^2}s^2)}{r_1^3}F_2^{\mathcal{P}}\right]$$

$$\mathcal{M}_{PV} = \frac{256\pi^2\alpha M\sqrt{M_{\mathcal{P}}M_{\mathcal{V}}}}{3s^6}\bar{v}(p_+)\gamma_\beta u(p_-)\varepsilon_{\beta\alpha\sigma\lambda}\varepsilon^{\alpha}_{\mathcal{V}}v_1^{\sigma}v_2^{\lambda}\Psi^0_{B^*_{\bar{b}c}}(0)\Psi^0_{B_{b\bar{c}}}(0)\times \tag{14}$$

$$\left[\frac{Q_c\alpha_s(\frac{m_2^2}{M^2}s^2)}{r_2^3}F_1^{\mathcal{P},\mathcal{V}} + \frac{Q_b\alpha_s(\frac{m_1^2}{M^2}s^2)}{r_1^3}F_2^{\mathcal{P},\mathcal{V}}\right],$$

$$\mathcal{M}_{VV} = \frac{256\pi^2\alpha M M_{\mathcal{V}}}{3s^6}\bar{v}(p_+)\gamma_\beta u(p_-)\Psi^0_{B^*_{\bar{b}c}}(0)\Psi^0_{B^*_{b\bar{c}}}(0)\times \tag{15}$$

$$\left[\frac{Q_c\alpha_s(\frac{m_2^2}{M^2}s^2)}{r_2^3}F_1^{\mathcal{V},\beta} - \frac{Q_b\alpha_s(\frac{m_1^2}{M^2}s^2)}{r_1^3}F_2^{\mathcal{V},\beta}\right],$$

$\varepsilon_{\mathcal{V}}$ is the polarization vector of spin 1 B_c meson. The functions $F_i^{\mathcal{P}}$, $F_i^{\mathcal{P},\mathcal{V}}$, $F_i^{\mathcal{V},\beta}$ can be written as series in specific relativistic factors

$C_{ij} = [(m_1 - \epsilon_1(p))/(m_1 + \epsilon_1(p))]^i[(m_2 - \epsilon_2(q))/(m_2 + \epsilon_2(q))]^j$

with $i + j \leq 2$.

Relativistic parameters can be expressed in terms of momentum integrals I^{nk} and calculated in the quark model:

$$I^{nk}_{B_{\bar{b}c},B_{b\bar{c}}} = \int_0^\infty q^2 R_{B_{\bar{b}c},B_{b\bar{c}}}(q)\sqrt{\frac{(\epsilon_1(q)+m_1)(\epsilon_2(q)+m_2)}{2\epsilon_1(q)\cdot 2\epsilon_2(q)}}\left(\frac{\epsilon_1(q)-m_1}{\epsilon_1(q)+m_1}\right)^n\left(\frac{\epsilon_2(q)-m_2}{\epsilon_2(q)+m_2}\right)^k dq, \quad (16)$$

$$\omega_{10}^{B_{\bar{b}c},B_{b\bar{c}}} = \frac{I^{10}_{B_{\bar{b}c},B_{b\bar{c}}}}{I^{00}_{B_{\bar{b}c},B_{b\bar{c}}}}, \quad \omega_{01}^{B_{\bar{b}c},B_{b\bar{c}}} = \frac{I^{01}_{B_{\bar{b}c},B_{b\bar{c}}}}{I^{00}_{B_{\bar{b}c},B_{b\bar{c}}}}, \quad \omega_{\frac{1}{2}\frac{1}{2}}^{B_{\bar{b}c},B_{b\bar{c}}} = \frac{I^{\frac{1}{2}\frac{1}{2}}_{B_{\bar{b}c},B_{b\bar{c}}}}{I^{00}_{B_{\bar{b}c},B_{b\bar{c}}}}, \quad (17)$$

$R_{B_{\bar{b}c},B_{b\bar{c}}}(q)$ is the radial wave function of the mesons $B_{\bar{b}c}$,$B_{b\bar{c}}$ in momentum space. The expansions can be extended to take into account terms of higher order in p and q. In the process of obtaining the necessary integrand functions we use different substitutions for $\mathbf{p}^2$ and $\mathbf{q}^2$. All of them can be obtained using the following expansion

$$|\mathbf{p}| = 2m_1\left[\sqrt{\frac{\epsilon_1-m_1}{\epsilon_1+m_1}}+\left(\frac{\epsilon_1-m_1}{\epsilon_1+m_1}\right)^{3/2}+\left(\frac{\epsilon_1-m_1}{\epsilon_1+m_1}\right)^{5/2}+\ldots\right] = \quad (18)$$

$$2m_2\left[\sqrt{\frac{\epsilon_2-m_2}{\epsilon_2+m_2}}+\left(\frac{\epsilon_2-m_2}{\epsilon_2+m_2}\right)^{3/2}+\left(\frac{\epsilon_2-m_2}{\epsilon_2+m_2}\right)^{5/2}+\ldots\right], \quad (19)$$

which allows you to save, if necessary, the symmetry of the particles.

Another source of relativistic corrections is related with the Hamiltonian of the heavy quark bound states which allows to calculate the bound state wave functions. The exact form of the bound state wave functions $\Psi^0_{B_c}(\mathbf{q})$ is important to obtain more reliable predictions for the production rates.

D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D **67**, 014027 (2003).

S. F. Radford and W. W. Repko, Phys. Rev. D **75**, 074031 (2007).

W. Lucha and F. F. Schöberl, Phys. Rev. A **51**, 4419 (1995).

$$H = H_0 + \Delta U_1 + \Delta U_2, \quad H_0 = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} - \frac{4\tilde{\alpha}_s}{3r} + (Ar + B), \tag{20}$$

$$\Delta U_1(r) = -\frac{\alpha_s^2}{3\pi r}\left[2\beta_0 \ln(\mu r) + a_1 + 2\gamma_E \beta_0\right], \quad a_1 = \frac{31}{3} - \frac{10}{9} n_f, \quad \beta_0 = 11 - \frac{2}{3} n_f, \tag{21}$$

$$\Delta U_2(r) = -\frac{2\alpha_s}{3m_1 m_2 r}\left[\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{rp})\mathbf{p}}{r^2}\right] + \frac{2\pi\alpha_s}{3}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\delta(\mathbf{r}) + \frac{4\alpha_s}{3r^3}\left(\frac{1}{2m_1^2} + \frac{1}{m_1 m_2}\right)(\mathbf{S}_1\mathbf{L}) + \tag{22}$$

$$+\frac{4\alpha_s}{3r^3}\left(\frac{1}{2m_2^2} + \frac{1}{m_1 m_2}\right)(\mathbf{S}_2\mathbf{L}) + \frac{32\pi\alpha_s}{9m_1 m_2}(\mathbf{S}_1\mathbf{S}_2)\delta(\mathbf{r}) + \frac{4\alpha_s}{3m_1 m_2 r^3}\left[\frac{3(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r})}{r^2} - (\mathbf{S}_1\mathbf{S}_2)\right] -$$

$$-\frac{\alpha_s^2(m_1 + m_2)}{m_1 m_2 r^2}\left[1 - \frac{4m_1 m_2}{9(m_1 + m_2)^2}\right],$$

$$\Delta V^{hfs}_{conf}(r) = f_V \frac{A}{8r}\left\{\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{16}{3m_1 m_2}(\mathbf{S}_1\mathbf{S}_2) + \frac{4}{3m_1 m_2}\left[\frac{3(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r})}{r^2} - (\mathbf{S}_1\mathbf{S}_2)\right]\right\}, \tag{23}$$

Total cross sections for the exclusive pair production B_c mesons:

$$\sigma_{PP} = \frac{4096\pi^3\alpha^2}{27s^{10}} M_P^2 |\Psi^0_{B_{\bar{b}c}}(0)|^2 |\Psi_{B_{b\bar{c}}}(0)|^2 (1 - \frac{4M^2_{B_{\bar{b}c}}}{s^2})^{3/2} \left[\frac{Q_c \alpha_s(\frac{m_2^2}{M^2}s^2)}{r_2^3} F_1^P - \frac{Q_b \alpha_s(\frac{m_1^2}{M^2}s^2)}{r_1^3} F_2^P \right]^2 , \tag{24}$$

$$\sigma_{PV} = \frac{2048\pi^3\alpha^2}{27s^8} \frac{M_P M_V}{M^2} \left[\left(1 - \frac{(M_P + M_V)^2}{s^2}\right) \left(1 - \frac{(M_P - M_V)^2}{s^2}\right) \right]^{3/2} \times \tag{25}$$

$$|\Psi^0_{B^*_{\bar{b}c}}(0)|^2 |\Psi_{B_{b\bar{c}}}(0)|^2 \left[\frac{Q_c \alpha_s \left(\frac{m_2^2}{M^2}s^2\right)}{r_2^3} F_1^{P,V} + \frac{Q_b \alpha_s \left(\frac{m_1^2}{M^2}s^2\right)}{r_1^3} F_2^{P,V} \right]^2 ,$$

$$\sigma_{VV} = \frac{512\pi^3\alpha^2}{27s^{10}} M_V^2 |\Psi^0_{B^*_{\bar{b}c}}(0)|^2 |\Psi_{B^*_{b\bar{c}}}(0)|^2 \left(1 - \frac{4M_V^2}{s^2}\right)^{3/2} (3F_A - F_B) . \tag{26}$$

Table: Numerical values of the relativistic parameters

$n^{2S+1}L_J$	M_{B_c}, GeV	$\tilde{R}(0)$, $\tilde{R}'(0)$	ω_{10}, $\tilde{\omega}_{10}$	ω_{01}, $\tilde{\omega}_{01}$	$\omega_{\frac{1}{2}\frac{1}{2}}$, $\tilde{\omega}_{\frac{1}{2}\frac{1}{2}}$	ω_{20}, $\tilde{\omega}_{20}$	ω_{02}, $\tilde{\omega}_{02}$	ω_{11}, $\tilde{\omega}_{11}$
1^1S_0	6.275	0.886	0.0728	0.0089	0.0254	0.0073	0.0001	0.0009
1^3S_1	6.317	0.750	0.0703	0.0086	0.0245	0.0069	0.0001	0.0009

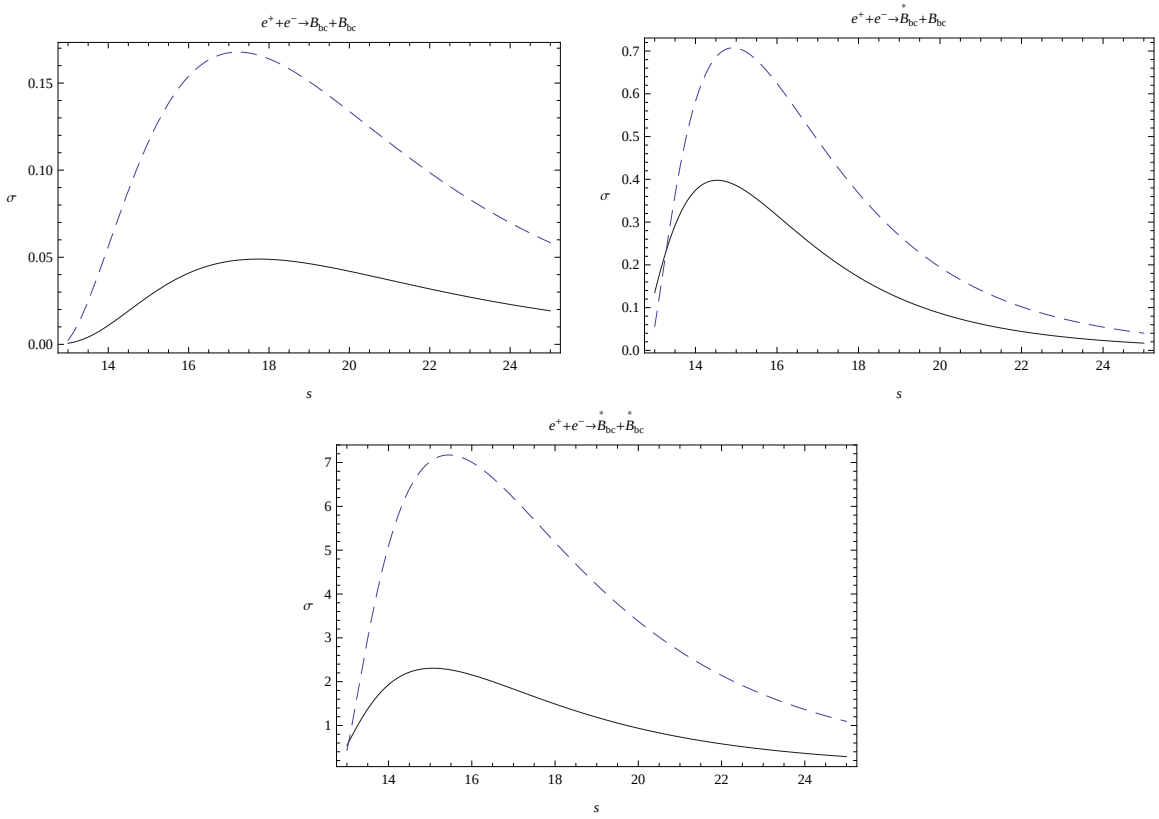

Figure: The cross section in fb of e^+e^- annihilation into a pair of pseudoscalar and vector B_c meson states as a function of the center-of-mass energy s (solid line). The dashed line shows nonrelativistic result without bound state and relativistic corrections.

Assuming that a luminosity at the B-factory $\mathcal{L} = 10^{34}\ cm^{-2} \cdot c^{-1}$ the yield of pairs of vector B_c mesons can be near 50 events per month at the center-of-mass energy $s = 16\ GeV$.

Table: The comparison of obtained results for the production cross sections with nonrelativistic calculation. In third column we present nonrelativistic result obtained in our model.

Final state $B_{\bar{b}c}B_{b\bar{c}}$	Center-of-mass energy s	Nonrelativistic cross section σ_{nr}	Relativistic cross section σ_r
$B^{+}_{\bar{b}c} + B^{-}_{b\bar{c}}$	22.0 GeV	0.10 fb	0.03 fb
$B^{*+}_{\bar{b}c} + B^{-}_{b\bar{c}}$	22.0 GeV	0.10 fb	0.04 fb
$B^{*+}_{\bar{b}c} + B^{*-}_{b\bar{c}}$	22.0 GeV	2.14 fb	0.58 fb

We take into account relativistic corrections of several types:

- Relativistic corrections from the transformation law of the bound state wave functions.
- The relativistic corrections to the production amplitude connected with the relative quark momenta **p** and **q**.
- The relativistic corrections to the quark-quark interaction operator which lead to the modification of the quark bound state wave functions $\Psi^{0}_{B_{\bar{b}c}}(\mathbf{p})$ as compared with nonrelativistic case.
- We also systematically account for the bound state corrections working with masses of B_c mesons.

Our total maximum theoretical errors are estimated in 30%.

Thank you for attention!