Introduction
Model description
Relativistic diquark-antidiquark model
Results
Threshold analysis
Experimental data
Conclusion
Publications

"Fully-heavy tetraquarks in the relativistic diquark-antidiquark picture"

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Introduction
Model description
Relativistic diquark-antidiquark model
Results
Threshold analysis
Experimental data
Conclusion
Publications

Introduction

- Ordinary hadrons: baryons qqq and mesons $q\bar{q}$.
- Exotic hadrons: tetraquarks $qq\bar{q}\bar{q}$, pentaquarks $qqqq\bar{q}$, etc.

- Searches for the $X_{cc\bar{c}\bar{c}}$, $X_{bb\bar{b}\bar{b}}$ are conducted on the Large Hadron Collider (LHC) by the LHCb and CMS Collaborations.

Model description

- $M_c = 1.55$ GeV, $M_b = 4.88$ GeV.
- We consider symmetric quark content: $cc\bar{c}\bar{c}$, $cb\bar{c}\bar{b}$, $bb\bar{b}\bar{b}$.
- Diquark QQ' – antidiquark $\bar{Q}\bar{Q}'$ bound state.
- Ground state (anti)diquarks can be in scalar $J = 0$ (S) or axialvector $J = 1$ (A) state.
- $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$ can contain only axialvector (anti)diquarks, $cb\bar{c}\bar{b}$ can contain both types of (anti)diquarks.

Relativistic diquark-antidiquark model I

- Relativistic Schrödinger-type quasipotential equation:

$$\left(\frac{b^2(M)}{2\mu_R(M)} - \frac{\mathbf{p}^2}{2\mu_R(M)}\right)\Psi_{T,d}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3}\, V(\mathbf{p},\mathbf{q};M)\Psi_{T,d}(\mathbf{q})$$

$$\mu_R = \frac{E_1E_2}{E_1+E_2} = \frac{M^4-(m_1^2-m_2^2)^2}{4M^3}$$

$$b^2(M) = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}$$

Relativistic diquark-antidiquark model II

- Quark-quark interaction quasipotential:

$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q)$$

$$\mathcal{V}(\mathbf{p},\mathbf{q};M) = \frac{1}{2}\Bigg[\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + \\ + V^V_{\text{conf.}}(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k}) + V^S_{\text{conf.}}(\mathbf{k})\Bigg]$$

- Diquark-antidiquark interaction quasipotential:

$$V(\mathbf{p},\mathbf{q};M) = \frac{<d(\mathcal{P})|J_\mu|d(\mathcal{Q})>}{2\sqrt{E_d}\sqrt{E_d}}\frac{4}{3}\alpha_s D^{\mu\nu}(\mathbf{k})\frac{<d'(\mathcal{P}')|J_\nu|d'(\mathcal{Q}')>}{2\sqrt{E_{d'}}\sqrt{E_{d'}}} \\ + \Psi^*_d(\mathcal{P})\Psi^*_{d'}(\mathcal{P}')[J_{d;\mu}J^\mu_{d'}V^V_{\text{conf.}}(\mathbf{k}) + V^S_{\text{conf.}}(\mathbf{k})]\Psi_d(\mathcal{Q})\Psi_{d'}(\mathcal{Q}')$$

Relativistic diquark-antidiquark model III

- Diquark-antidiquark interaction quasipotential in configuration space:

$$
\begin{aligned}
V(r) = {} & \Bigg[V_{\text{Coul.}}(r) + V_{\text{conf.}}(r) + \frac{1}{E_1 E_2}\Bigg\{ \mathbf{p}\Big[V_{\text{Coul.}}(r) + V^{V}_{\text{conf.}}(r)\Big]\mathbf{p} - \frac{1}{4}\Delta V^{V}_{\text{conf.}}(r) + V'_{\text{Coul.}}(r)\frac{\mathbf{L}^2}{2r}\Bigg\}\Bigg]_a \\
& + \Bigg[\Bigg\{ \frac{1}{2}\Big[\frac{1}{E_1(E_1+M_1)} + \frac{1}{E_2(E_2+M_2)}\Big]\frac{V'_{\text{Coul.}}(r)}{r} - \frac{1}{2}\Big[\frac{1}{M_1(E_1+M_1)} + \frac{1}{M_2(E_2+M_2)}\Big]\frac{V'_{\text{conf.}}(r)}{r} \\
& + \frac{\mu_d}{4}\Big[\frac{1}{M_1^2} + \frac{1}{M_2^2}\Big]\frac{V'^{V}_{\text{conf.}}(r)}{r} + \frac{1}{E_1 E_2}\Big[V'_{\text{Coul.}}(r) + \frac{\mu_d}{4}\Big(\frac{E_1}{M_1} + \frac{E_2}{M_2}\Big)V'^{V}_{\text{conf.}}(r)\Big]\frac{1}{r}\Bigg\}\mathbf{L}(\mathbf{S_1}+\mathbf{S_2}) \\
& + \Bigg\{ \frac{1}{2}\Big[\frac{1}{E_1(E_1+M_1)} - \frac{1}{E_2(E_2+M_2)}\Big]\frac{V'_{\text{Coul.}}(r)}{r} - \frac{1}{2}\Big[\frac{1}{M_1(E_1+M_1)} - \frac{1}{M_2(E_2+M_2)}\Big]\frac{V'_{\text{conf.}}(r)}{r} \\
& + \frac{\mu_d}{4}\Big[\frac{1}{M_1^2} - \frac{1}{M_2^2}\Big]\frac{V'^{V}_{\text{conf.}}(r)}{r} + \frac{1}{E_1 E_2}\frac{\mu_d}{4}\Big(\frac{E_1}{M_1} - \frac{E_2}{M_2}\Big)\frac{V'^{V}_{\text{conf.}}(r)}{r}\Bigg\}\mathbf{L}(\mathbf{S_1}-\mathbf{S_2})\Bigg]_b \\
& + \Bigg[\frac{1}{3E_1E_2}\Bigg\{\frac{1}{r}V'_{\text{Coul.}}(r) - V''_{\text{Coul.}}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\Big(\frac{1}{r}V'^{V}_{\text{conf.}}(r) - V''^{V}_{\text{conf.}}(r)\Big)\Bigg\} \times \Big[\frac{3}{r^2}\big(\mathbf{S_1r}\big)\big(\mathbf{S_2r}\big) - \mathbf{S_1S_2}\Big]\Bigg]_c \\
& + \Bigg[\frac{2}{3E_1E_2}\Bigg\{\Delta V_{\text{Coul.}}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\Delta V^{V}_{\text{conf.}}(r)\Bigg\}\mathbf{S_1S_2}\Bigg]_d
\end{aligned}
$$

Results for the $X_{cc\bar{c}\bar{c}}$, $X_{bb\bar{b}\bar{b}}$

Table 1: Masses $M_{QQ'\bar{Q}\bar{Q}'}$ of the ground (1S) and excited (1P, 2S, 1D, 2P, 3S) $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ states. d and $\bar{d}'$ are the axialvector (A) or scalar (S) diquark and antidiquark, respectively. S is the total spin of the diquark-antidiquark system. All masses are given in MeV.

$d\bar{d}'$	nL	n_r	L	S	J	J^{PC}	M
$A\bar{A}$	1S	0	0	0	0	0^{++}	6190
				1	1	1^{+-}	6271
				2	2	2^{++}	6367
	1P	0	1	0	1	1^{--}	6631
				1	0	0^{-+}	6628
					1	1^{-+}	6634
					2	2^{-+}	6644
				2	1	1^{--}	6635
					2	2^{--}	6648
					3	3^{--}	6664
	2S	1	0	0	0	0^{++}	6782
				1	1	1^{+-}	6816
				2	2	2^{++}	6868
	1D	0	2	0	2	2^{++}	6921
				1	1	1^{+-}	6909
					2	2^{+-}	6920
					3	3^{+-}	6932
				2	0	0^{++}	6899
					1	1^{++}	6904
					2	2^{++}	6915
					3	3^{++}	6929
					4	4^{++}	6945
	2P	1	1	0	1	1^{--}	7091
				1	0	0^{-+}	7100
					1	1^{-+}	7099
					2	2^{-+}	7098
				2	1	1^{--}	7113
					2	2^{--}	7113
					3	3^{--}	7112
	3S	2	0	0	0	0^{++}	7259
				1	1	1^{+-}	7287
				2	2	2^{++}	7333

$d\bar{d}'$	nL	n_r	L	S	J	J^{PC}	M
$A\bar{A}$	1S	0	0	0	0	0^{++}	19315
				1	1	1^{+-}	19320
				2	2	2^{++}	19331
	1P	0	1	0	1	1^{--}	19536
				1	0	0^{-+}	19533
					1	1^{-+}	19535
					2	2^{-+}	19539
				2	1	1^{--}	19534
					2	2^{--}	19538
					3	3^{--}	19545
	2S	1	0	0	0	0^{++}	19680
				1	1	1^{+-}	19682
				2	2	2^{++}	19687
	1D	0	2	0	2	2^{++}	19715
				1	1	1^{+-}	19710
					2	2^{+-}	19714
					3	3^{+-}	19720
				2	0	0^{++}	19705
					1	1^{++}	19707
					2	2^{++}	19711
					3	3^{++}	19717
					4	4^{++}	19724
	2P	1	1	0	1	1^{--}	19820
				1	0	0^{-+}	19821
					1	1^{-+}	19821
					2	2^{-+}	19822
				2	1	1^{--}	19823
					2	2^{--}	19823
					3	3^{--}	19824
	3S	2	0	0	0	0^{++}	19941
				1	1	1^{+-}	19943
				2	2	2^{++}	19947

Results for the $X_{cb\bar{c}\bar{b}}$

Table 2: Masses $M_{QQ'\bar{Q}\bar{Q}'}$ of the ground (1S) and excited (1P, 2S, 1D, 2P, 3S) $cb\bar{c}\bar{b}$ states. d and $\bar{d}'$ are the axialvector (A) or scalar (S) diquark and antidiquark, respectively. S is the total spin of the diquark-antidiquark system. All masses are given in MeV.

$d\bar{d}'$	nL	n_r	L	S	J	J^{PC}	M
$A\bar{A}$	1S	0	0	0	0	0^{++}	12838
				1	1	1^{+-}	12855
				2	2	2^{++}	12883
	1P	0	1	0	1	1^{--}	13103
				1	0	0^{-+}	13100
					1	1^{-+}	13103
					2	2^{-+}	13108
				2	1	1^{--}	13103
					2	2^{--}	13109
					3	3^{--}	13116
	2S	1	0	0	0	0^{++}	13247
				1	1	1^{+-}	13256
				2	2	2^{++}	13272
	1D	0	2	0	2	2^{++}	13306
				1	1	1^{+-}	13299
					2	2^{+-}	13304
					3	3^{+-}	13311
				2	0	0^{++}	13293
					1	1^{++}	13296
					2	2^{++}	13301
					3	3^{++}	13308
					4	4^{++}	13317
	2P	1	1	0	1	1^{--}	13428
				1	0	0^{-+}	13431
					1	1^{-+}	13431
					2	2^{-+}	13431
				2	1	1^{--}	13434
					2	2^{--}	13435
					3	3^{--}	13436
	3S	2	0	0	0	0^{++}	13558
				1	1	1^{+-}	13566
				2	2	2^{++}	13580

$d\bar{d}'$	nL	n_r	L	S	J	J^{PC}	M
$\frac{1}{\sqrt{2}}\left(A\bar{S} \pm S\bar{A}\right)$	1S	0	0	1	1	$1^{+\pm}$	12863
	1P	0	1		0	$0^{-\pm}$	13096
					1	$1^{-\pm}$	13099
					2	$2^{-\pm}$	13104
	2S	1	0		1	$1^{+\pm}$	13257
	1D	0	2		1	$1^{+\pm}$	13293
					2	$2^{+\pm}$	13298
					3	$3^{+\pm}$	13305
	2P	1	1		0	$0^{-\pm}$	13426
					1	$1^{-\pm}$	13426
					2	$2^{-\pm}$	13427
	3S	2	0		1	$1^{+\pm}$	13566
$S\bar{S}$	1S	0	0	0	0	0^{++}	12856
	1P	0	1		1	1^{--}	13095
	2S	1	0		0	0^{++}	13250
	1D	0	2		2	2^{++}	13293
	2P	1	1		1	1^{--}	13420
	3S	2	0		0	0^{++}	13559

Threshold analysis I

Introduction
Model description
Relativistic diquark-antidiquark model
Results
Threshold analysis
Experimental data
Conclusion
Publications

$$\Delta = M_{QQ'\bar{Q}\bar{Q}'} - M_{\text{threshold}}$$

- Many masses lie well above thresholds with $\Delta > 300$ MeV.
- Significant amount of masses lie in the $100 < \Delta < 300$ MeV interval.
- Few masses lie in the $0 < \Delta < 100$ MeV interval.
- Such behavior is seen across all excitations and all quark compositions.

Threshold analysis II

Example:

Table 3: Masses M of the ground (1S) and excited (2S, 1D) $cb\bar{c}\bar{b}$ states composed from the axialvector diquarks and the corresponding meson-meson thresholds. d and $\bar{d}'$ are the axialvector (A) or scalar (S) diquark and antidiquark, respectively. S is the total spin of the diquark-antidiquark system. M_{thr} is the corresponding meson-meson threshold. Δ is the difference between the tetraquark mass and threshold: $\Delta = M - M_{thr}$. All masses are given in MeV.

$QQ\overline{QQ}'$	$d\bar{d}'$	nL	S	J^{PC}	M	M_{thr}	Δ	Meson pair
$cb\bar{c}\bar{b}$	$A\bar{A}$	1S	0	0^{++}	12838	12383	455	$\eta_c(1S)\eta_b(1S)$
			1	1^{+-}	12855	12444	411	$\eta_c(1S)\Upsilon(1S)$
			2	2^{++}	12883	12557	326	$J/\psi(1S)\Upsilon(1S)$
		2S	0	0^{++}	13247	12383	864	$\eta_c(1S)\eta_b(1S)$
			1	1^{+-}	13256	12444	812	$\eta_c(1S)\Upsilon(1S)$
			2	2^{++}	13272	12557	715	$J/\psi(1S)\Upsilon(1S)$
		1D	0	2^{++}	13306	12557	749	$J/\psi(1S)\Upsilon(1S)$
			1	1^{+-}	13299	12444	855	$\eta_c(1S)\Upsilon(1S)$
				2^{+-}	13304	13148	156	$\eta_c(1S)\Upsilon_2(1D)$
						13222	82	$\psi_2(3823)\eta_b(1S)$
				3^{+-}	13311	13241	70	$\psi_3(3842)\eta_b(1S)$
			2	0^{++}	13293	12383	910	$\eta_c(1S)\eta_b(1S)$
				1^{++}	13296	12557	739	$J/\psi(1S)\Upsilon(1S)$
				2^{++}	13301	12557	744	$J/\psi(1S)\Upsilon(1S)$
				3^{++}	13308	13261	47	$J/\psi(1S)\Upsilon_2(1D)$
						13284	24	$\psi_2(3823)\Upsilon(1S)$
						13303	5	$\psi_3(3842)\Upsilon(1S)$
				4^{++}	13317	13303	14	$\psi_3(3842)\Upsilon(1S)$

Threshold analysis: bottom

- The only exceptions are the two $X_{bb\bar{b}\bar{b}}$ states lying approximately 100 MeV below any possible threshold:

Table 4: Tetraquark states lying under fall-apart thresholds.

$X_{QQ'\bar{Q}\bar{Q}'}$	nL	S	J^{PC}	M, MeV	Δ, MeV	threshold
$X_{bb\bar{b}\bar{b}}$	1D	1	3^{+-}	19720	-92	$h_b(1P)\chi_{b2}(1P)$
		2	4^{++}	19724	-100	$\chi_{b2}(1P)\chi_{b2}(1P)$

- The fall-apart decays into a pair of heavy mesons are forbidden, thus they can be narrow states.

Experimental data I

- In 2020 the LHCb Collaboration announced the discovery of the narrow resonance X(6900).
- Several other broad structures peaking at about 6.4 and 7.2 GeV were reported.
- In 2022 CMS and ATLAS Collaborations presented preliminary data confirming X(6900) and giving hints of few more states including structures at 6.4 and 7.2 GeV.

Experimental data II

Current observation status and our predictions:

Table 5: Exotic X states observed by the LHCb, CMS and ATLAS Collaborations in di-J/ψ invariant mass spectra and our candidates. All masses are given in MeV.

Collaboration	State	Mass, MeV	Width, MeV	Our candidates			
				nL	S	J^{PC}	Mass, MeV
ATLAS	X(6200)	$6220 \pm 50^{+40}_{-50}$	$310 \pm 120^{+70}_{-80}$	1S	0	0^{++}	6190
LHCb	X(6400)	≈ 6400		1S	2	2^{++}	6367
CMS	X(6600)	$6552 \pm 10 \pm 12$	$124 \pm 29 \pm 34$	1S	2	2^{++}	6367
ATLAS		$6620 \pm 30^{+20}_{-10}$	$310 \pm 90^{+60}_{-110}$	2S	0	0^{++}	6782
LHCb	X(6900)	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	2S	2	2^{++}	6868
		$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	1D	0	2^{++}	6921
CMS		$6927 \pm 9 \pm 5$	$122 \pm 22 \pm 19$	1D	2	0^{++}	6899
				1D	2	1^{++}	6904
ATLAS		$6870 \pm 30^{+60}_{-10}$	$120 \pm 40^{+30}_{-10}$	1D	2	2^{++}	6915
LHCb	X(7200)	≈ 7200		3S	0	0^{++}	7259
ATLAS		$7220 \pm 30^{+20}_{-30}$	$100^{+130+60}_{-70-50}$				
CMS	X(7300)	$7287 \pm 19 \pm 5$	$95 \pm 46 \pm 20$	3S	0	0^{++}	7259
				3S	2	2^{++}	7333

Conclusion I

Introduction
Model description
Relativistic diquark-antidiquark model
Results
Threshold analysis
Experimental data
Conclusion
Publications

- Masses of ground and excited states of fully-heavy tetraquarks were calculated.
- The finite diquark size was taken into account.

Conclusion II

Introduction
Model description
Relativistic diquark-antidiquark model
Results
Threshold analysis
Experimental data
Conclusion
Publications

- The predicted masses are consistent with the results of experimental searches for the $X_{bb\bar{b}\bar{b}}$ state by the LHC and CMS (which found nothing).
- However, the two $X_{bb\bar{b}\bar{b}}$ excitations can still be narrow states.

Conclusion III

Introduction
Model description
Relativistic diquark-antidiquark model
Results
Threshold analysis
Experimental data
Conclusion
Publications

- Masses of resonances in the di-J/ψ production detected at the LHCb, CMS and ATLAS agree with our predictions for the ground and excited $X_{cc\bar{c}\bar{c}}$ states.
- Tetraquark states which are most convenient for the experimental detection are identified.

Publications

Introduction
Model description
Relativistic diquark-antidiquark model
Results
Threshold analysis
Experimental data
Conclusion
Publications

- This talk is based on the following publications:
  - Masses of the $QQ\bar{Q}\bar{Q}$ tetraquarks in the relativistic diquark-antidiquark picture, Physical Review D, 2020, vol. 102, № 11, p. 114030;
  - Heavy Tetraquarks in the Relativistic Quark Model, Universe, 2021, vol. 7, № 4, p. 94;
  - Fully Heavy Tetraquark Spectroscopy in the Relativistic Quark Model, Symmetry, 2022, vol. 14, № 12, p. 2504.