Associated quarkonia production

in a single boson e^+e^- annihilation

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Contents

Studied processes

- The cross sections of charmonia pair production, bottomonia pair production, and also an associated charmonium and bottomonium production in electron-positron annihilation has been studied.
- The single boson annihilation have been considered, including the annihilation into both photon and Z-boson.
- Processes of pseudoscalar 1^1S_0 (η_c, η_b) and vector 1^3S_1 $(J/\psi, \Upsilon)$ states production have been taken into account.

$$CC: \begin{cases} e^+e^- \xrightarrow{\gamma^*,\ Z^*} J/\psi\ \eta_c, \\ e^+e^- \xrightarrow{Z^*} J/\psi\ J/\psi, \\ e^+e^- \not\to \eta_c\ \eta_c; \end{cases}$$

$$BB: \begin{cases} e^+e^- \xrightarrow{\gamma^*,\ Z^*} \Upsilon\ \eta_b, \\ e^+e^- \xrightarrow{Z^*} \Upsilon\ \Upsilon, \\ e^+e^- \not\to \eta_b\ \eta_b; \end{cases}$$

$$CB: \begin{cases} e^+e^- \xrightarrow{\gamma^*,\ Z^*} J/\psi\ \eta_b, \\ e^+e^- \xrightarrow{\gamma^*,\ Z^*} \Upsilon\ \eta_c, \\ e^+e^- \xrightarrow{Z^*} J/\psi\ \Upsilon, \\ e^+e^- \not\to \eta_c\ \eta_b. \end{cases}$$

Motivation

- Recent experiments on LHC, BELLE-II and BES-III have led to various discoveries in heavy quark physics.
- Research of meson pair production: theoretical predictions for cross-section have underestimated the real yield [1–3] by the order of magnitude;
- Additional interest to this topic has been caused by the recent observation of the structure in spectrum at large statistics by LHCb collaboration [4];
- Experiments at the LHC (e.g. search for H and Z-bosons decay into J/ψ or Υ pairs) [5];
- Researches on future colliders.

Relevance for future e^+e^- colliders

Collider	$\sqrt{s}$, GeV		
BaBar	~ 10		
BELLE-II	~ 10.5		
BES-III	4.63		
ILC	250	500	
FCC-ee	90 ÷ 350		
CEPC	91.2	160	240
CLIC	380	1500	3000
Muon	$(3 \div 14) \times 10^3$		

$\sqrt{s_{th}} \sim 12.5$ GeV

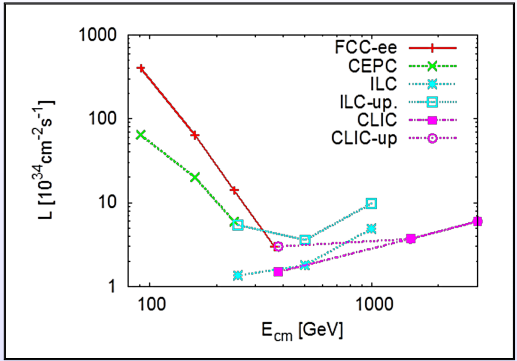


Fig. 1 – Collision energy dependence of luminosity for future e^+e^- colliders and for their potential improvements. [6]

Heavy quark physics

Heavy quark physics is extremely promising as a verification tool for various theories describing interactions (e.g. QCD).

NRQCD

Physical meaning is the separation of hard and soft interactions in the hierarchy assumption: $m_q \gg m_q v \gg m_q v^2$.

$$\text{Forces:} \begin{cases} \text{Hard} - \text{perturbative production of } q\bar{q}\text{-pairs} - \mathcal{A}_{Q_1 Q_2}(P, Q); \\ \text{Soft} - \text{bound state dynamics} - R_{Q_1}(0) \text{ and } R_{Q_2}(0). \end{cases}$$

In the current work only color singlet states have been considered $\Rightarrow$ CSM.

Selection rules

- C-parity prohibitions:
  $\gamma^* \nrightarrow PP$, $\gamma^* \nrightarrow VV$, $Z^*_{\text{axial}} \nrightarrow VP$,
  $Z^*_{\text{vector}} \nrightarrow PP$, $Z^*_{\text{vector}} \nrightarrow VV$;
- Combined CP-parity prohibitions: $Z^*_{\text{axial}} \nrightarrow PP$.

Therefore, the production of the pseudoscalar meson pair in single boson e^+e^- annihilation is suppressed due to the selection rules.

- The only acceptable production mechanism for a vector pair is: $e^+e^- \to Z^*_{\text{axial}} \to VV$.

The leading order in case of charmonia pair or bottomonia pair production is a tree-level QCD order.
In case of associated charmonium and bottomonium production, the tree-level QCD diagrams are prohibited $\Rightarrow$ The one-loop QCD order is the leading one.

Amplitude contributions in case of charmonia pair or bottomonia pair production (CC, BB)

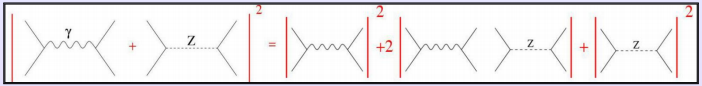
The total amplitude consists of several contributions, corresponding to a certain set of production mechanism, order of the coupling constant (topology) and intermediate boson.

- On the one hand, topology:

$$\text{QCD mechanism} \rightarrow \begin{cases} \text{LO} \sim \mathcal{O}\left(e^2 g_s^2\right); \\ \text{NLO} \sim \mathcal{O}\left(e^2 g_s^4\right); \end{cases} \qquad \text{EW mechanism} \rightarrow \begin{cases} \text{LO} \sim \mathcal{O}\left(e^4\right); \\ \text{NLO} \sim \mathcal{O}\left(e^4 g_s^2\right); \end{cases}$$

$$\mathcal{A} = \mathcal{A}^{QCDLO} + \mathcal{A}^{QCDNLO} + \mathcal{A}^{EWLO} + \mathcal{A}^{EWNLO}$$

- On the other hand, the intermediate boson:



$$\Rightarrow |\mathcal{A}|^2 = |\mathcal{A}_\gamma|^2 + |\mathcal{A}_Z|^2 + 2Re\left(\mathcal{A}_\gamma^* \, \mathcal{A}_Z\right)$$

Amplitude contributions in case of charmonia pair or bottomonia pair production (CC, BB)

Considering all the contributions above we obtain 26 terms for the squared amplitude:

$$|\mathcal{A}|^2 = |\mathcal{A}_\gamma^{QCDLO}|^2 + 2Re\left(\mathcal{A}_\gamma^{QCDLO*}\mathcal{A}_Z^{QCDLO}\right) + |\mathcal{A}_Z^{QCDLO}|^2 + |\mathcal{A}_\gamma^{EWLO}|^2 + 2Re\left(\mathcal{A}_\gamma^{EWLO*}\mathcal{A}_Z^{EWLO}\right) + |\mathcal{A}_Z^{EWLO}|^2 +$$
$$+ 2Re\left(\mathcal{A}_\gamma^{QCDLO*}\mathcal{A}_\gamma^{EWLO}\right) + 2Re\left(\mathcal{A}_\gamma^{QCDLO*}\mathcal{A}_Z^{EWLO}\right) + 2Re\left(\mathcal{A}_Z^{QCDLO*}\mathcal{A}_\gamma^{EWLO}\right) + 2Re\left(\mathcal{A}_Z^{QCDLO*}\mathcal{A}_Z^{EWLO}\right) +$$

$$2Re\left(\mathcal{A}_\gamma^{QCDLO*}\mathcal{A}_\gamma^{QCDNLO}\right) + 2Re\left(\mathcal{A}_\gamma^{QCDLO*}\mathcal{A}_Z^{QCDNLO}\right) + 2Re\left(\mathcal{A}_Z^{QCDLO*}\mathcal{A}_\gamma^{QCDNLO}\right) + 2Re\left(\mathcal{A}_Z^{QCDLO*}\mathcal{A}_Z^{QCDNLO}\right) +$$
$$+ 2Re\left(\mathcal{A}_\gamma^{EWLO*}\mathcal{A}_\gamma^{QCDNLO}\right) + 2Re\left(\mathcal{A}_\gamma^{EWLO*}\mathcal{A}_Z^{QCDNLO}\right) + 2Re\left(\mathcal{A}_Z^{EWLO*}\mathcal{A}_\gamma^{QCDNLO}\right) + 2Re\left(\mathcal{A}_Z^{EWLO*}\mathcal{A}_Z^{QCDNLO}\right) +$$
$$+ 2Re\left(\mathcal{A}_\gamma^{QCDLO*}\mathcal{A}_\gamma^{EWNLO}\right) + 2Re\left(\mathcal{A}_\gamma^{QCDLO*}\mathcal{A}_Z^{EWNLO}\right) + 2Re\left(\mathcal{A}_Z^{QCDLO*}\mathcal{A}_\gamma^{EWNLO}\right) + 2Re\left(\mathcal{A}_Z^{QCDLO*}\mathcal{A}_Z^{EWNLO}\right) +$$
$$+ 2Re\left(\mathcal{A}_\gamma^{EWLO*}\mathcal{A}_\gamma^{EWNLO}\right) + 2Re\left(\mathcal{A}_\gamma^{EWLO*}\mathcal{A}_Z^{EWNLO}\right) + 2Re\left(\mathcal{A}_Z^{EWLO*}\mathcal{A}_\gamma^{EWNLO}\right) + 2Re\left(\mathcal{A}_Z^{EWLO*}\mathcal{A}_Z^{EWNLO}\right) + \ldots$$

In the following analysis squared NLO amplitude terms are not considered, as they exceed the required order of accuracy.

Amplitude contributions in case of the associated charmonium and bottomonium production (CB)

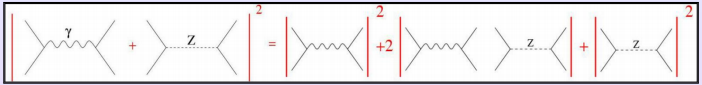
The total amplitude consists of several contributions, corresponding to a certain set of production mechanism, order of the coupling constant (topology) and intermediate boson. In this case, the tree-level QCD diagrams are prohibited So the one-loop QCD order is the leading one.

- On the one hand, topology:

QCD mechanism → One loop $\sim \mathcal{O}\left(e^2 g_s^4\right)$; EW mechanism → Tree level $\sim \mathcal{O}\left(e^4\right)$;

$$\mathcal{A} = \mathcal{A}^{QCD} + \mathcal{A}^{EW}$$

- On the other hand, the intermediate boson:



$$\Rightarrow |\mathcal{A}|^2 = |\mathcal{A}_\gamma|^2 + |\mathcal{A}_Z|^2 + 2Re\left(\mathcal{A}_\gamma^* \ \mathcal{A}_Z\right)$$

Amplitude contributions in case of the associated charmonium and bottomonium production (CB)

Considering all the contributions above we obtain 10 terms for the squared amplitude:

$$|\mathcal{A}|^2 = |\mathcal{A}_\gamma^{EW}|^2 + 2Re\left(\mathcal{A}_\gamma^{EW*}\mathcal{A}_Z^{EW}\right) + |\mathcal{A}_Z^{EW}|^2 + |\mathcal{A}_\gamma^{QCD}|^2 + 2Re\left(\mathcal{A}_\gamma^{QCD*}\mathcal{A}_Z^{QCD}\right) + |\mathcal{A}_Z^{QCD}|^2 + 2Re\left(\mathcal{A}_\gamma^{EW*}\mathcal{A}_\gamma^{QCD}\right) + 2Re\left(\mathcal{A}_\gamma^{EW*}\mathcal{A}_Z^{QCD}\right) + 2Re\left(\mathcal{A}_Z^{EW*}\mathcal{A}_\gamma^{QCD}\right) + 2Re\left(\mathcal{A}_Z^{EW*}\mathcal{A}_Z^{QCD}\right) + \dots$$

In the following analysis squared NLO amplitude terms are not considered, as they exceed the required order of accuracy.

Projectors

According to the selection rules, quarks form two final bound states which are either two vectors or a vector and a pseudoscalar.

Bound state	Projection operator	
	Vector	Pseudoscalar
$\Psi_{c\bar{c}}$	$\Pi_{J/\psi}(P_c, m_c) = \frac{\not{P}-2m_c}{2\sqrt{2m_c}} \not{\epsilon}^{J/\psi} \otimes \frac{1}{\sqrt{N_c}}$	$\Pi_{\eta_c}(Q_c, m_c) = \frac{\not{Q}-2m_c}{2\sqrt{2m_c}}\gamma^5 \otimes \frac{1}{\sqrt{N_c}}$
$\Psi_{b\bar{b}}$	$\Pi_{\Upsilon}(P_b, m_b) = \frac{\not{P}-2m_b}{2\sqrt{2m_b}} \not{\epsilon}^{\Upsilon} \otimes \frac{1}{\sqrt{N_c}}$	$\Pi_{\eta_b}(Q_b, m_b) = \frac{\not{Q}-2m_b}{2\sqrt{2m_b}}\gamma^5 \otimes \frac{1}{\sqrt{N_c}}$

$\epsilon^{J/\psi}$ and ϵ^{Υ} – polarizations of J/ψ and Υ mesons satisfying:

$$\epsilon^{J/\psi} \cdot \epsilon^{J/\psi *} = -1, \ \epsilon^{J/\psi} \cdot P_c = 0, \ \epsilon^{\Upsilon} \cdot \epsilon^{\Upsilon *} = -1 \text{ and } \epsilon^{\Upsilon} \cdot P_b = 0.$$

Software and packages

The whole calculation technique has been performed in Wolfram Mathematica using the following additional toolchain:

FeynArts → FeynCalc(TIDL) → Apart → FIRE → X

- FeynArts [7] – generation of diagrams and corresponding amplitudes;
- FeynCalc [8] – calculation of tree-level amplitudes, algebraic calculations with Dirac and colour matrices, including the trace evaluation;
- TIDL, Apart [9] – additional simplifications;
- FIRE [10] – reduction to master integrals [11];
- Package-X [12] – evaluation of master integrals;

Diagram compilation technique – production of charmonia pair or bottomonia pair

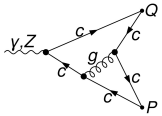
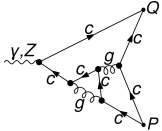
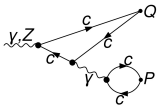
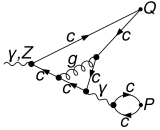
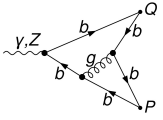
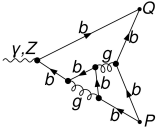
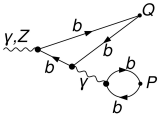
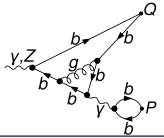
QCDLO $\sim \mathcal{O}\left(e^2 g_s^2\right)$	QCDNLO $\sim \mathcal{O}\left(e^2 g_s^4\right)$	EWLO $\sim \mathcal{O}\left(e^4\right)$	EWNLO $\sim \mathcal{O}\left(e^4 g_s^2\right)$
			
			

Fig. 2 – Examples of Feynman diagram compilation for the following processes: $e^+e^- \to J/\psi\ \eta_c$ (top figures) and $e^+e^- \to \Upsilon\ \eta_b$ (bottom figures).

Diagram compilation technique – associated production of charmonium and bottomonium

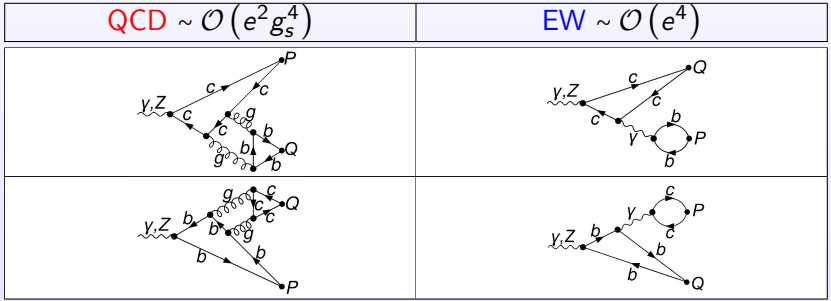


Fig. 3 – Examples of Feynman diagram compilation for the following processes: $e^+e^- \to J/\psi\ \eta_b$ and $e^+e^- \to \Upsilon\ \eta_c$.

Diagram compilation technique – number of diagrams

Subprocess	QCD		EW	
	LO	NLO	LO	NLO
1. $\gamma^* \to J/\psi\ \eta_c$	4	80	6	10
2. $Z^* \to J/\psi\ \eta_c$	4	92	6	10
3. $\gamma^* \to J/\psi\ J/\psi$	0	0	0	0
4. $Z^* \to J/\psi\ J/\psi$	4	86	8	20
1. $\gamma^* \to \Upsilon\ \eta_b$	4	80	6	10
2. $Z^* \to \Upsilon\ \eta_b$	4	92	6	10
3. $\gamma^* \to \Upsilon\ \Upsilon$	0	0	0	0
4. $Z^* \to \Upsilon\ \Upsilon$	4	86	8	20

Subprocess	QCD	EW
1. $\gamma^* \to J/\psi\ \eta_b$	6	2
2. $Z^* \to J/\psi\ \eta_b$	6	2
3. $\gamma^* \to \Upsilon\ \eta_c$	6	2
4. $Z^* \to \Upsilon\ \eta_c$	6	2
5. $\gamma^* \to \Upsilon\ J/\psi$	0	0
6. $Z^* \to \Upsilon\ J/\psi$	0	4

Number of topologically non-equivalent Feynman diagrams with non-zero contributions to total amplitude in case of each production mechanism and each intermediate boson.

Regularization and renormalization

CDR regularization scheme:

All momenta are considered in $D = 4 - 2\varepsilon$ dimensions:

$$\{\gamma^\mu \gamma^\nu\} = 2g^{\mu\nu}, \; g_{\mu\nu} g^{\mu\nu} = D$$

γ^5 interpretation:

Traces with an odd number of γ^5 are left with a single γ^5 to the right, interpreted as:

$$\gamma^5 = \frac{-i}{24} \varepsilon_{\alpha\beta\sigma\rho} \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\rho,$$

where $\varepsilon_{\alpha\beta\sigma\rho}$ is either 4-dimensional, or D-dimensional.

"On shell" scheme for mass and spinor renormalization, $\overline{MS}$ scheme for coupling constant:

$$Z_m^{OS} = 1 - \frac{\alpha_s}{4\pi} C_F C_\epsilon \left[\frac{3}{\epsilon_{UV}} + 4 \right] + \mathcal{O}\left(\alpha_s^2\right)$$

$$Z_2^{OS} = 1 - \frac{\alpha_s}{4\pi} C_F C_\epsilon \left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} + 4 \right] + \mathcal{O}\left(\alpha_s^2\right)$$

$$Z_g^{\overline{MS}} = 1 - \frac{\beta_0}{2} \frac{\alpha_s}{4\pi} \left[\frac{1}{\epsilon_{UV}} - \gamma_E + \log(4\pi) \right] + \mathcal{O}\left(\alpha_s^2\right)$$

- Automatic tools do not distinguish ε_{IR} and ε_{UV}
- Singular parts carry poles of order $\sim 1/\epsilon$ only.

$$Z_2^2 \mathcal{A}^{LO} \Big|_{\substack{m \to Z_m m, \\ g_s \to Z_g g_s}} = \mathcal{A}^{LO} + \mathcal{A}^{CT} + \mathcal{O}(g_s^6).$$

Features of charmonia pair and bottomonia pair production (CC, BB)

- Tree-level amplitudes of charmonia pair and bottomonia pair production are interchangeable when replacing quark masses and charges for each of the production mechanisms.
- Consideration of diagrams with secondary intermediate Z-boson instead of a photon producing a vector boson leads to negligible corrections of order $\mathcal{O}\left(\frac{m_q^2}{M_Z^2-4m_q^2}\right)$. They are investigated more precisely in [13].
- Amplitude asymptotic behavior for $s \to \infty$:

$$\mathcal{A}^{QCDLO} \sim \frac{1}{s^2}; \quad \frac{\mathcal{A}^{QCDNLO}}{\mathcal{A}^{QCDLO}} \sim \alpha_s \left(c_2 \log^2 s + c_1 \log s + c_0 + c_\mu \log \mu + c_{m_q} \log m_q\right);$$

$$\mathcal{A}^{EWLO} \sim \frac{1}{s}; \quad \frac{\mathcal{A}^{EWNLO}}{\mathcal{A}^{EWLO}} \sim \alpha_s \left(d_1 \log s + d_0\right).$$

Production of charmonia pairs (CC)

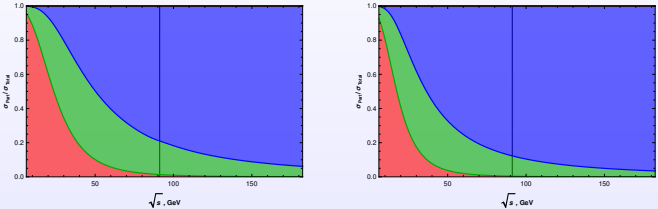


Fig. 4 – Relative contribution of different amplitude terms to the total cross section in case of $J/\psi\ \eta_c$ pair production (left) and $J/\psi\ J/\psi$ pair production (right) as functions of collision energy.

$$|\mathcal{A}|^2 = |\mathcal{A}^{QCD}|^2 + 2Re(\mathcal{A}^{QCD}\mathcal{A}^{EW*}) + |\mathcal{A}^{EW}|^2$$

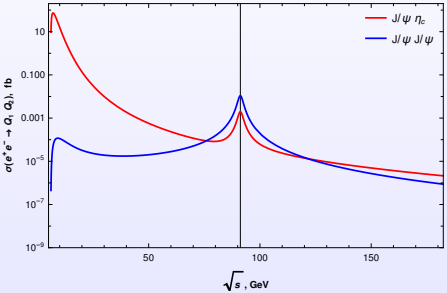


Fig. 5 – The total cross sections dependence on the collision energy for charmonia pair production.

Production of bottomonia pairs (BB)

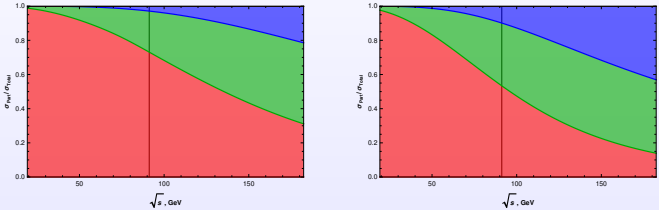


Fig. 6 – Relative contribution of different amplitude terms to the total cross section in case of $\Upsilon\ \eta_b$ pair production (left) and $\Upsilon\ \Upsilon$ production (right) as functions of collision energy.

$$|\mathcal{A}|^2 = |\mathcal{A}^{QCD}|^2 + 2Re(\mathcal{A}^{QCD}\mathcal{A}^{EW*}) + |\mathcal{A}^{EW}|^2$$

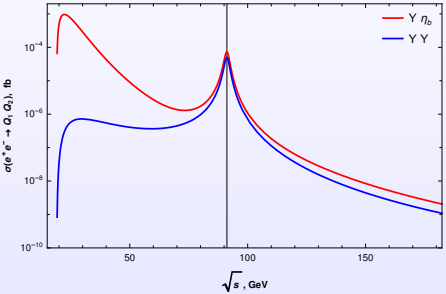


Fig. 7 – The total cross sections dependence on the collision energy for bottomonia pair production.

NLO corrections for charmonia pair and bottomonia pair production (CC, BB)

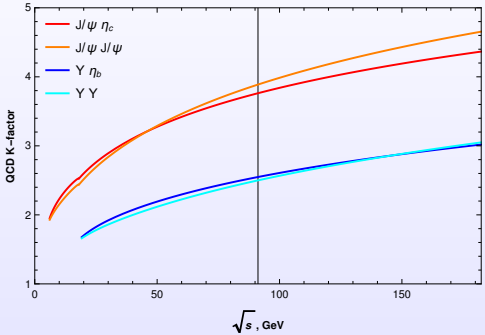


Fig. 8 – The total QCDNLO and QCDLO cross sections ratio (QCD K-factor) dependence on the collision energy.

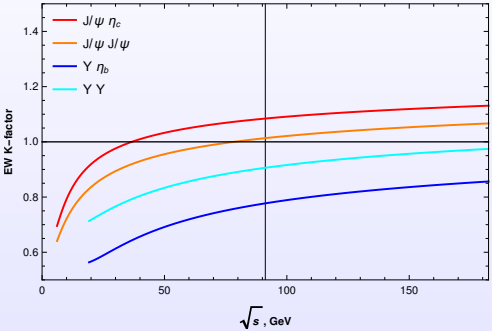


Fig. 9 – The total EWNLO and EWLO cross sections ratio (EW K-factor) dependence on the collision energy.

Features of associated charmonium and bottomonium production (CB)

- Amplitudes of EW production mechanism in case of $J/\psi\ \eta_b$ and $\Upsilon\ \eta_c$ pairs are interchangeable when replacing quark masses and charges.
- Consideration of diagrams with secondary intermediate Z-boson instead of a photon producing a vector boson leads to negligible corrections of order $\mathcal{O}\left(\frac{m_q^2}{M_Z^2-4m_q^2}\right)$. They are investigated more precisely in [13].
- Amplitude asymptotic behavior for $s \to \infty$:

$$\mathcal{A}^{EW} \sim \frac{1}{s^2}; \quad \mathcal{A}^{QCD} \sim \frac{1}{s^3}\log s;$$

- Cross section asymptotic behavior for $s \to \infty$:

$$\sigma_{PV} \sim \frac{1}{s^4}\left[1+\mathcal{O}\left(\frac{\log s}{s}\right)+\mathcal{O}\left(\left(\frac{\log s}{s}\right)^2\right)\right]; \quad \sigma_{VV} \sim \frac{1}{s^4};$$

Associated production of charmonium and bottomonium (CB)

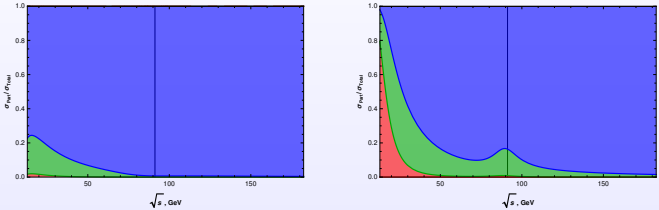


Fig. 10 – Relative contribution of different amplitude terms to the total cross section in case of $J/\psi\ \eta_b$ pair production (left) and $\Upsilon\ \eta_c$ production (right) as functions of collision energy. In case of $J/\psi\ \Upsilon$ pair production only EW diagrams contribute to the total amplitude.

$$|\mathcal{A}|^2 = |\mathcal{A}^{QCD}|^2 + 2Re(\mathcal{A}^{QCD}\mathcal{A}^{EW*}) + |\mathcal{A}^{EW}|^2$$

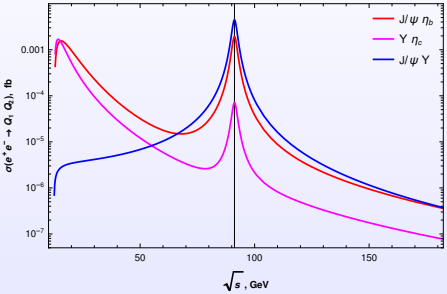


Fig. 11 – The total cross sections dependence on the collision energy for associated charmonium and bottomonium production.

Numerical values of parameters

$$m_c = 1.5 \text{ GeV}; \quad m_b = 4.7 \text{ GeV}; \quad M_Z = 91.2 \text{ GeV}; \quad \Gamma_Z = 2.5 \text{ GeV};$$
$$R^2_{J/\psi}(0) = R^2_{\eta_c}(0) = 1.1 \text{ GeV}^3; \quad R^2_{\Upsilon}(0) = R^2_{\eta_b}(0) = 5.9 \text{ GeV}^3; \quad \sin^2\theta_w = 0.23 \ .$$

$\sigma(s)$, fbn	$\sqrt{s} = 0.25 M_Z$	$\sqrt{s} = 0.5 M_Z$	$\sqrt{s} = M_Z$	$\sqrt{s} = 2M_Z$	$\sqrt{s_{max}}$, GeV	$\sigma(s_{max})$
$J/\psi\ \eta_c$	$5.75 \cdot 10^{-2}$	$9.86 \cdot 10^{-4}$	$1.97 \cdot 10^{-3}$	$2.13 \cdot 10^{-6}$	7.0	$7.31 \cdot 10^{1}$
$J/\psi\ J/\psi$	$2.62 \cdot 10^{-5}$	$1.82 \cdot 10^{-5}$	$1.09 \cdot 10^{-2}$	$8.73 \cdot 10^{-7}$	9.2	$1.17 \cdot 10^{-4}$
$\Upsilon\ \eta_b$	$9.45 \cdot 10^{-4}$	$1.89 \cdot 10^{-5}$	$7.35 \cdot 10^{-5}$	$2.06 \cdot 10^{-9}$	22.2	$9.59 \cdot 10^{-4}$
$\Upsilon\ \Upsilon$	$3.62 \cdot 10^{-7}$	$4.63 \cdot 10^{-7}$	$4.91 \cdot 10^{-5}$	$1.09 \cdot 10^{-9}$	29.5	$7.21 \cdot 10^{-7}$
$J/\psi\ \eta_b$	$6.29 \cdot 10^{-4}$	$4.73 \cdot 10^{-5}$	$1.94 \cdot 10^{-3}$	$3.58 \cdot 10^{-7}$	15.4	$1.56 \cdot 10^{-3}$
$\Upsilon\ \eta_c$	$3.00 \cdot 10^{-4}$	$1.85 \cdot 10^{-5}$	$7.12 \cdot 10^{-5}$	$7.72 \cdot 10^{-8}$	14.3	$1.68 \cdot 10^{-3}$
$\Upsilon J/\psi$	$3.64 \cdot 10^{-6}$	$5.82 \cdot 10^{-6}$	$4.45 \cdot 10^{-3}$	$3.72 \cdot 10^{-7}$	–	–

Results and conclusions

In the presented work within the considered models for productions of charmonia pairs, bottomonia pairs and also for the associated production of charmonium and bottomonium in e^+e^- annihilation the following results have been achieved:

- The selection rules have been compiled and analytically proven;
- Analytical expressions for QCD and EW amplitudes and cross sections contributions have been obtained in both cases of tree and single loop diagrams;
- Dependencies on collision energies for total cross sections and for contributions from QCD and EW production mechanisms have been numerically calculated within NLO accuracy;

Also, it has been shown that the production of the pseudoscalar meson pair in single boson e^+e^- annihilation is suppressed due to the selection rules.
The production cross sections of investigated processes have been studied in a wide range of energies (from thresholds up to $2M_Z$) that is of interest in terms of researches on future e^+e^- colliders.
Different production mechanisms have been taken into account.

Thank you for attention!

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Features of charmonia pair and bottomonia pair production (CC, BB) – QCDNLO correction dependence on inner quark mass

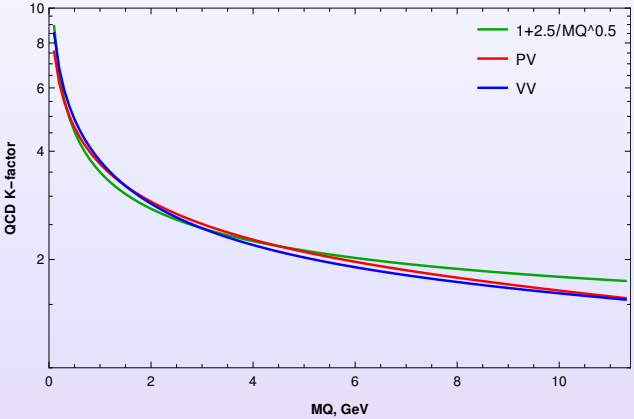

Fig. 12 – The total QCDNLO and QCDLO cross sections ratio dependence on the quark mass for $\sqrt{s} = M_Z/2$.
Cross section relation for $\sqrt{s} = M_Z/2$:

$$\frac{\sigma^{QCDNLO}}{\sigma^{QCDLO}}\Big|_{s=M_Z/2} \sim 1+\frac{2.5\ GeV^{1/2}}{m_q^{1/2}}.$$