NLL BFKL for Mueller-Navelet dijets with large Δy and jet veto

ANATOLII EGOROV (1,2) , V.T. KIM (1,2)

1 - NRC KI Petersburg Nuclear Physics Institute, Gatchina, Russia

2 - Peter the Great St. Petersburg Polytechnic University, St.Petersburg, Russia

DGLAP vs BFKL

Bjorken limit

$$\sqrt{s} \to \infty; p_T \to \infty; x \sim \frac{p_T}{\sqrt{s}} \sim 1$$

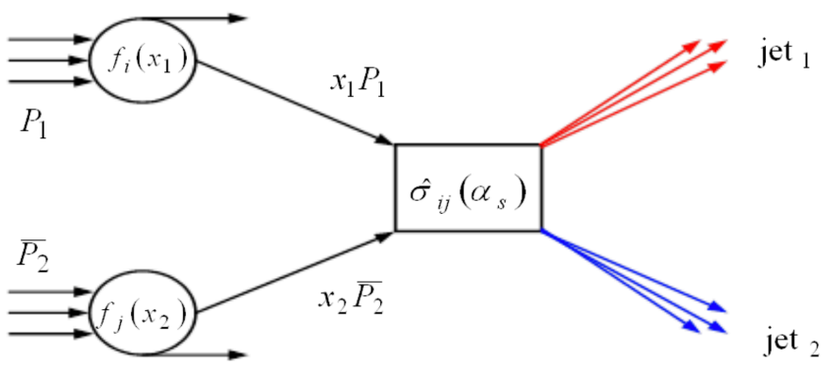
$$k_{Tn} \gg k_{Tn-1} \gg \ldots \gg k_{T2} \gg k_{T1}$$

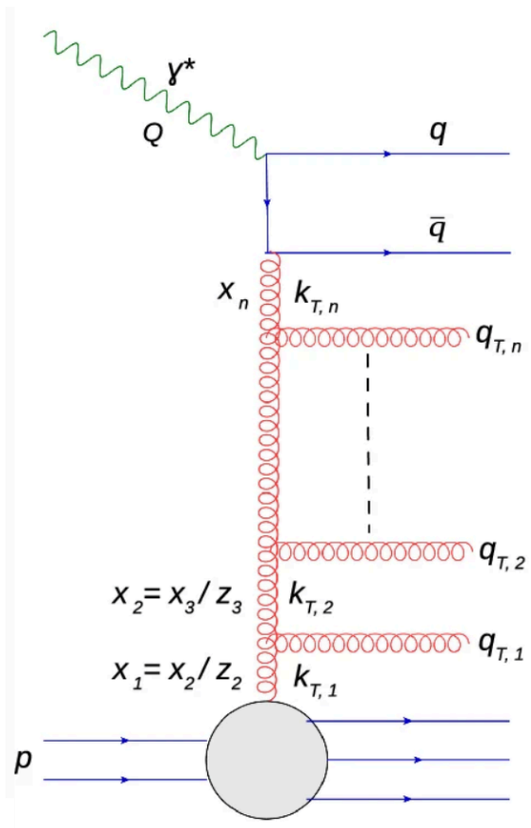
$$[\alpha_s \log Q^2]^n$$

DGLAP evolution

$$\frac{df_i}{d\log\mu^2} = \frac{\alpha_s}{2\pi}[P_{qq} \otimes f_i + P_{qg} \otimes f_g]$$

$$\frac{df_g}{d\log\mu^2} = \frac{\alpha_s}{2\pi}[P_{gq} \otimes \sum_i f_i + P_{gg} \otimes f_g]$$





BFKL kinematics (LLA): Regge-Gribov limit

$$\sqrt{s} \to \infty; p_T - finite; x \sim \frac{p_T}{\sqrt{s}} \to 0;$$

c $p_T \gg \Lambda_{QCD}$

$$x_n \gg x_{n-1} \gg \ldots \gg x_2 \gg x_1$$

$$k_{Tn} \sim k_{Tn-1} \sim \ldots \sim k_{T2} \sim k_{T1}$$

BFKL evolution

$$[\alpha_s \log(1/x)]^n$$

$$\frac{\partial f_g}{\partial \log 1/x} = K \otimes f_g = \omega f_g$$

$$\omega_{\max} = \alpha_{I\!P}(0) - 1$$

LL BFKL provides too large intercept

$$\alpha_{I\!P}^{LL}(0) \approx 1.5$$

NLL BFKL: $\alpha_{I\!P}^{NLL} \approx 1.2$

S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, G.B. Pivovarov

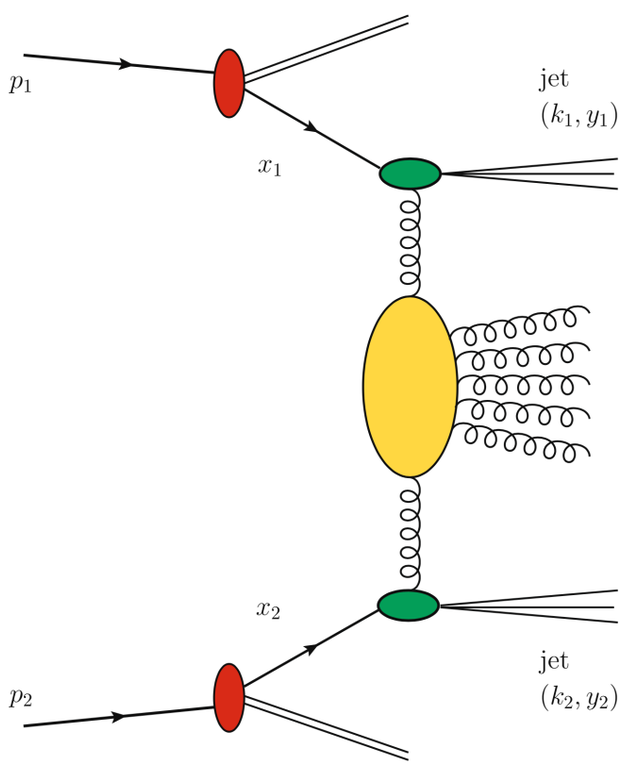
[JETP Lett. 70 (1999) 155-160]

Mueller-Navelet dijets

$\sqrt{s} = 2.76$ TeV

A. H. Mueller and H. Navelet [Nucl. Phys. B 282 (1987) 727]

σ^{MN}



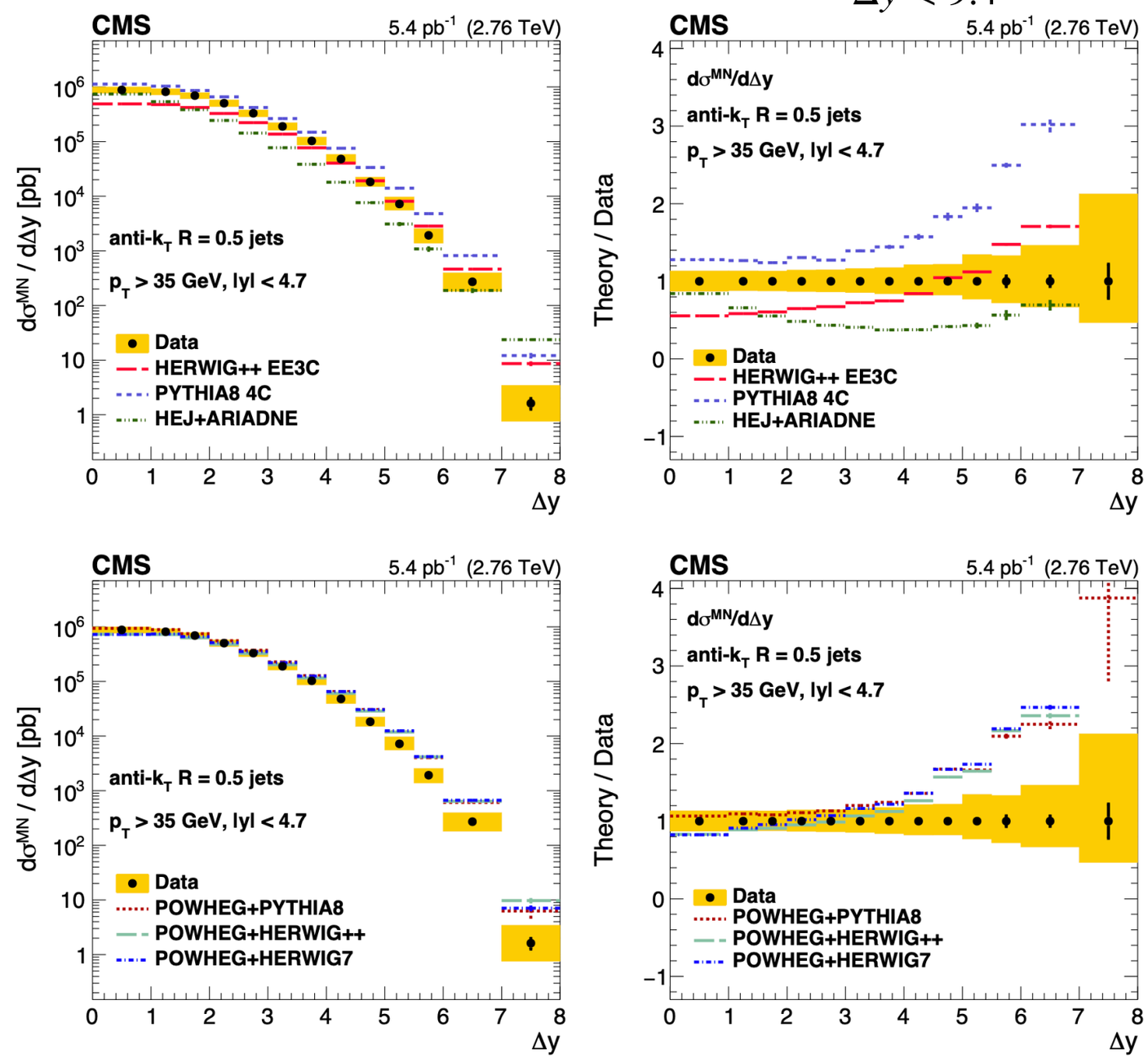
MN jet pair is a pair of jets with p_T above $p_{T\,\mathrm{min}}$ most separated in rapidity

$\Delta y = |y_1 - y_2|$

CMS [JHEP03(2022)189]

$p_{T\mathrm{min}} > 35$ GeV

$\Delta y < 9.4$



RATIOS of X-sections with VETO

A. H. Mueller and H. Navelet
[Nucl. Phys. B 282 (1987) 727]

$$\frac{\sigma^{MN}}{\sigma^{Born}}$$ MN K factor

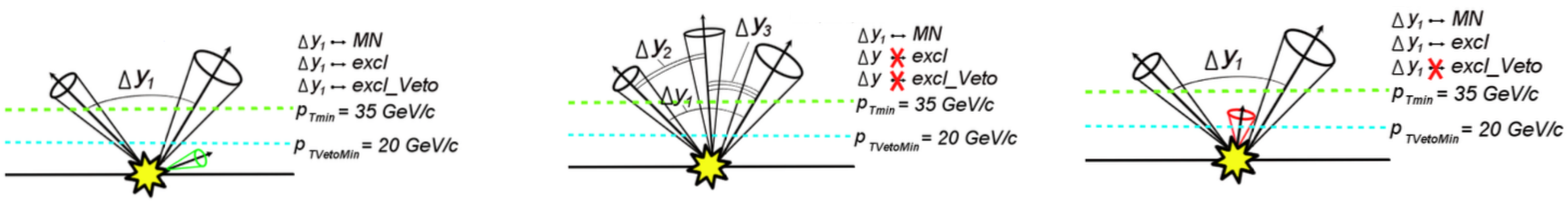
σ^{Born} - impossible to measure

σ^{excl} "Exclusive" cross section: events with just one pair of jets with p_T above $p_{T\,\mathrm{min}}$

σ^{excl}_{veto} "Exclusive" with veto - "exclusive" events with Veto on additional jets with p_T above $p_{T\ veto}$

$$R^{\mathrm{MN}} = \frac{\sigma^{\mathrm{MN}}}{\sigma^{\mathrm{excl}}}$$ MN "K-factor"

$$R^{\mathrm{MN}}_{\mathrm{veto}} = \frac{\sigma^{\mathrm{MN}}}{\sigma^{\mathrm{excl}}_{\mathrm{veto}}}$$ MN "K-factor" with veto



R^{MN} were measured in pp by CMS at 7 TeV [Eur. Phys. J. C 72 (2012) 2216]

σ^{MN}, R^{MN} and $R^{\mathrm{MN}}_{\mathrm{veto}}$ were measured in pp by CMS at 2.76 TeV [JHEP03(2022)189]

NLL BFKL for MN dijets (1)

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \sum_{ij} \int f_i(x_1,\mu_F) f_j(x_2,\mu_F) \frac{d\hat{\sigma}_{ij}(x_1 x_2 s, \mu_F, \mu_R)}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}}$$

Large Δy: $$f^{\rm eff}(x,\mu_F) = \frac{C_A}{C_F} f_g(x,\mu_F) + \sum_{i=q,\bar{q}} f_i(x,\mu_F),$$

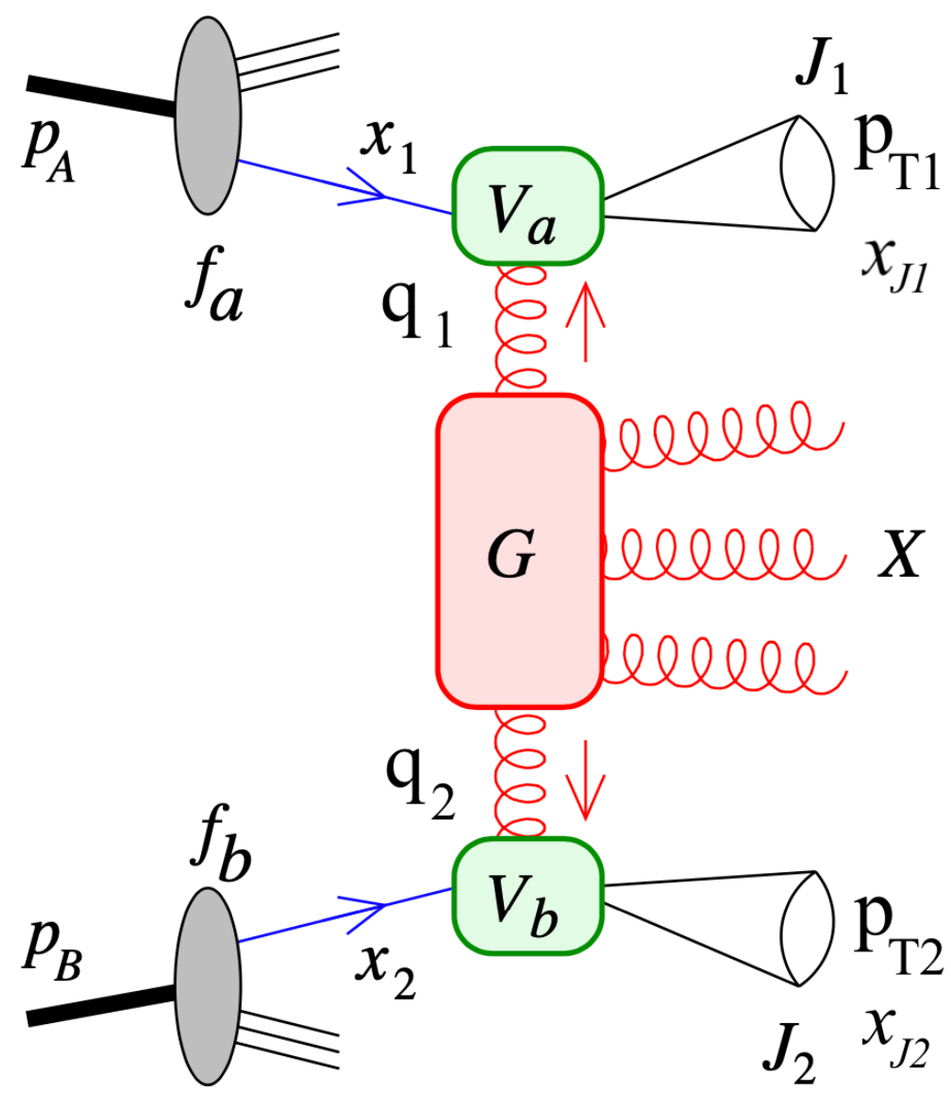
NLL BFKL

$$\frac{d\hat{\sigma}_{gg}}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \frac{x_{J1} x_{J2}}{(2\pi)^2} \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} V_1(\vec{q}_1, x_1, \vec{p}_{T1}, x_{J1}) \times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} V_2(-\vec{q}_2, x_2, \vec{p}_{T2}, x_{J2}) \times \int_C \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2),$$

$$\Phi(\vec{q}, \vec{p}_T, x_J, \omega) \equiv \sum_i \int_0^1 dx f_i(x,\mu_F) \left(\frac{x}{x_J}\right)^{\omega} V_i(\vec{q}, x, \vec{p}_T, x_J),$$

$$\Phi_{1,2}(n,\nu,\vec{p}_{T1,2}, x_{J1,2}, \omega) = \alpha_s(\mu_R)[c_{1,2}(n,\nu) + \bar{\alpha}_s(\mu_R) c_{1,2}^{(1)}(n,\nu)]$$

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{p}_{T1}| d|\vec{p}_{T2}| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathscr{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\mathscr{C}_n\right]$$



NLL BFKL for MN dijets (2)

NLL BFKL

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{p}_{T1}| d|\vec{p}_{T2}| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathscr{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\mathscr{C}_n\right]$$

$$\mathscr{C}_n = \frac{x_{J1}x_{J2}}{|\vec{p}_{T1}||\vec{p}_{T2}|}\int_{-\infty}^{+\infty} d\nu e^{(Y-Y_0)\bar{\alpha}_s(\mu_R)\chi(n,\nu)}\alpha_s^2(\mu_R)c_1(n,\nu)c_1(n,\nu)\left[1+\bar{\alpha}_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)}+\frac{\bar{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right.\right.$$
$$\left.\left.+\frac{\beta_0}{2N_c}\left(\frac{5}{3}+\ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}+f(\nu)\right)\right)+\bar{\alpha}_s^2(\mu_r)\ln\frac{s}{s_0}\left\{\bar{\chi}(n,\nu)+\frac{\beta_0}{4N_c}\chi(n,\nu)\left(-\frac{\chi(n,\nu)}{2}+\frac{5}{3}+\ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}+f(\nu)\right)\right\}\right]$$

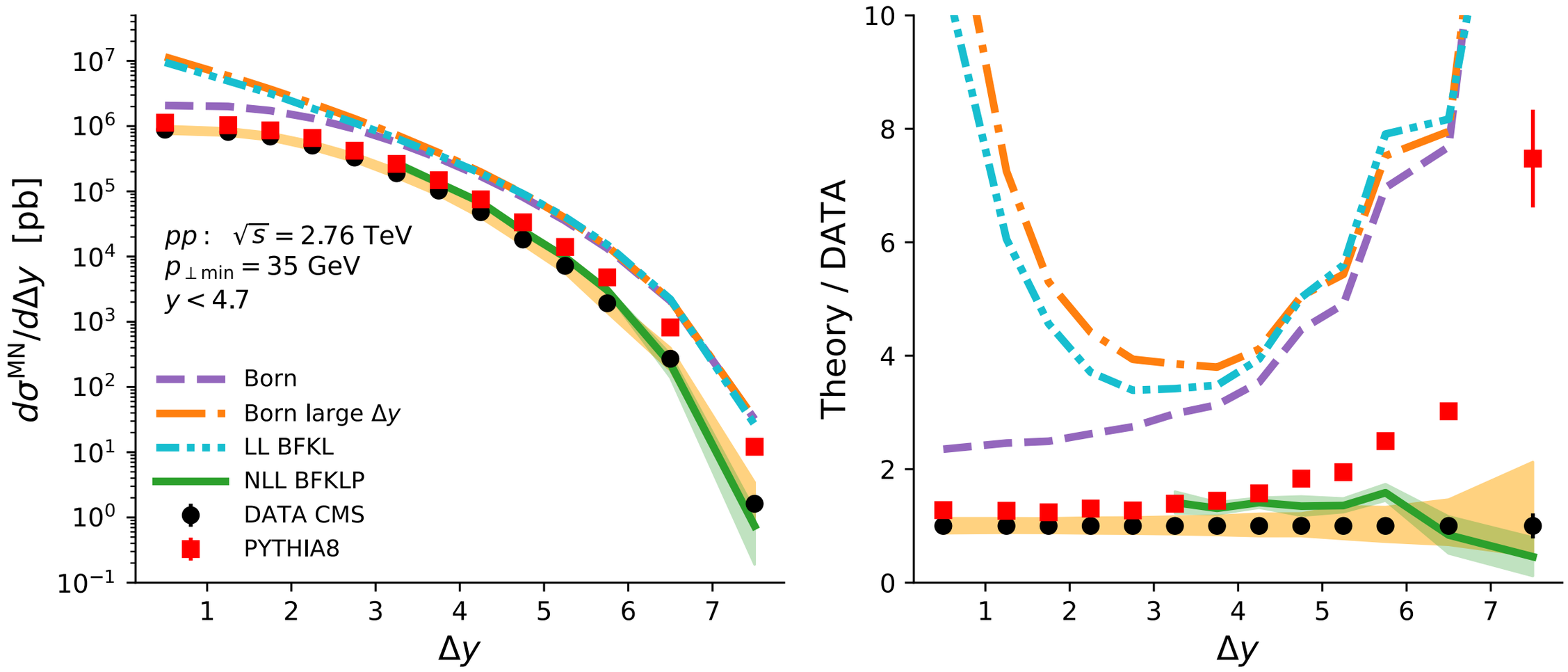
where

$$Y = y_1 - y_2 = \ln\frac{x_{J1}x_{J2}s}{|\vec{p}_{T1}||\vec{p}_{T2}|} \quad \text{and} \quad Y_0 = \ln\frac{s_0}{|\vec{p}_{T1}||\vec{p}_{T2}|}$$

BFKLP [JETP Lett. 70 (1999) 155-160]

- transform to MOM scheme;
- choose scale to make β_0 terms vanish

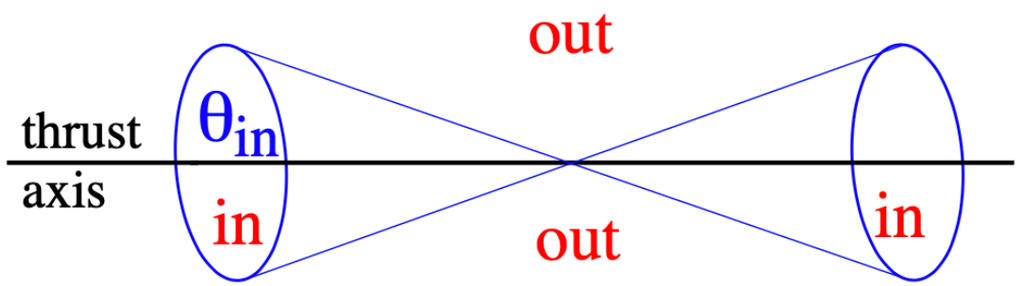
RESULTS of NLL BFKLP for MN dijets @ 2.76 TeV



The BMS equation

Banfi-Marchesini-Smye (BMS) (2002) [JHEP 08 (2002) 006]

$$P_{out}(Q, E_{out}) = \sum_n \int \frac{d\sigma_n}{\sigma_T} \Theta\left(E_{out} - \sum_{out} q_{ti}\right)$$



Soft gluons, large N_c, energy ordering;

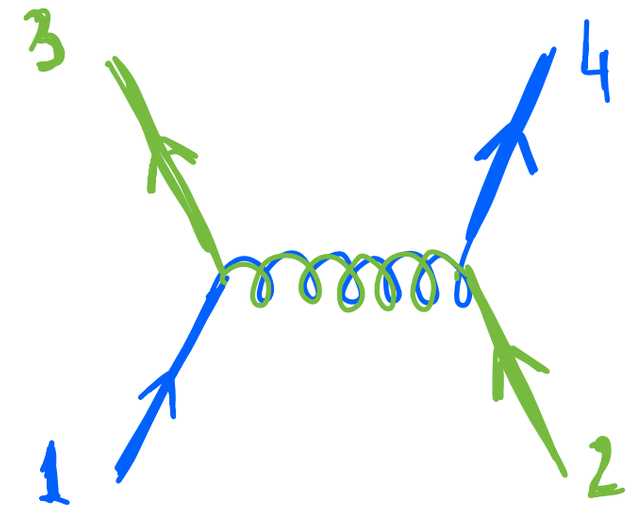
Large angle emission, Sudakov and non-global logarithms;

$$\partial_\tau P_{ab}(\tau) = -(\partial_\tau R_{ab}) P_{ab} + \int_{in} \frac{d\Omega_q}{4\pi} w_{ab}(q) [P_{aq}(\tau) P_{qb}(\tau) - P_{ab}(\tau)]$$

$$\tau = \int_{Q_0}^{Q} \frac{dq_t}{q_t} \frac{\alpha_s(q_t) C_A}{\pi}; \qquad Q_0 = E_{out}; \qquad w_{ab}(q) = \frac{1 - \cos\theta_{ab}}{(1 - \cos\theta_{aq})(1 - \cos\theta_{qb})}$$

$$R_{ab}(\tau) = \int_{E_{out}}^{Q} \frac{dq_t}{q_t} \bar{\alpha}_s(q_t) \int_{out} \frac{d\Omega_q}{4\pi} w_{ab}(q) \approx \tau f_{ab};$$

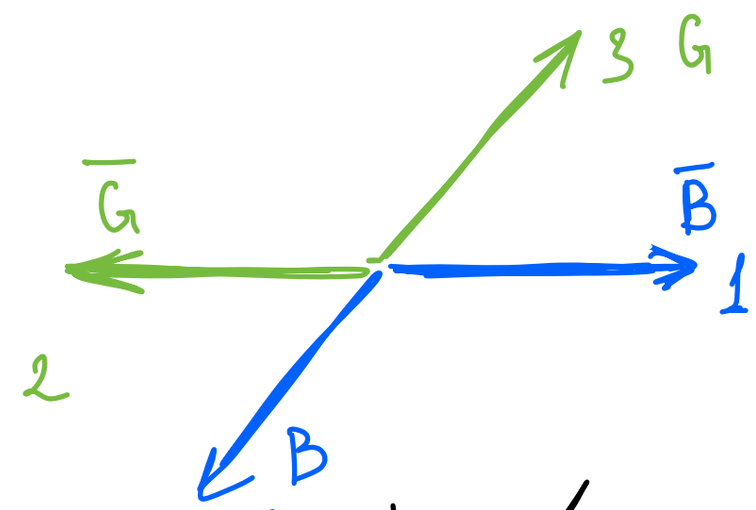
INTER-JET Veto



$$\frac{d\sigma^{veto}}{d\Delta y d^2 p_T} = \sum_{ij}^{q,\bar{q},g} \int_{\bar{y}_{min}(p_T,\Delta y)}^{\bar{y}_{max}(p_T,\Delta y)} d\bar{y} x_1 f_i(x_1, p_T) x_2 f_j(x_2, p_T) \frac{1}{\pi} \frac{d\hat{\sigma}_{ij}^{veto}}{d\hat{t}}$$

at large N_c limit, and in one-gluon exchange,

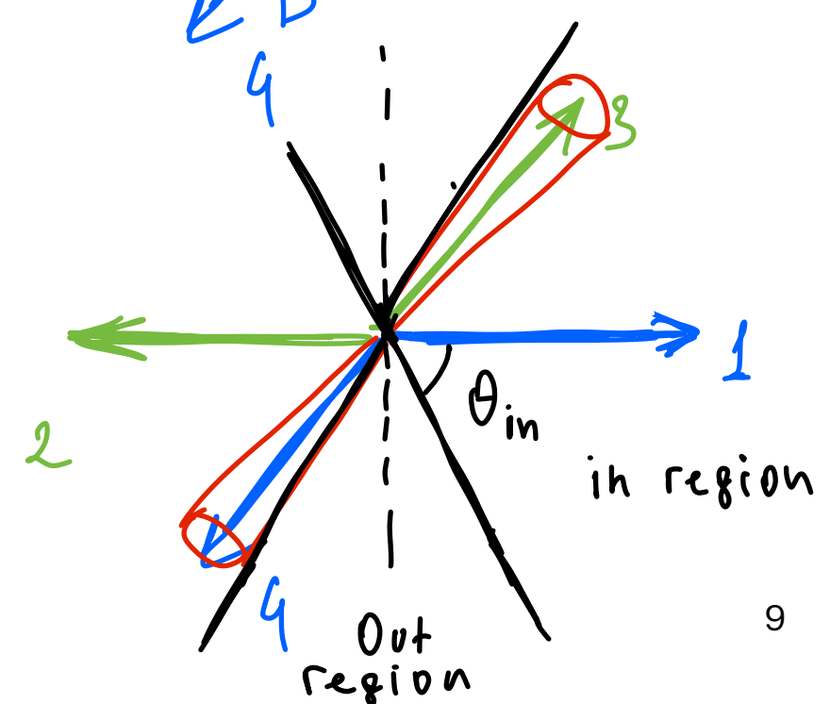
color flows as $1 \to 4$ and $2 \to 3$



$$\frac{d\hat{\sigma}_{qq'}^{veto}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2}(h^A(\hat{s},\hat{t},\hat{u})P_{14}P_{23} + h^A(\hat{s},\hat{u},\hat{t})P_{13}P_{24})$$

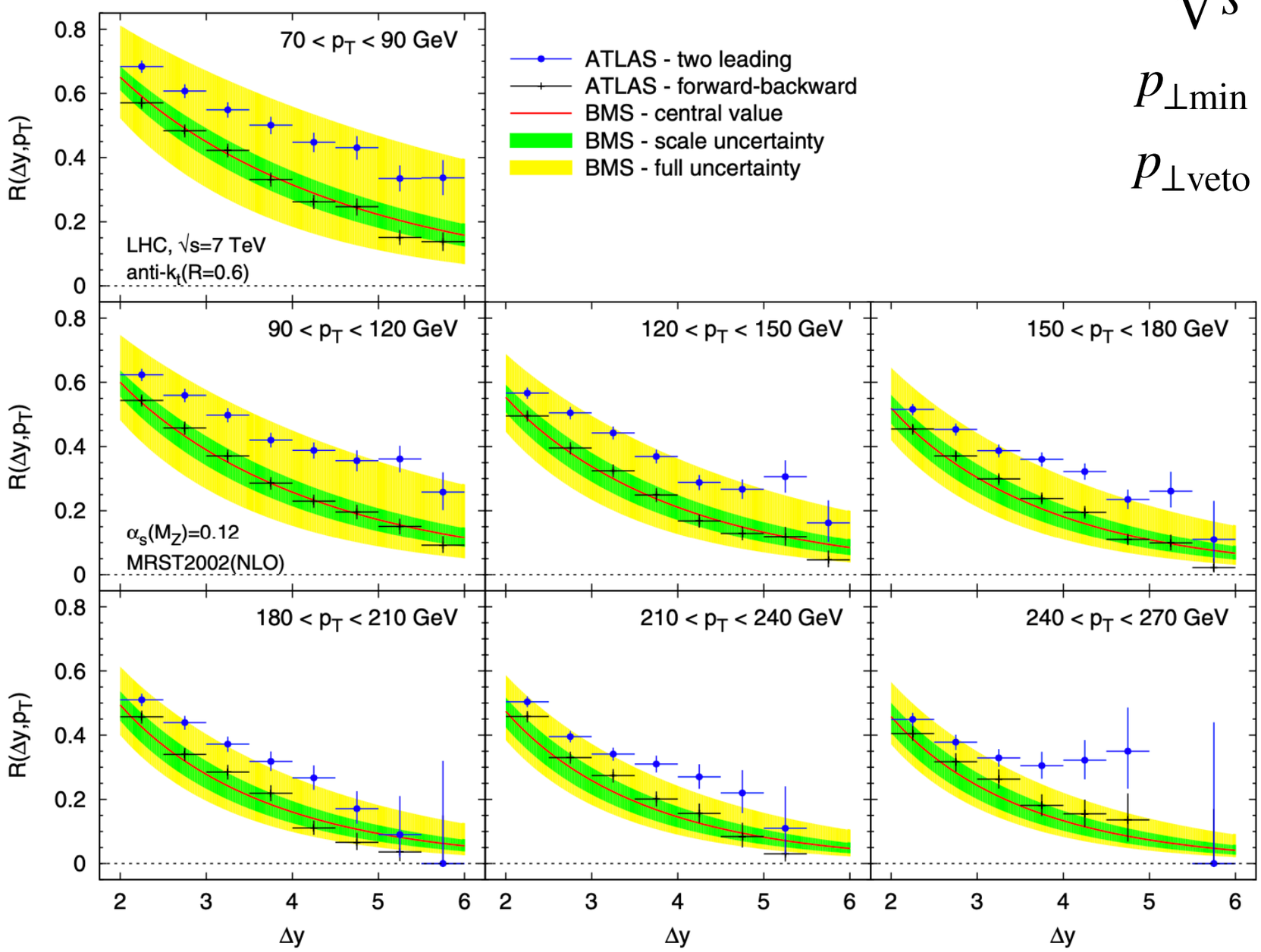
$$h^A(\hat{s},\hat{t},\hat{u}) = g^4 \frac{C_F}{N_c}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)$$

$$y_{in} = \frac{\Delta y}{2} - R_{jet} \qquad\qquad y_{in} = -\log\tan\left(\frac{\theta_{in}}{2}\right)$$



BMS vs ATLAS @ 7 TeV

Y.Hatta et al. (2013) [Phys. Rev. D 87, 054016]



$$\sqrt{s} = 7 \text{ TeV}$$

$$p_{\perp\text{min}} = 70 \text{ GeV}$$

$$p_{\perp\text{veto}} = 20 \text{ GeV}$$

Modifications of the BMS approach (1)

Born x-section $\Rightarrow$ NLL BFKL x-section

Large Δy $\qquad \hat{s}, \hat{u} \gg \hat{t}$

$$\frac{d\hat{\sigma}_{ij}(x_1 x_2 s, \mu_F, \mu_R)}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} \xrightarrow{\Delta y \to \infty} \frac{d\hat{\sigma}_{gg}}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}}$$

P_{ab} can be averaged over $f^{\text{eff}}(x, Q)$

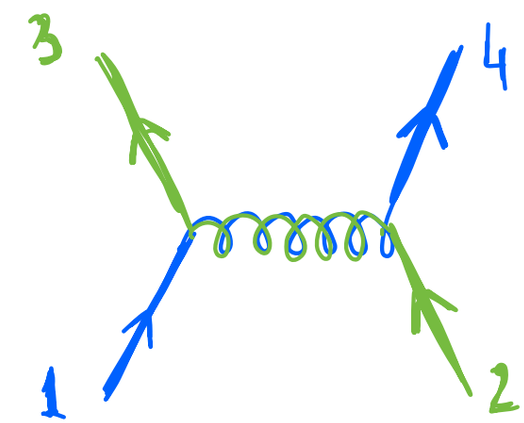
$$P^{\text{eff}} = \frac{1}{f^{\text{eff}}(x_1) f^{\text{eff}}(x_2)} \Bigg[\left(\frac{C_A}{C_F}\right)^2 f_g(x_1) f_g(x_2) P_{gg} + \frac{C_A}{C_F} \left(f_g(x_1) \sum_{i=q,\bar{q}} f_i(x_2) + f_g(x_2) \sum_{i=q,\bar{q}} f_i(x_1) \right) P_{gq}$$
$$+ \left(\sum_{\substack{i=q \\ j=q}} f_i(x_1) f_j(x_1) + \sum_{\substack{i=\bar{q} \\ j=\bar{q}}} f_i(x_1) f_j(x_1) \right) P_{qq} + \left(\sum_{\substack{i=q \\ j=\bar{q}}} f_i(x_1) f_j(x_2) + \sum_{\substack{i=\bar{q} \\ j=\bar{q}}} f_i(x_1) f_j(x_2) \right) P_{q\bar{q}} \Bigg]$$

$$P_{gg} = \frac{1}{2}\left(P_{12}P_{13}P_{24}P_{34} + P_{14}P_{24}P_{13}P_{23}\right)$$
$$P_{gq} = \frac{1}{2}\left(P_{24}P_{12}P_{34} + P_{24}P_{14}P_{23}\right)$$
$$P_{qq} = P_{14}P_{23}$$
$$P_{q\bar{q}} = P_{12}P_{34},$$

Modifications of the BMS approach (2)

Notice that gluon is represented by two dipoles stretched above Δy

$$\frac{d\hat{\sigma}^{veto}_{qq'}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2}(h^A(\hat{s},\hat{t},\hat{u})P_{14}P_{23}+h^A(\hat{s},\hat{u},\hat{t})P_{13}P_{24})$$

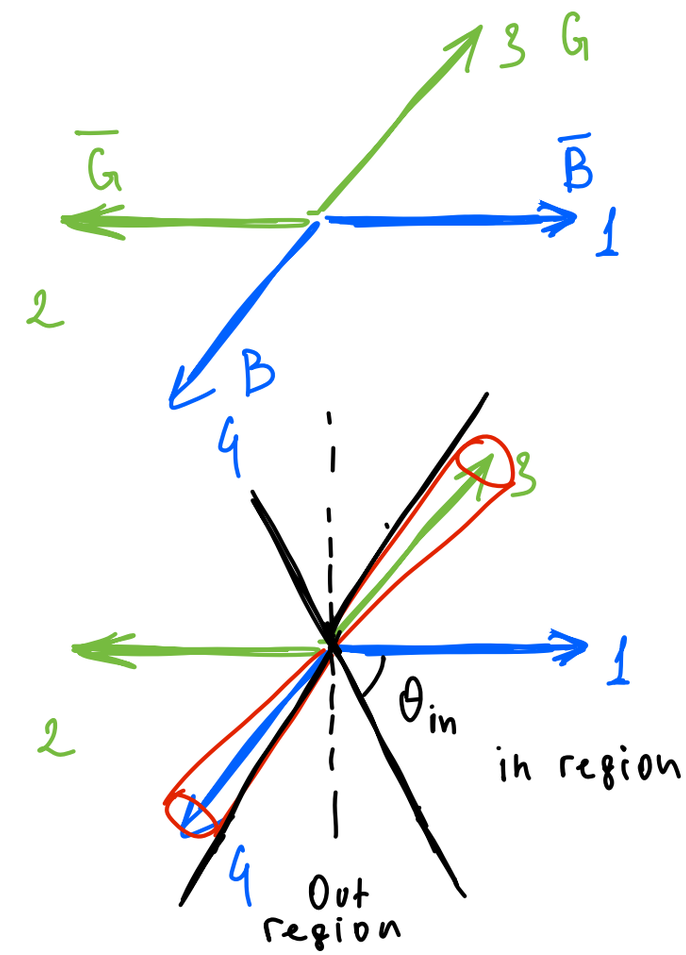


each dipole connects its ends in color singlet state

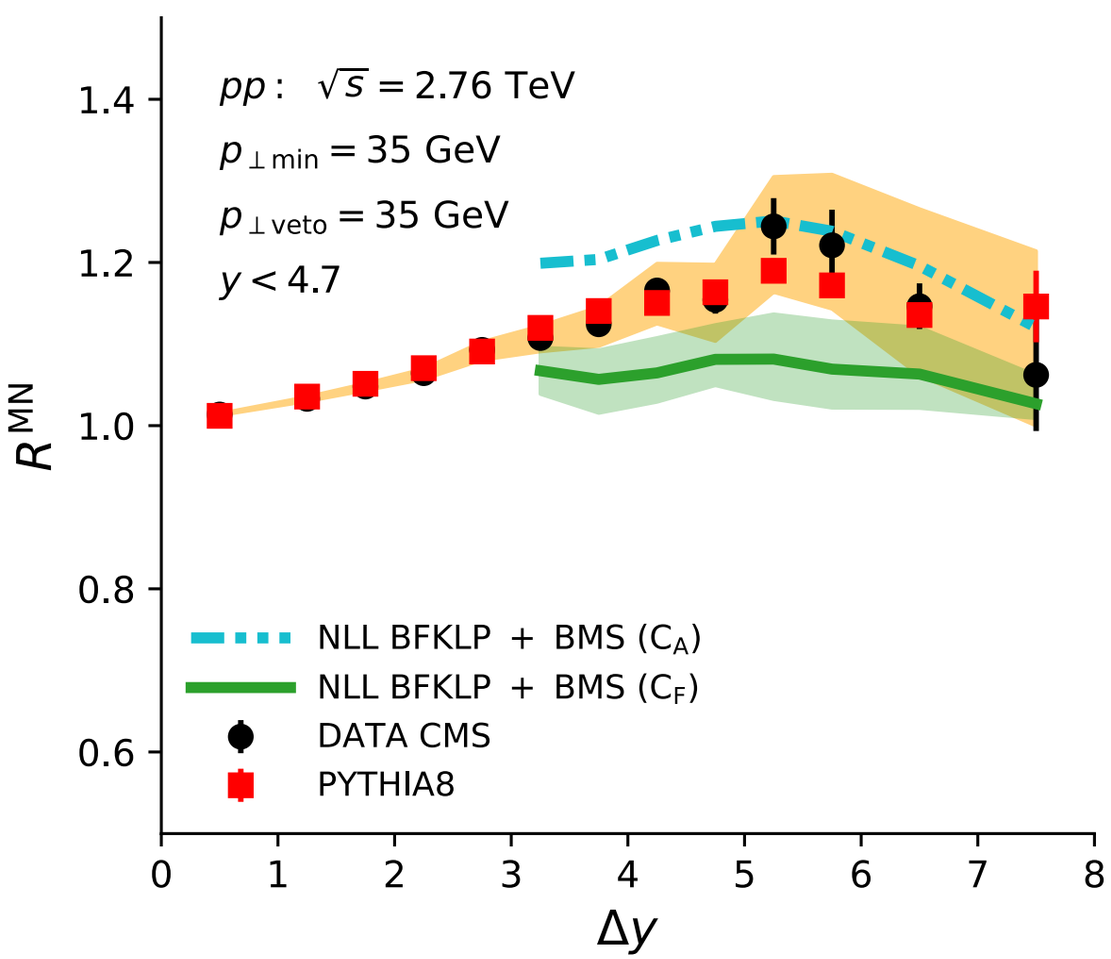
so each dipole should emit $\propto \alpha_s C_F$,

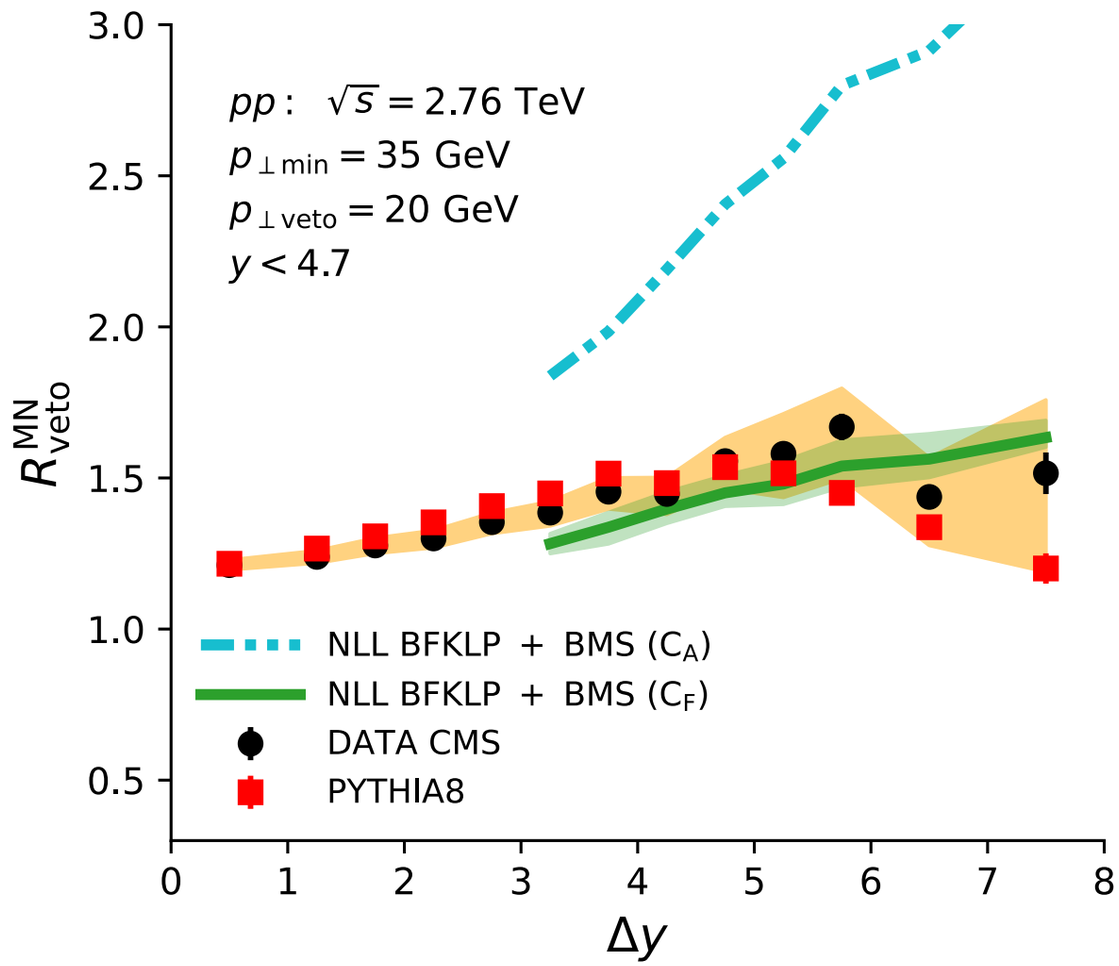
then two of them emit $\propto \alpha_s(C_F + C_F) = \alpha_s(C_A + 1/N_c)$

$$\tau = \int_{Q_0}^{Q} \frac{dq_t}{q_t}\frac{\alpha_s(q_t)C_A}{\pi} \quad \Rightarrow \tau = \int_{Q_0}^{Q} \frac{dq_t}{q_t}\frac{\alpha_s(q_t)C_F}{\pi}$$



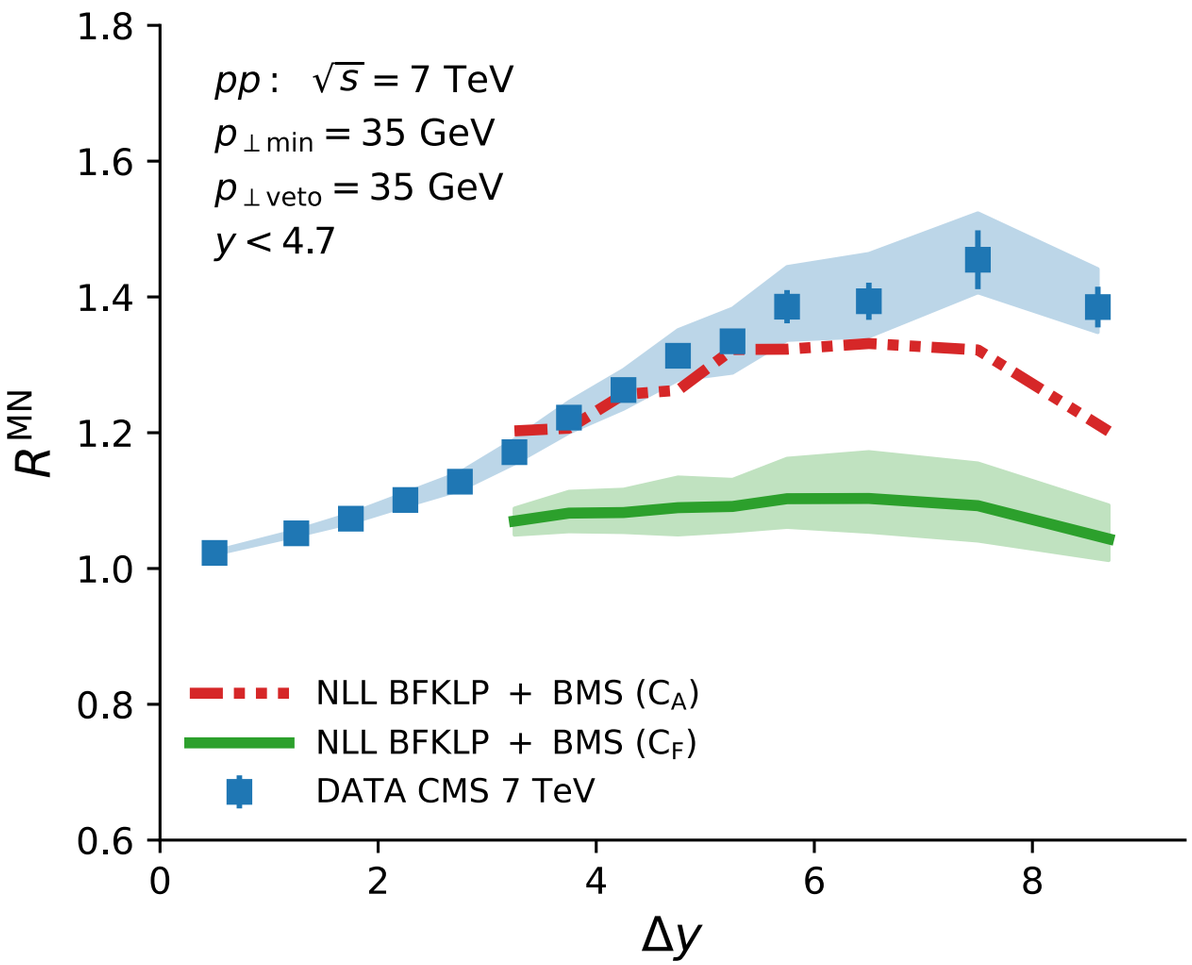
Results for Ratios with Veto @ 2.76 TeV





As the left plot shows C_A does overshot the emission. C_F should be used

Results for Ratios with Veto @ 7 TeV



BMS emission is not enough

BFKL is needed

Summary

- The NLL BFKL calculation with BFKLP scale setting agrees MN x-section measurements at $\sqrt{s} = 2.76$ TeV
- BMS agrees to veto measurements when C_A is replaced with C_F
- BMS predicts not enough emission when there is no phase space for development of cascade ordered in p_T
- All this argues in favour of BFKL evolution
- The development of a BFKL based method for Veto calculation is needed

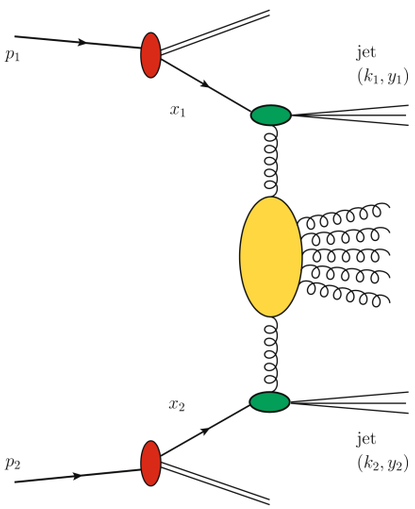
THANK YOU!

BACKUP

Mueller-Navelet dijets

A. H. Mueller and H. Navelet [Nucl. Phys. B 282 (1987) 727]

σ^{MN}



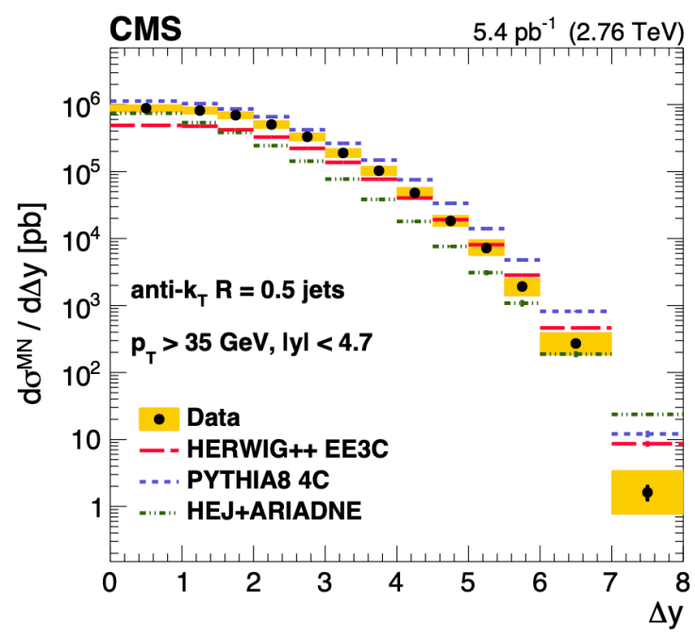
MN jet pair is a pair of jets with p_T above $p_{T\,\mathrm{min}}$ most separated in rapidity $\Delta y = |y_1 - y_2|$

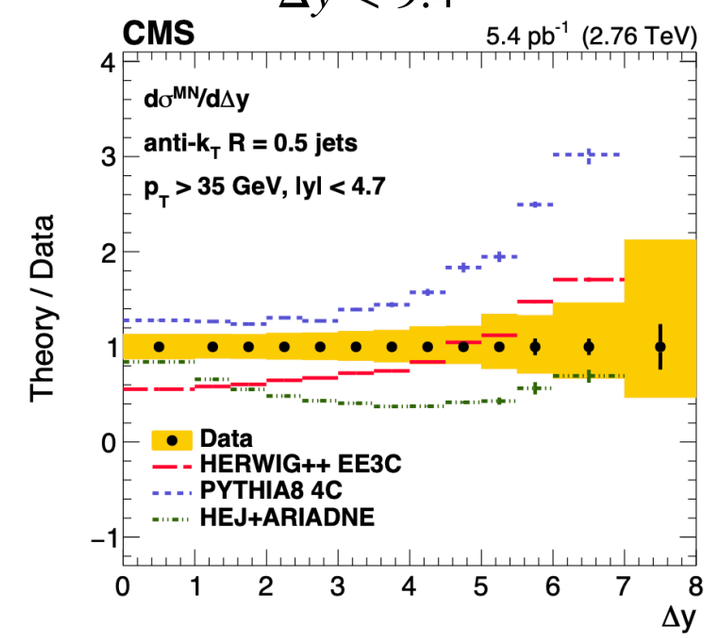
$\sqrt{s} = 2.76$ TeV

CMS [JHEP03(2022)189]

$p_{T\mathrm{min}} > 35$ GeV

$\Delta y < 9.4$



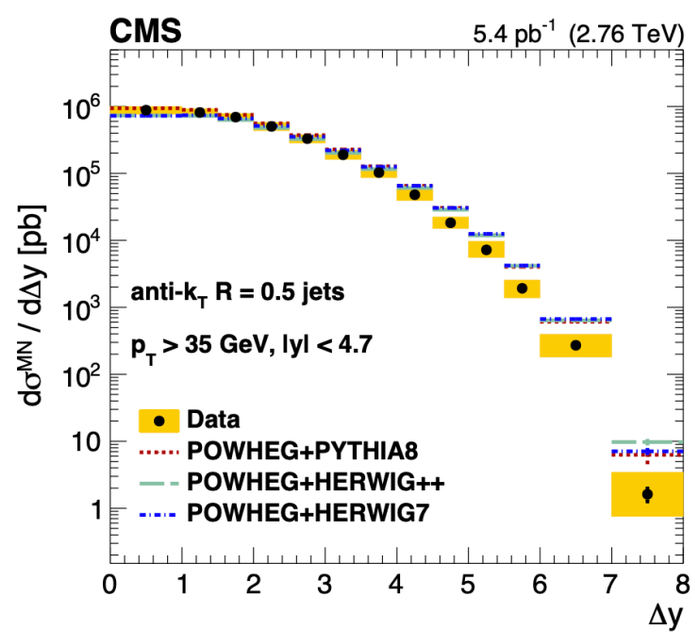


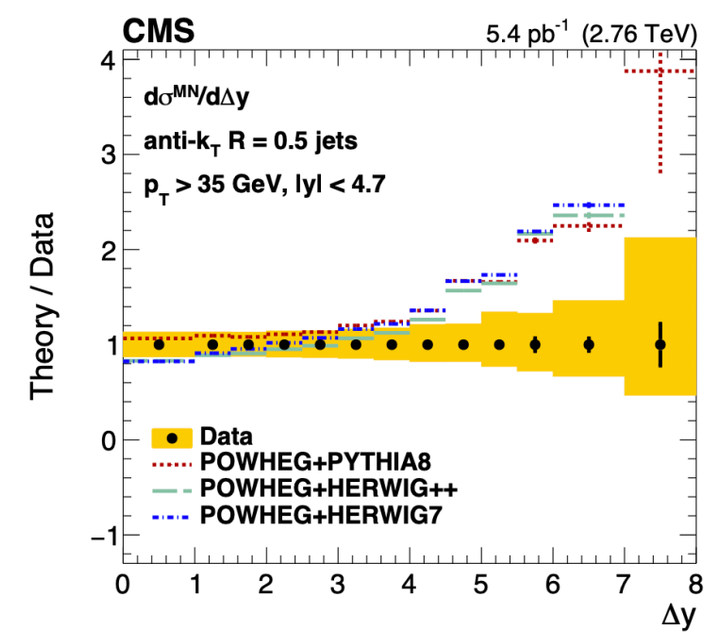
DGLAP generators

- PYTHIA - LO + LL DGLAP + color dipole
- HERWIG - LO + LL DGLAP + angular ordering
- POWHEG+PYTHIA - NLO + LL DGLAP + color dipole
- POWHEG+HERWIG - NLO + LL DGLAP + angular ordering
- SHERPA - LO + N-Real + LL DGLAP

BFKL generators

- HEJ - LL BFKL (parton level)
- HEJ+ARIADNE - LL BFKL (hadron level)





RATIOS with Veto at LHC

Djet cross section ratios ("K factor") at $\sqrt{s} = 2.76$ TeV

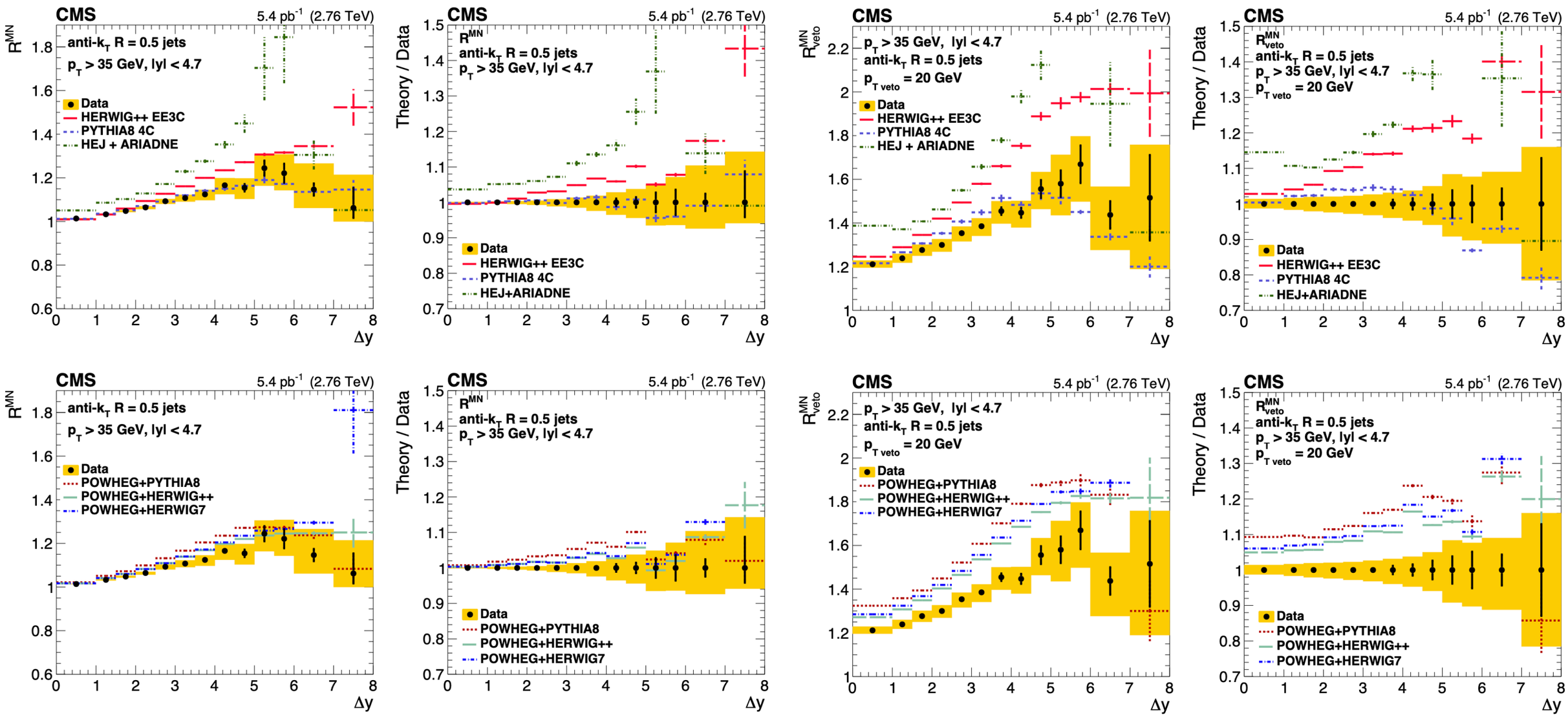
CMS [JHEP03(2022)189]

$$R^{MN} = \frac{d\sigma^{MN}/d\Delta y}{d\sigma^{excl}/d\Delta y}$$

$p_T > 35$ ГэВ

$\Delta y < 9.4$

$$R^{MN}_{veto} = \frac{d\sigma^{MN}/d\Delta y}{d\sigma^{excl}_{veto}/d\Delta y}$$



LL BFKL for MN dijets

$$\omega_{\max} = \alpha_{I\!P}(0) - 1$$

LL BFKL provides too large intercept

$$\alpha_{I\!P}^{LL}(0) \approx 1.5$$

NLL BFKL: $\alpha_{I\!P}^{NLL} \approx 1.2$

S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, G.B. Pivovarov (BFKLP)

[JETP Lett. 70 (1999) 155-160]

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \sum_{ij} \int f_i(x_1,\mu_F) f_j(x_2,\mu_F) \frac{d\hat{\sigma}_{ij}(x_1 x_2 s,\mu_F,\mu_R)}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}}$$

Large Δy $\qquad \hat{s}, \hat{u} \gg \hat{t}$

$$\frac{d\hat{\sigma}_{ij}(x_1 x_2 s,\mu_F,\mu_R)}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} \xrightarrow{\Delta y \to \infty} \frac{d\hat{\sigma}_{gg}}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}}$$

$$f^{\text{eff}}(x,\mu_F) = \frac{C_A}{C_F} f_g(x,\mu_F) + \sum_{i=q,\bar{q}} f_i(x,\mu_F),$$

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \int f^{\text{eff}}(x_1,\mu_F) f^{\text{eff}}(x_2,\mu_F) \left(\frac{C_F}{C_A}\right)^2 \frac{d\hat{\sigma}_{gg}(x_1 x_2 s,\mu_F,\mu_R)}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}}$$

LL BFKL

$$\frac{d\hat{\sigma}_{gg}}{d^2 p_{T1} d^2 p_{T2}} = \left[\frac{C_A \alpha_s}{p_{T1}^2}\right] f_g(\Delta y, p_{T1}, p_{T2}) \left[\frac{C_A \alpha_s}{p_{T2}^2}\right]$$

$$\frac{d\hat{\sigma}_{gg}}{dp_{T1}^2 dp_{T2}^2 d\phi} = \frac{C_A^2 \alpha_s^2}{4\pi p_{T1}^3 p_{T2}^3} \sum_n e^{in(\phi-\pi)} \int_0^\infty d\nu e^{\omega(n,\nu)\Delta y} \cos\left(\nu \ln \frac{p_{T1}^2}{p_{T2}^2}\right)$$

$$\omega(n,\nu) = \frac{2C_A \alpha_s}{\pi} \left[\psi(1) - \text{Re}\psi\left(\frac{|n|+1}{2} + i\nu\right)\right]$$

$$\frac{d\hat{\sigma}_{gg}}{dp_{T1}^2 dp_{T2}^2} = \frac{C_A^2 \alpha_s^2}{2 p_{T1}^3 p_{T2}^3} \int_0^\infty d\nu e^{\omega(n,\nu)\Delta y} \cos\left(\nu \ln \frac{p_{T1}^2}{p_{T2}^2}\right)$$

Calculation of ratios of x-sections with veto in the BMS approach

veto to inclusive

$$\mathscr{R}(\Delta y, p_T) = \frac{d\sigma^{veto}/d\Delta y d^2 p_T}{d\sigma^{incl}/d\Delta y d^2 p_T}$$

inclusive x-section:

$$\frac{d\sigma^{incl}}{d\Delta y d^2 p_T} = \sum_{ij}^{q,\bar{q},g} \int_{\bar{y}_{min}(p_T,\Delta y)}^{\bar{y}_{max}(p_T,\Delta y)} d\bar{y} x_1 f_i(x_1, p_T) x_2 f_j(x_2, p_T) \frac{1}{\pi} \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$

qq′ scattering with one gluon exchange

$$\frac{d\hat{\sigma}_{qq'}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2} h^A(\hat{s}, \hat{t}, \hat{u}) \qquad h^A(s,t,u) = g^4 \frac{C_F}{N_c}\left(\frac{s^2+u^2}{t^2}\right)$$

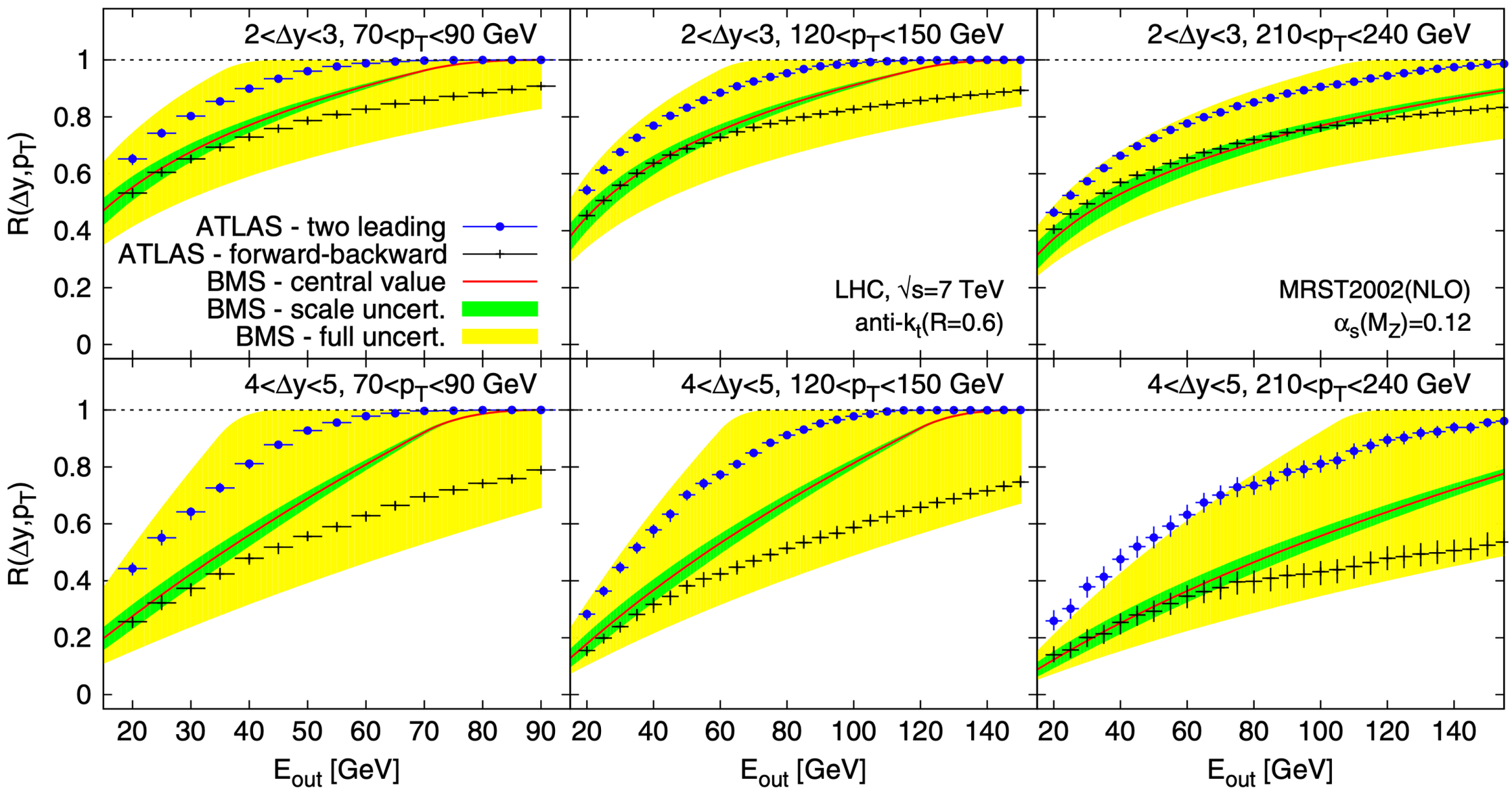
if consider only $\Delta y > 0$

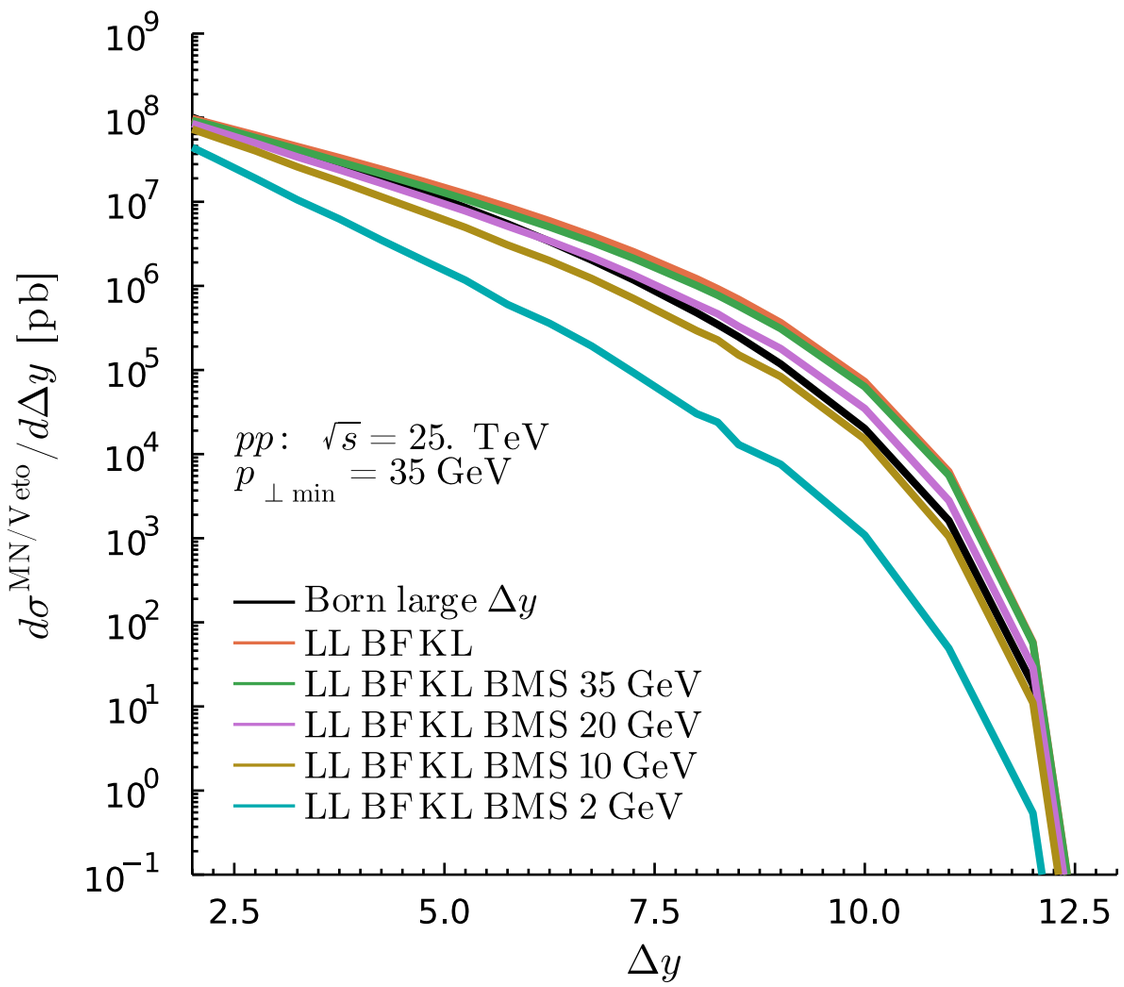
$$\frac{d\hat{\sigma}_{qq'}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2}(h^A(\hat{s}, \hat{t}, \hat{u}) + h^A(\hat{s}, \hat{u}, \hat{t}))$$

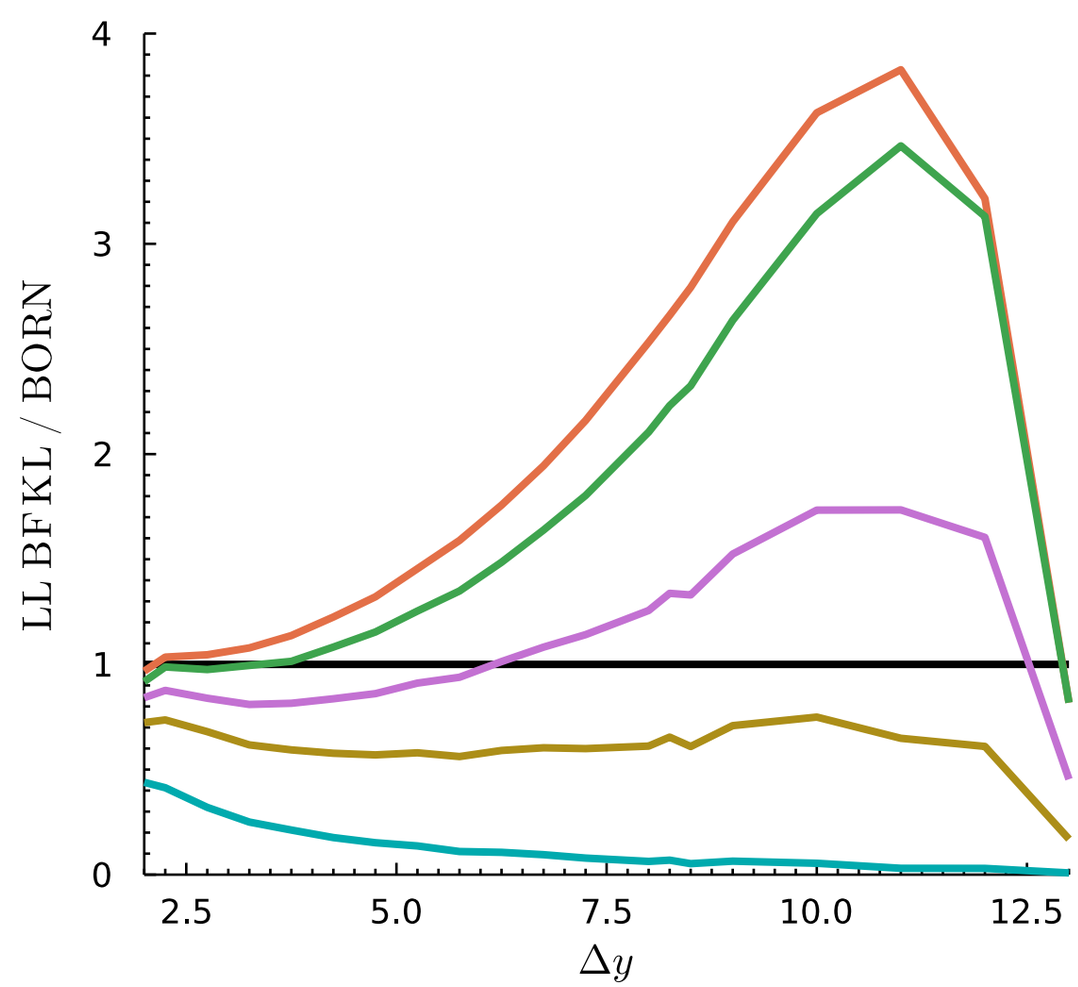
BMS vs ATLAS @ 7 TeV (2)

Y.Hatta et al. (2013) [Phys. Rev. D 87, 054016]

$$\sqrt{s} = 7 \text{ TeV}$$
$$p_{\perp\text{min}} = 70 \text{ GeV}$$
$$p_{\perp\text{veto}} = 20 \text{ GeV}$$




$d\sigma^{\mathrm{MN/Veto}}/d\Delta y$ [pb]
pp: $\sqrt{s} = 25.$ TeV
$p_{\perp\,\mathrm{min}} = 35$ GeV
Born large Δy
LL BFKL
LL BFKL BMS 35 GeV
LL BFKL BMS 20 GeV
LL BFKL BMS 10 GeV
LL BFKL BMS 2 GeV
Δy


LL BFKL / BORN
Δy