A non-minimal approximation of the type I see-saw

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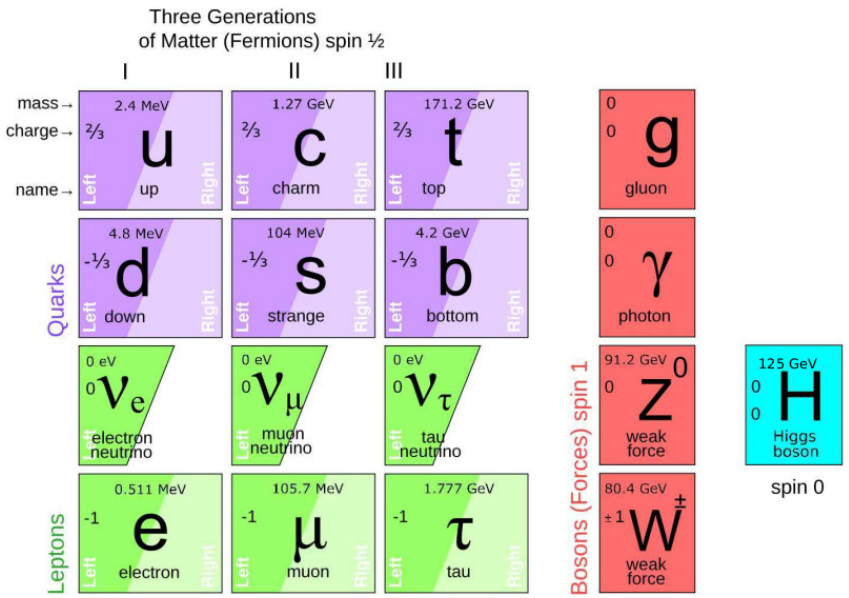


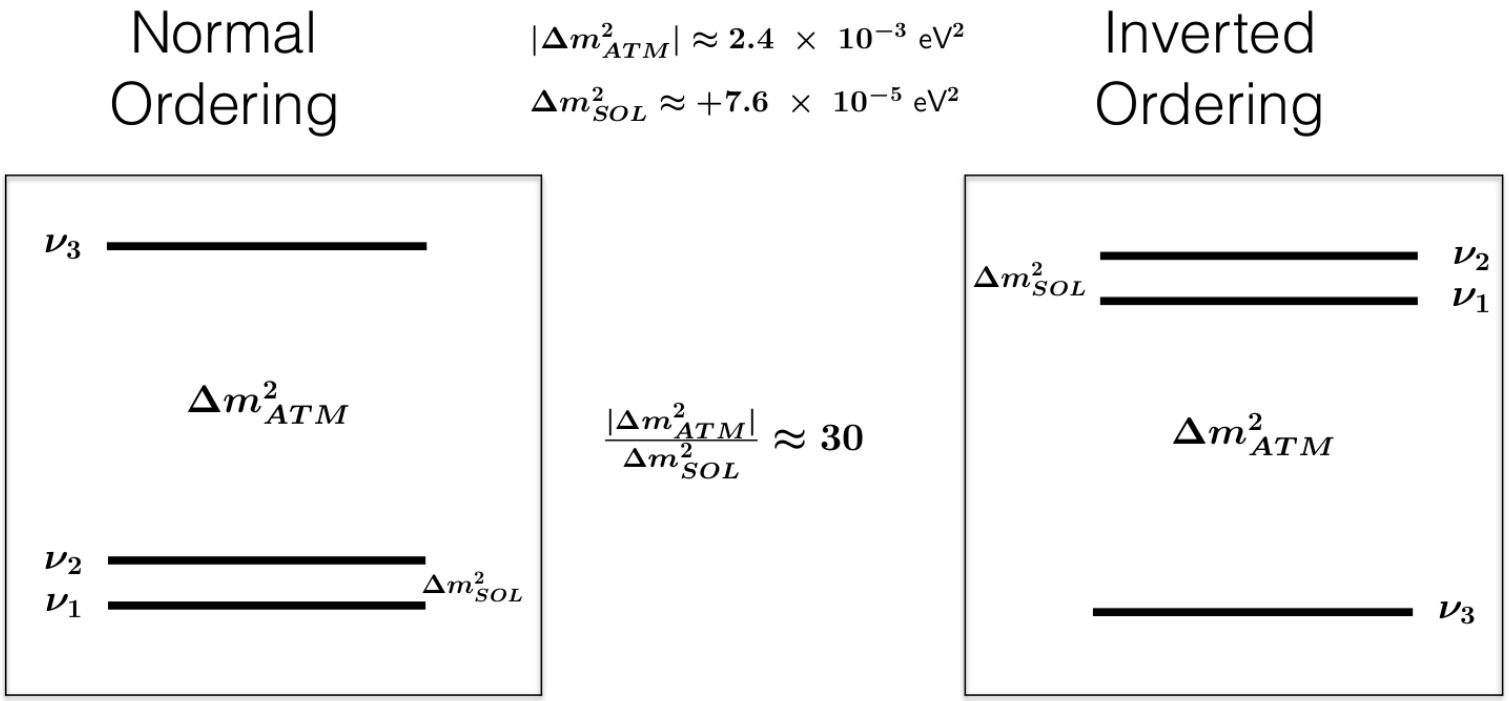
M. Shaposhnikov, J. Phys. Conf. Ser. 408 (2013) 012015

SM can not solve
– the problem of neutrino masses
– the DM problem
– the problem of baryon asymmetry in the Universe (BAU)
– ...

The most economical and natural way to solve them is to extend the SM with three right-handed neutrinos (HNL, sterile neutrinos, gauge-singlet fermions)

- the symmetry between right- and left-handed neutrinos is restored
- no new physical principles and symmetries are needed
- no new energy scales (in the νMSM)
- the smallness of m_ν by seesaw mechanism (type I) and neutrino oscillations
- BAU

- Experimentally observed neutrino oscillations consequence on nonzero neutrino masses
- the smallness of their masses



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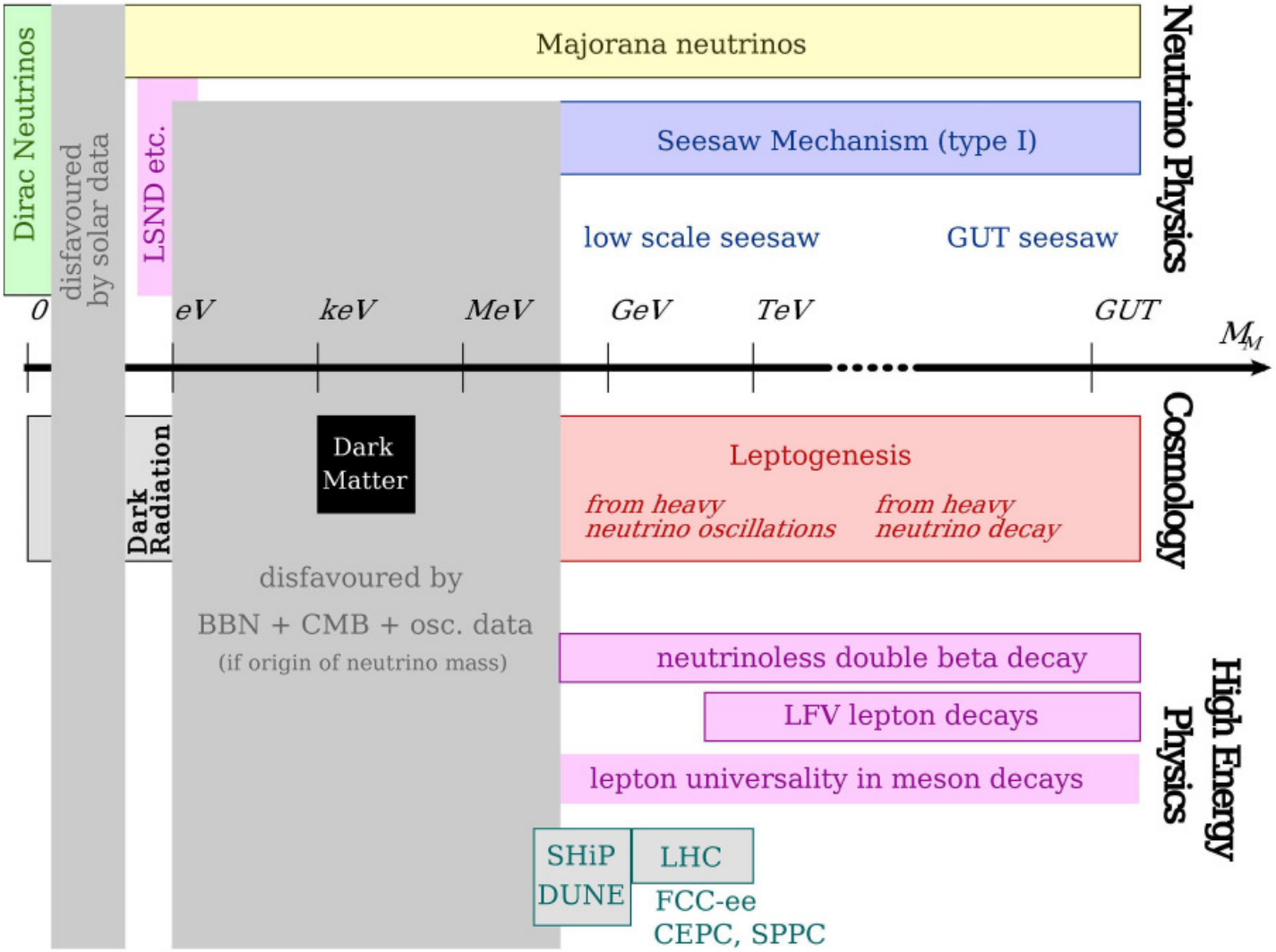
The dynamical mechanism behind the neutrino masses are not known



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νMSM: $\mathcal{O}(\text{eV}) < M_I \leq m_H$

Lagrangian

Flavour basis

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{\nu}_R \partial_\mu \gamma^\mu \nu_R - \left(Y \bar{l}_L \nu_R \tilde{H} + \frac{M_M}{2} \overline{\nu^c}_R \nu_R + h.c \right), \tag{1}$$

where

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \qquad \tilde{H} = \epsilon_{ij} H^\dagger \tag{2}$$

Spontaneous EW-symmetry breaking

$$\frac{1}{2} (\overline{\nu}_L \overline{\nu^c}_R) \begin{pmatrix} 0 & M_D \\ M_D^T & M_M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}, \tag{3}$$

where $M_D = Y \langle H \rangle$ – Dirac mass matrix

Mass basis

$$\mathcal{U}^\dagger \mathcal{M} \mathcal{U}^* = \begin{pmatrix} \hat{m} & 0 \\ 0 & \hat{M} \end{pmatrix}, \tag{4}$$

where $\hat{m} = \mathrm{diag}(m_1, m_2, m_3)$, $\hat{M} = \mathrm{diag}(M_1, M_2, M_3)$,

$$\mathcal{U} = W \cdot \begin{pmatrix} U_\nu & 0 \\ 0 & U_N^* \end{pmatrix}, \qquad W = \exp \begin{pmatrix} 0 & \theta \\ -\theta^\dagger & 0 \end{pmatrix}, \qquad \theta^\dagger = -\theta \tag{5}$$

$$\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = P_L \mathcal{U} \begin{pmatrix} \nu \\ N \end{pmatrix} \tag{6}$$

Common approach

$$W = \exp\begin{pmatrix} 0 & \theta \\ -\theta^\dagger & 0 \end{pmatrix} \simeq \begin{pmatrix} 1-\frac{1}{2}\theta\theta^\dagger & \theta \\ -\theta^\dagger & 1-\frac{1}{2}\theta^\dagger\theta \end{pmatrix}, \qquad \theta \ll I \tag{7}$$

$$W^\dagger \begin{pmatrix} 0 & M_D \\ M_D^T & M_M \end{pmatrix} W^* = \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix} \equiv \tilde{\mathcal{M}} \tag{8}$$

$$(12): \qquad \theta \simeq M_D M_M^{-1}, \tag{9}$$

$$(11): \qquad m_\nu \simeq -M_D M_M^{-1} M_D^T, \tag{10}$$

$$(22): \qquad M_N \simeq M_M + \frac{1}{2}(\theta^\dagger\theta M_M + M_M^T\theta^T\theta^*) \simeq M_M \tag{11}$$

Using Eq. (6)

$$\nu_L \simeq \left(1-\frac{1}{2}\theta\theta^\dagger\right) U_\nu \nu_L + \theta U_N^* N_L, \tag{12}$$

$$U_{\rm PMNS} \simeq (1+\eta) U_\nu, \qquad \Theta_{\rm m} \simeq \theta U_N^* \tag{13}$$

where $\eta = -1/2\theta\theta^\dagger$

Non-minimal approximation

$$W \simeq \begin{pmatrix} 1-\frac{1}{2}\theta\theta^\dagger & \theta-\frac{1}{6}\theta\theta^\dagger\theta \\ -\theta^\dagger+\frac{1}{6}\theta^\dagger\theta\theta^\dagger & 1-\frac{1}{2}\theta^\dagger\theta \end{pmatrix}, \qquad \theta < I \tag{14}$$

$$(12): \qquad M_D - \theta M_M - \frac{1}{2}(\theta\theta^\dagger M_D + M_D\theta^T\theta^*) - \theta M_D^T\theta^* + \frac{1}{2}\theta M_M\theta^T\theta^* + \frac{1}{6}\theta\theta^\dagger\theta M_M \simeq 0, \tag{15}$$

$$(11): \qquad -(\theta M_D^T + M_D\theta^T) + \theta M_M\theta^T + \frac{1}{6}(\theta\theta^\dagger\theta M_D^T + M_D\theta^T\theta^*\theta^T) + \frac{1}{2}(\theta\theta^\dagger M_D\theta^T + \theta M_D^T\theta^*\theta^T) \simeq m_\nu, \tag{16}$$

$$(22): \qquad M_M + (\theta^\dagger M_D + M_D^T\theta^*) - \frac{1}{2}(\theta^\dagger\theta M_M + M_M\theta^T\theta^*) - \frac{1}{2}\theta^\dagger(\theta M_D^T + M_D\theta^T)\theta^* - \frac{1}{6}(\theta^\dagger\theta\theta^\dagger M_D + M_D^T\theta^*\theta^T\theta^*) \simeq M_N, \tag{17}$$

where θ, M_D, M_M are unknown.

The condition $\tilde{\mathcal{M}}_{12} = 0$ [Eq.(15)]

$$\begin{aligned} M_M &- \frac{1}{2} M_M \theta^T \theta^* - \frac{1}{6} \theta^\dagger \theta M_M \\ &\simeq \theta^{-1} M_D \left(1 - \frac{1}{2}\theta^T \theta^*\right) - \frac{1}{2}\theta^\dagger M_D - M_D^T \theta^*, \end{aligned} \tag{18}$$

i.e.

$$M_M \sim \mathcal{O}(\theta^{-1} M_D), \tag{19}$$

$$\mathcal{O}(\theta^2 M_M) \sim \mathcal{O}(\theta M_D), \tag{20}$$

so in the following we will omitted terms $\sim \mathcal{O}(\theta^n M_D)$, $n > 1$, in $\tilde{\mathcal{M}}_{11}$ and $\tilde{\mathcal{M}}_{22}$. Expressions for

$$m_\nu \simeq -(\theta M_D^T + M_D \theta^T) + \theta M_M \theta^T, \tag{21}$$

$$M_N \simeq M_M + (\theta^\dagger M_D + M_D^T \theta^*) - \frac{1}{2}(\theta^\dagger \theta M_M + M_M \theta^T \theta^*) \tag{22}$$

are the same as in the common approach but with the relation (18) instead of $M_M = \theta^{-1} M_D$.

$$m_\nu \simeq -\theta M_D^T \simeq -M_D \theta^T, \tag{23}$$

$$M_N \simeq \theta^{-1} M_D + \frac{1}{6}\theta^\dagger M_D - \frac{1}{2} M_D^T \theta^* \simeq \left(\theta^{-1} - \frac{1}{3}\theta^\dagger\right) M_D, \tag{24}$$

$$M_M \simeq \theta^{-1} M_D - \frac{1}{3}\theta^\dagger M_D - M_D^T \theta^* \simeq \left(\theta^{-1} - \frac{4}{3}\theta^\dagger\right) M_D. \tag{25}$$

Using Eq. (6)

$$\nu_L \simeq \left(1 - \frac{1}{2}\theta\theta^\dagger\right) U_\nu \boldsymbol{\nu}_L + \left(\theta - \frac{1}{6}\theta\theta^\dagger\theta\right) U_N^* N_L, \tag{26}$$

$$U_{\rm PMNS} \simeq \left(1 - \frac{1}{2}\theta\theta^\dagger\right) U_\nu, \qquad \Theta_{\rm nm} \simeq \left(\theta - \frac{1}{6}\theta\theta^\dagger\theta\right) U_N^*. \tag{27}$$

Parametrization of the mixing matrix Θ

Approximation $M_M \sim \mathcal{O}(\theta^{-1} M_D)$

$$I = M_N M_N^{-1} \simeq -M_D^T U_{\rm PMNS}^* \hat{m}^{-1} U_{\rm PMNS}^\dagger M_D U_N \hat{M}^{-1} U_N^T, \tag{28}$$

$$I \simeq -(\sqrt{\hat{M}^{-1}} U_N^T M_D^T U_{\rm PMNS}^* \sqrt{\hat{m}^{-1}})(\sqrt{\hat{m}^{-1}} U_{\rm PMNS}^\dagger M_D U_N \sqrt{\hat{M}^{-1}}) = \Omega_{\rm m}^T \Omega_{\rm m}, \tag{29}$$

$$M_D = -i U_{\rm PMNS} \sqrt{\hat{m}} \Omega_{\rm m} \sqrt{\hat{M}} U_N^\dagger, \tag{30}$$

known as the Casas-Ibarra parametrization [Casas J., Ibarra A., Nucl.Phys.B 618 (2001) 171]. So,

$$\Theta_{\rm m} \simeq \theta U_N^* \simeq -i U_{\rm PMNS} \sqrt{\hat{m}} \Omega_{\rm m} \sqrt{\hat{M}^{-1}}. \tag{31}$$

Approximation $M_M \sim \mathcal{O}(\theta M_D)$

$$-\sqrt{\hat{M}^{-1}} U_N^T \left(\theta^{-1} - \frac{1}{3}\theta^\dagger\right) U_\nu \hat{m} U_\nu^T (\theta^T)^{-1} U_N \sqrt{\hat{M}^{-1}} \simeq I, \tag{32}$$

$$\Omega_{\rm nm} = i\sqrt{\hat{m}} U_\nu^T (\theta^T)^{-1} U_N \sqrt{\hat{M}^{-1}} \tag{33}$$

$$\Omega_{\rm nm}^{-1} = \Omega_{\rm nm}^{T} + \frac{1}{3}\hat{M}^{-1}(\Omega_{\rm nm}^{-1})^{*}\hat{m}, \quad (34)$$

$$\theta = -iU_{\nu}\sqrt{\hat{m}}(\Omega_{\rm nm}^{-1})^{T}\sqrt{\hat{M}^{-1}}U_{N}^{T}, \quad (35)$$

$$\Theta_{\rm nm} \simeq \theta U_{N}^{*} \simeq -iU_{\nu}\sqrt{\hat{m}}(\Omega_{\rm nm}^{-1})^{T}\sqrt{\hat{M}^{-1}} \quad (36)$$

Charged and neutral neutrino currents

$$\mathcal{L}_{NC}^{\nu} = \frac{g}{2c_W}\overline{\nu}_L\gamma^{\mu}U_{\rm PMNS}^{\dagger}U_{\rm PMNS}\nu_L Z_{\mu}, \quad (37)$$

$$\mathcal{L}_{CC}^{\nu} = -\frac{g}{\sqrt{2}}\bar{l}_L\gamma^{\mu}U_{\rm PMNS}\nu_L W_{\mu}^{-} + h.c., \quad (38)$$

$$\mathcal{L}_{NC}^{N} = -\frac{g}{2c_W}\overline{N}_L\gamma^{\mu}\Theta^{\dagger}\Theta N_L Z_{\mu} - \left(\frac{g}{2c_W}\overline{\nu}_L U_{\rm PMNS}^{\dagger}\gamma^{\mu}\Theta N_L Z_{\mu} + h.c.\right), \quad (39)$$

$$\mathcal{L}_{CC}^{N} = -\frac{g}{\sqrt{2}}\bar{l}_L\gamma^{\mu}\Theta N_L W_{\mu}^{-} + h.c., \quad (40)$$

where Θ is the minimal $\Theta_{\rm m}$ or nonminimal $\Theta_{\rm nm}$ mixing matrix

Effective mass m_ν

$$(m_\nu)_{ee} < 2 \text{ eV}, \qquad (m_\nu)_{\mu\mu} < 0.19 \text{ MeV}, \qquad (m_\nu)_{\tau\tau} < 18.2 \text{ MeV},$$

K. Olive, et al., Review of particle physics, Chin. Phys. C 38 (2014) 090001

$\sum_i m_{\nu_i} < 0.2 - 1.0$ eV 1602.04816v2 [hep-ph]

Approximation $M_M \sim \mathcal{O}(\theta^{-1} M_D)$

$$\begin{aligned} m_\nu &\simeq -M_D\theta^T \simeq -\theta M_N \theta^T \\ &\simeq -\theta U_N^* \hat{M} U_N^\dagger \theta^T = -\Theta \hat{M} \Theta^T \simeq U_{\rm PMNS} \hat{m} U_{\rm PMNS}^T, \end{aligned} \tag{41}$$

where Eqs. (9), (11), (31) and $\Omega_{\rm m}\Omega_{\rm m}^T = I$ were used, correspondingly.

Approximation $M_M \sim \mathcal{O}(\theta M_D)$

Using (23), (24), (35), (4), (34) and (36), correspondingly, one can find

$$m_\nu \simeq -M_D\theta^T \simeq -\left[\theta - \frac{1}{3}(\theta^\dagger)^{-1}\right] M_N \theta^T \tag{42}$$

$$= iU_\nu \sqrt{\hat{m}} \left[(\Omega_{\rm nm}^{-1})^T + \frac{1}{3}\hat{m}^{-1}\Omega_{\rm nm}^* \hat{M}\right] \sqrt{\hat{M}^{-1}} U_N^T M_N \theta^T \tag{43}$$

$$\simeq U_{\rm PMNS} \hat{m} U_{\rm PMNS}^T$$

Numerical estimations

For simplicity, we assume that

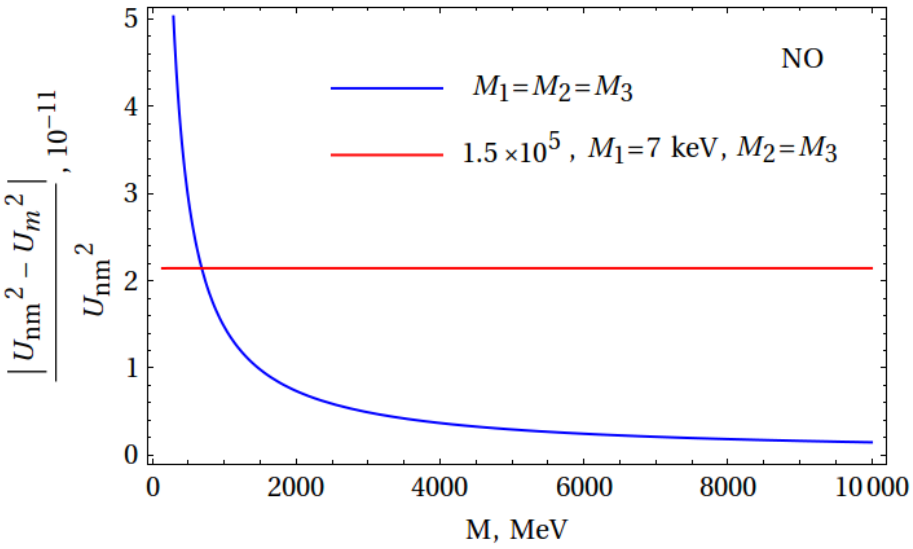
$$\Omega_{\rm m} = I, \qquad \Omega_{\rm nm} = {\rm diag}(\omega_1, \omega_2, \omega_3), \qquad \omega_i = \sqrt{1 - \frac{1}{3}\frac{m_i}{M_i}}, \tag{44}$$

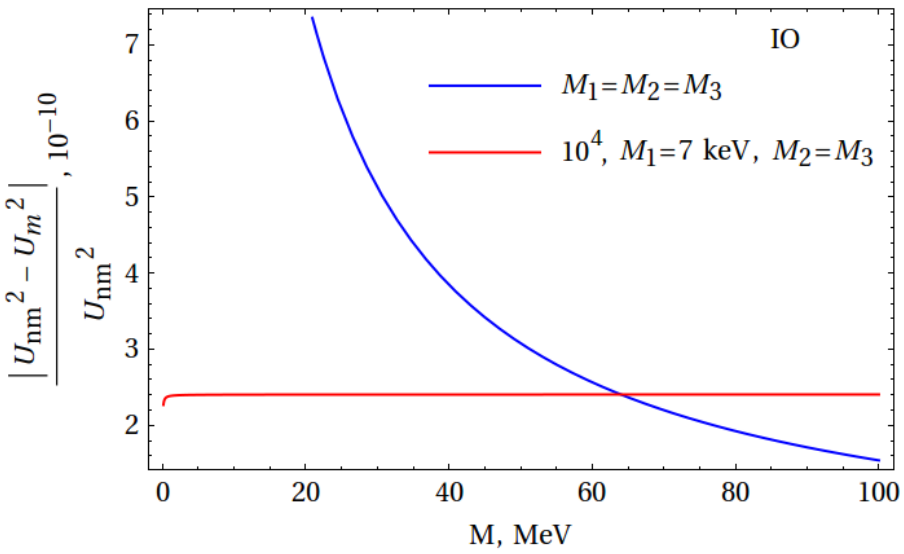
which satisfy requirements (29) and (34) and

- m_0 [NO (IO)] $= 0.03$ eV (0.015 eV)
- (i) $M_1 = 7$ keV, $M_2 = M_3 = M$; (ii) $M_1 = M_2 = M_3 = M$

Phenomenologically convenient quantities:

$$U^2_{\alpha i} = |\Theta_{\alpha I}|^2, \qquad U^2_i = \sum_\alpha U^2_{\alpha I}, \qquad U^2 = \sum_i U^2_i \tag{45}$$





Prospects and conclusions

- We proposed a modified seesaw type I expression for the mixing matrix

$$\Theta_{\rm nm} \simeq \theta U_N^* \simeq -iU_\nu \sqrt{\hat{m}} (\Omega_{\rm nm}^{-1})^T \sqrt{\hat{M}^{-1}}, \tag{46}$$

  where the matrix $\Omega_{\rm nm}$ satisfies

$$\Omega_{\rm nm}^{-1} = \Omega_{\rm nm}^T + \frac{1}{3}\hat{M}^{-1}(\Omega_{\rm nm}^{-1})^* \hat{m}. \tag{47}$$

- The contributions of the order of $\mathcal{O}(\theta M_D)$ coming from the expansion of rotation matrix W up to $\mathcal{O}(\theta^3)$ terms to the Majorana matrix representation M_M were obtained. It was shown that these contributions are of the same order as for the case $W \sim \mathcal{O}(\theta^2)$ and, therofore, they should be included in relevant analysis.

- It was shown that the effective mass m_ν has the same representation for both approximations $M_M \sim \mathcal{O}(\theta^{-1} M_D)$ and $M_M \sim \mathcal{O}(\theta M_D)$.

- The absolute value of relative difference of $|\Theta|^2$ in approximations $M_M \sim \mathcal{O}(\theta^{-1} M_D)$ and $M_M \sim \mathcal{O}(\theta M_D)$ is negligible for phenomenology on accelerators. However, it can be important for processes in the early Universe.

Thank you for your attention