Estimators study for centrality determination in the BM@N experiment

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Motivation

● Evolution of matter produced in heavy-ion collisions depends on its initial geometry

● Goal of centrality determination:
  map (on average) the collision geometry parameters
to experimental observables (centrality estimators)
  ○ Glauber model is commonly used to build such connection
  ○ Model parameters are fixed by minimizing
    the difference between the model and real data distributions

● Centrality class: group of events corresponding to
  a given fraction (%) of the total cross section:

\[ C_b = \frac{1}{\sigma_{\text{inel}}^{AA}} \int_0^b \frac{d\sigma}{db'} \, db' \]
Why several alternative centrality estimators

- **MC-Glauber x NBD multiplicity fitting procedure** is standard method for centrality determination.
- **BM@N** needs this method to compare data in the least experiment dependent way.
- **Innovative Γ-fit method** is also being considered for centrality determination based on multiplicity.
Why several alternative centrality estimators

Anticorrelation between charge of the spectator fragments (FW) and particle multiplicity (hits)

A number of produced protons is stronger correlated with the number of produced particles (track & RPC+TOF hits) than with the total charge of spectator fragments (FW)

Avoid self-correlation biases when using spectators fragments for centrality estimation
Centrality Estimators in BM@N

Produced charged particles

DCM-QGSM-SMM
Au-Au @ $p_{lab} = 12A$ GeV/c
($\sqrt{s_{NN}} = 4.95$ GeV)

0-10%

Multiplicity

Target spectators
(not measured)

Projectile spectators

DCM-QGSM-SMM
Au-Au @ $p_{lab} = 12A$ GeV/c
($\sqrt{s_{NN}} = 4.95$ GeV)

0-10%

Spectators energy, GeV
BM@N subsystems for centrality determination

Simulation setup
- DCM-QGSM-SMM
- Xe-Cs @ 4A GeV
- Transport: GEANT4

Subsystem
- Participants: Tracking system
  - GEM+STS, BD, SiMD
- Spectators: FHCal, Hodoscope,
  ScWall, FD

M. Baznat et al. PPNL 17 (2020) 3, 303
MC-Glauber + NBD fitting procedure

Produced charged particles

Extract relation between geometry parameters and multiplicity

MC-Glauber multiplicity distribution, $dN/dM_{GI}$

Full Monte-Carlo (real data) multiplicity distribution, $dN/dM_{MC/data}$

Code repository: https://git.cbm.gsi.de/pwg-c2f/analysis/centrality
MC-Glauber + NBD fitting procedure

MC-Glauber calculations for a set of fit parameters

Sample $N_{\text{part}}$, $N_{\text{coll}}$ from MC-Glauber

Evaluate number of ancestors (sources of produced particles)

$N_{a} = f N_{\text{part}} + (1 - f) N_{\text{coll}}$

(other functional forms will be also investigated)

Sample $N_{a}$ times from NBD($\mu$, $k$) and calculate multiplicity of produced particles

Iterate steps above $N_{\text{events}}$ times to produce the MC-Glauber multiplicity distribution

Extract relation between geometry parameters and multiplicity

MC-Glauber multiplicity distribution, $dN/dM_{\text{Gi}}$

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MC-Glauber multiplicity distribution, $dN/dM_{\text{Gl}}$

Full Monte-Carlo (real data) multiplicity distribution, $dN/dM_{\text{MC/data}}$

Evaluate $\chi^2$ between $dN/dM_{\text{MC/data}}$ and $dN/dM_{\text{Gl}}$

Scan phase space of $f$, $\mu$ and $k$ to find their values for minimum of $\chi^2$

Extract relation between geometry parameters and multiplicity

Implementation for MPD: https://github.com/FlowNICA/CentralityFramework
MC-Glauber fit result Xe-Cs @ 4.0 AGeV

\( \chi^2 = 1.31 \pm 0.07; \)
\( f = 0.9, \)
\( \mu = 0.786293, \)
\( k = 1; \)
\( \text{MinFitBin} = 10, \)
\( \text{MaxFitBin} = 250 \)

- Fit result is good
- Impact parameter distributions in different centrality classes reproduces ones from DCM-QGSM-SMM
The Bayesian inversion method ($\Gamma$-fit): main assumptions

Relation between multiplicity $N_{ch}$ and impact parameter $b$ is defined by the fluctuation kernel:

$$P(N_{ch}|c_b) = \frac{1}{\Gamma(k(c_b)) \theta^k} N_{ch}^{k(c_b)-1} e^{-n/\theta}$$

$$c_b = \int_0^b P(b')db' \approx \frac{\pi b^2}{\sigma_{inel}}$$

- centrality based on impact parameter

The dependence of the average value of multiplicity on centrality and the results of its fit

$$\frac{\sigma^2}{\langle N_{ch} \rangle} = \theta \approx \text{const}$$

$$\langle N_{ch} \rangle = N_{knee} \exp\left(\sum_{j=1}^{3} a_j c_b^j\right), \quad k = \frac{\langle N_{ch} \rangle}{\theta}$$

Five fit parameters: $N_{knee}, \theta, a_j$
Reconstruction of $b$

- Normalized multiplicity distribution $P(N_{ch})$
  
  \[ P(N_{ch}) = \int_{0}^{1} P(N_{ch}|c_{b}) dc_{b} \]

- Find probability of $b$ for fixed range of $N_{ch}$ using Bayes’ theorem:

  \[ P(b|n_{1} < N_{ch} < n_{2}) = P(b) \frac{\int_{n_{1}}^{n_{2}} P(b|N_{ch})dN_{ch}}{\int_{n_{1}}^{n_{2}} P(N_{ch})dN_{ch}} \]

- The Bayesian inversion method consists of 2 steps:
  - Fit normalized multiplicity distribution with $P(N_{ch})$
  - Construct $P(b|N_{ch})$ using Bayes’ theorem with parameters from the fit

Implementation for MPD and BM@N by D. Idrisov: [https://github.com/Dim23/GammaFit](https://github.com/Dim23/GammaFit)
Example of application in MPD: P. Parfenov et al., Particles 4 (2021) 2, 275-287
MC-Glauber Π-fit result Xe-Cs @ 4.0 AGeV

- Fit result is good
- Impact parameter distributions in different centrality classes reproduces ones from DCM-QGSM-SMM
Possibilities of spectators fragments as estimators

- FHCal energy and Hodoscope charge distributions have partial correlation with impact parameter.
- For example, impact parameter at 6 fm might be used as threshold for simulations.
- Corresponding physical threshold could be Hodoscope signal $E_{\text{Hodo}} = 0.04$. 

![Graphs showing correlation between FHCal energy and Hodoscope charge distributions with impact parameter.](image)
Possibilities of spectators fragments as estimators

- FHCal energy distribution improved and has more linear correlation with impact parameter (for range $Q_{\text{Hodo}} < 0.04$)

- There is good correlation between Hodoscope charge and impact parameter (for range $Q_{\text{Hodo}} > 0.04$)

BM@N simulations
Xe-Cs @ 4AGeV
FHCal ($Q_{\text{Hodo}} < 0.04$)

FHCal energy distribution improved and has more linear correlation with impact parameter (for range $Q_{\text{Hodo}} < 0.04$)

There is good correlation between Hodoscope charge and impact parameter (for range $Q_{\text{Hodo}} > 0.04$)

BM@N simulations
Xe-Cs @ 4AGeV
Hodoscope

$b \sim 6 \text{ fm}$

$q_{\text{Hodo}} = 0.04$
Possibilities of spectators fragments as estimators

NA61/SHINE data 
PbPb @ 13AGeV 
\[ E_{PSD} = E_{PSD1} + E_{PSD2} + E_{PSD3} \]

BM@N simulations 
DCM-QGSM-SMM 
Geant4 
XeCs @ 4AGeV 

\[ E^{tot}_{frag} = k_1 E_{Hodo} + k_2 E_{FHCal} \]

\[ k_1 = 4690, \quad k_2 = 40 \]
Comparison of different estimators and methods

- Impact parameter distributions in different centrality classes are similar for different centrality estimators.
- These distributions for spectators energy is wider because of the width of b and energy correlation.
Summary

- MC Glauber and inverse bayesian fitting procedures are developed for multiplicity
- Relation between impact parameter and centrality classes is extracted
- Software implementation of the procedures is ready and also used in MPD

- Possibilities of using of forward detectors for centrality determination was studied
- Main tasks was detected: improvement of the width of impact parameter and energy distributions, validation of FHCal x Hodoscope energy distribution

Work in progress

- Apply centrality determination procedures based on multiplicity for simulations of lower energies collisions
- Continue work on preparing of centrality determination procedure based on spectators energy
Backup
MC Glauber model

MC Glauber model provides a description of the initial state of a heavy-ion collision
  ○ Independent straight line trajectories of the nucleons
  ○ A-A collision is treated as a sequence of independent binary NN collisions
  ○ Monte-Carlo sampling of nucleons position for individual collisions

Main model parameters
- Colliding nuclei
  - Inelastic nucleon-nucleon cross section ($\sigma_{\text{inel}}^{\text{NN}}$) (depends on collision energy)
- Nuclear charge densities (Wood-Saxon distribution)

\[ \rho(r) = \rho_0 \cdot \frac{1 + w(r/R)^2}{1 + \exp \left( \frac{r-R}{a} \right)} \]

Geometry parameters
  b – impact parameter
  $N_{\text{part}}$ – number of nucleons participating in the collision
  $N_{\text{spec}}$ – number of spectator nucleons in the collision
  $N_{\text{coll}}$ – number of binary NN collisions

Glauber Modeling in High Energy Nuclear Collisions:
ARNPS57:205-243,2007
Result of the fitting
SMM description of the ALADIN’s fragmentation data

A.S. Botvina et al. NPA 584 (1995) 737

R.Ogul et al. PRC 83, 024608 (2011)
Mass number of fragments sampling for given event: new procedure

1. Fill set of 2D probability distribution of $A_{\text{frag}}$ and $N_A$ for each set of $(A_{\text{tot}}, N_{\text{spec}}, b)$
2. Reset all pairs $(A_{\text{frag}}^i, N_A^i), N_U = 0$
3. Generated pair $(A_{\text{frag}}^i, N_A^i)$, if:
   a. $A_{\text{frag}}$ already among existing pairs from $(1, N_U -1)$, then skip
   b. $(\sum_{i=(1,N_U)}^N A_{\text{frag}}^i) > A_{\text{tot}}$ then go to step 2
   else:
      Add $(A_{\text{frag}}^i, N_A^i)$ to the list of pairs, increment $N_U = N_U + 1$
Result: a set of $N_U$ pairs $(A_{\text{frag}}, N_A)$ with $A_{\text{tot}} = N^1 A_{\text{frag}} + N^2 A_{\text{frag}} + \ldots N^N A_{\text{frag}}$

1. Sample $(N_{\text{spec}}, b)$ from Glauber Model
2. $(A_T) - \text{ at } t=\infty$
3. $F_{\text{spec}}$ for $(A_T, N_{\text{spec}}, b)$
4. $(E_{Af} Y_{Af})$ for $(A_T)$
5. $E_{\text{PSD}}^{Afj}$ for $(E,Y)_{Afj}$
6. Result: total $E_{\text{PSD}}$
Population of fragments with energy and rapidity

- Energy and rapidity distributions have different shapes for different fragment mass
- Shapes are used as input for sampling energy & rapidity values for each fragment
Respond of FHCal detector

- Mean of signal has linear dependency with beam energy

\[
\frac{\sigma_k}{E} = \sqrt{\left(\frac{0.54}{\sqrt{E}}\right)^2 + (0.041)^2 + \left(\frac{3.6}{E}\right)^2}
\]
NA61/SHINE experimental setup

PSD detector layout
Full mode procedure (example for NA61)

- Scaling along both X and Y axis is applied
- Form of energy distribution is reproducible
MC-Glauber+Spectators fitting procedure

- Sample $N_{\text{part}}$ from MC-Glauber
- Evaluate number of ancestors (sources of produced particles) $N_a = A_{\text{proj}} - N^{\text{proj}}_{\text{part}}$
- Sample $N_a$ times from Gauss($\mu$, $\kappa$) and calculate multiplicity of produces particles
- Iterate steps above $N_{\text{events}}$ times to produce the MC-Glauber energy distribution

- MC-Glauber energy distribution, $dN/dE_{\text{Gl}}$
- Full Monte-Carlo (real data) energy distribution, $dN/dE_{\text{MC/data}}$

- Evaluate $\chi^2$ between $dN/dE_{\text{MC/data}}$ and $dN/dE_{\text{Gl}}$
- Scan phase space of parameters to find their values for minimum of $\chi^2$

Extract relation between geometry parameters and energy
Light mode procedure fit (example for NA61)

$\chi^2 = 18.1891 \pm 0.365028$; 
$\mu = 12.4943$, 
k = 8.9; 
MinFitBin = 17 (200 GeV), 
MaxFitBin = 250 (3000 GeV)

- Produced particles affect form of full PSD distribution
- Light mode maybe needs some additional parameters
$\mu = 0.85$

MC Glauber fit results are in good agreement with simulated input
Centrality determination using STS multiplicity

Distribution provides connection between centrality class (multiplicity range, $M \pm \Delta M$) and impact parameter range ($b \pm \sigma_b$).