Fractal Entropy of Nuclear Medium Probed by K_S^0 Mesons Produced in Au+Au Collisions at RHIC

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**NPI, Řež, Czech Republic

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MT & I. Zborovský
Nucl. Phys. A 1025 (2022) 122492

Motivation & Goals

Search for new symmetries in Nature

Systematic analysis of inclusive cross sections of particle production in p+p, p+A and A+A collisions to search for general features of hadron structure, constituent interactions and fragmentation process over a wide scale range.

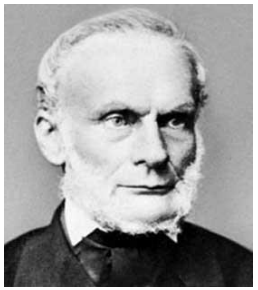
z-Scaling as a tool in high energy physics

Development of z-scaling approach for description of processes with strange particle production in inclusive reactions, verification of self-similarity principle, search for signatures of phase transition and critical point.

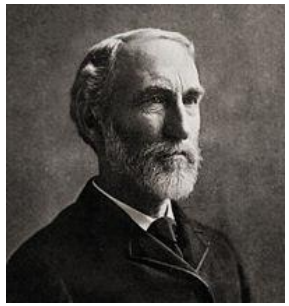
Anomaly of fractal entropy of K_S^0 - meson production in Au+Au collisions as a possible signature of phase transition in nuclear matter.

Entropy and thermodynamic potentials

Thermodynamics — ἐντροπία — Statistical physics

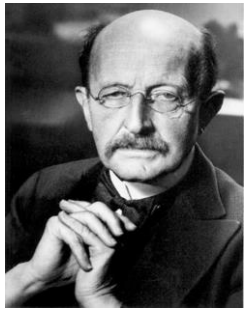


Rudolf Julius Emanuel Clausius



Josiah Willard Gibbs

Ludwig Eduard Boltzmann



Max Karl Ernst Ludwig Planck

Entropy is a function of state

$$dS = dU/T + pdV/T - \mu dN/T$$

$$S = S(U,V,N), \quad T=T(U,V,N)$$

Thermodynamic quantities and potentials are expressed via entropy

$$U=TS-pV+\mu N$$
$$H=U+pV$$
$$F=U-TS$$
$$G=U+pV-TS$$
$$\Omega=U-TS-\mu N$$

$$1/T = \partial S/\partial U|_{V,N}$$
$$p/T = \partial S/\partial V|_{U,N}$$
$$c_V = T\partial S/\partial T|_V$$
$$\partial p/\partial T|_{V,N} = \partial S/\partial V|_{T,N}$$
$$\partial V/\partial T|_{p,N} = - \partial S/\partial p|_{T,N}$$

$$S = k \cdot \ln W$$

k - Boltzmann constant
W - number of microstate

Various forms of entropy:

Clausius	(1865)	S_C
Gibbs	(1876)	S_G
Boltzmann	(1872)	S_B
Tsallis	(1988)	S_q
Sharma, Mittal	(1975)	S_{SM}
Kaniadakis	(2001)	S_k
……		

von Neumann	(1932)	
Shannon	(1948)	S_S
Khinchin	(1957)	
Kolmogorov	(1954)	
Rényi	(1961)	S_R
……		

The principles of evolution of thermodynamic system

L. Onsager
(1903-1976)

"… the discovery of the reciprocal relations…."

1968

I. Prigogine
(1917-2003)

"… dissipative structures, complex systems, and irreversibility"

1977

Stationary non-equilibrium systems

Principle of least dissipation of energy

If in a system a whole set of states is admissible that agrees with the conservation laws and other principles, as well as with the connections imposed on the system, then the state is realized that corresponds to the minimum energy dissipation.

Principle of minimum entropy production

The entropy production in the system, being in a stationary state with fixed external parameters, is constant in time and minimal in magnitude.

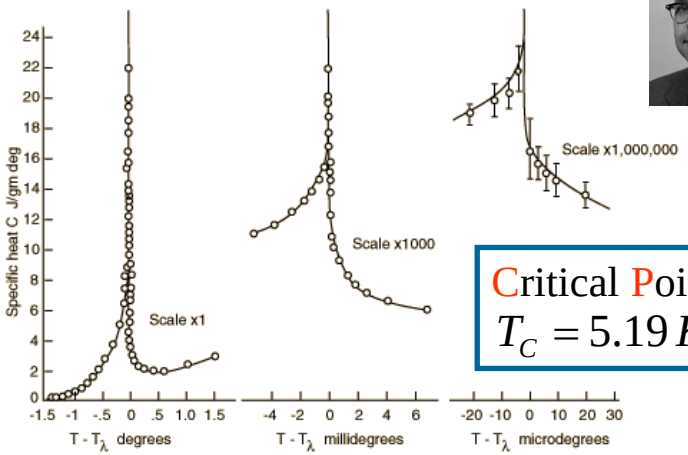
Correspondence between these principles

Properties of entropy

- Entropy is function of state of a (thermodynamic) system.
- Entropy is smooth function of thermodynamic parameters.
- For reversible processes $\oint dS = 0$.
- Basic concept in 2. and 3. law of thermodynamics.
- Entropy of phase transition.
- Entropy of extensive and non-extensive systems.
- Fractal entropy - entropy of systems with fractal objects.
- Entropy in quantum statistical mechanics.
- Entropy of entanglement, black hole, Big Band, Universe, …
- Entropy and information content of the human genome, …
- ……..

Singularity of specific heat near a Critical Point

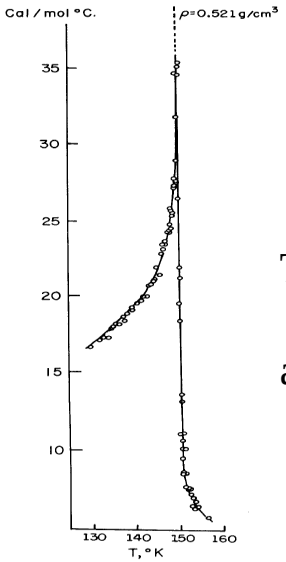
Specific heat of liquid ^{4}He
Superfluid transition



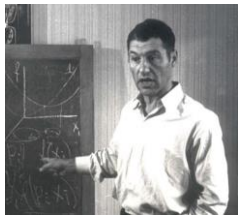
Critical Point
$T_C = 5.19\,K$

M. J. Buckingham and W. M. Fairbank, 1961
H.E. Stanley, 1971

Heat capacity of Ar



Critical Point
$T_C = 150.8\,K$



The isochoric heat capacity C_v of argon becomes infinite at the vapor-liquid critical point.

A.V. Voronel' et al.
Zh. Exp. Teor. Fiz. 43, 728 (1962).

- Near a critical point the singular part of thermodynamic potentials is a Generalized Homogeneous Function (GHF).
- The Helmholtz potential $F(\lambda^{a_\varepsilon}\varepsilon, \lambda^{a_V}V) = \lambda F(\varepsilon, V)$ is GHF of (ε, V).

$$c_V \sim |\varepsilon|^{-\alpha} \qquad \varepsilon \equiv (T-T_c)/T_c \qquad c_V = -T(\partial^2 F/\partial T^2)|_V \qquad c_V = T\partial S/\partial T|_V$$

Critical exponents define the behavior of thermodynamic quantities nearby the Critical Point.

Phase transitions & Critical phenomena

- Critical phenomena reveal unusual characteristic behavior of substances in the vicinity of phase transition points.
- They are observed due to an increase in the characteristic sizes of different fluctuations.
- In these phenomena, the self-similarity of a system arises spontaneously.
- This scale property is characteristic of fractal structures.
- Second order transition is accompanied by a spontaneous symmetry breaking.

Signatures of critical phenomena:

- increase in compressibility (liquid-vapor equilibrium)
- increase in magnetic and dielectric susceptibility in the vicinity of the Curie points of ferromagnets and ferroelectrics
- anomaly in heat capacity at the point of transition of helium to the superfluid state
- slowing of the mutual diffusion of substances near the critical points of mixtures of stratifying liquids
- anomaly in the propagation of ultrasound (absorption of sound and an increase in its dispersion)
- anomalies in viscosity, thermal conductivity, slowdown in the establishment of thermal equilibrium, etc.

These anomalies are described by power laws with critical indices.
Strong fluctuations and infinite correlation radii in such systems confirm self-similarity.

z-Scaling:

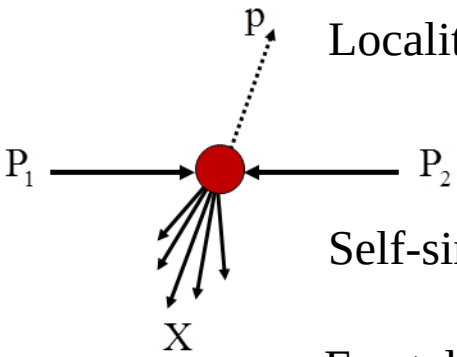
ideas, definitions, hypothesis,…

Basic principles:

locality, self-similarity, fractality,…

z-Scaling

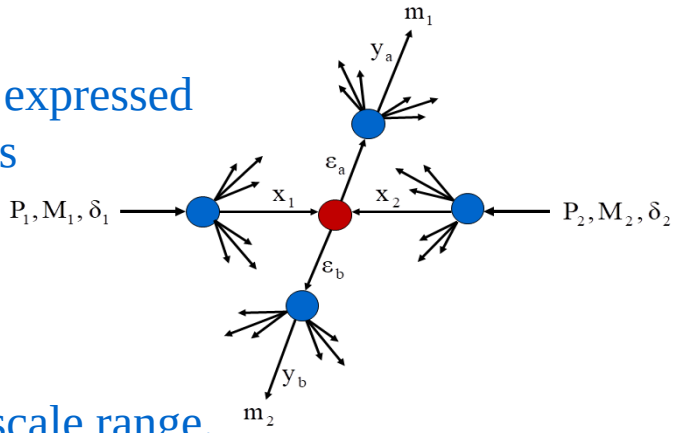
Principles: locality, self-similarity, fractality



Locality: collisions of hadrons and nuclei are expressed via interactions of their constituents (partons, quarks and gluons,...).

Self-similarity: interactions of constituents are mutually similar.

Fractality: self-similarity is valid over a wide scale range.



Hypothesis of z-scaling :

$s^{1/2}$, p_T, θ_{cms}

Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.

x_1, x_2, y_a, y_b

$\delta_1, \delta_2, \varepsilon_a, \varepsilon_b, c$

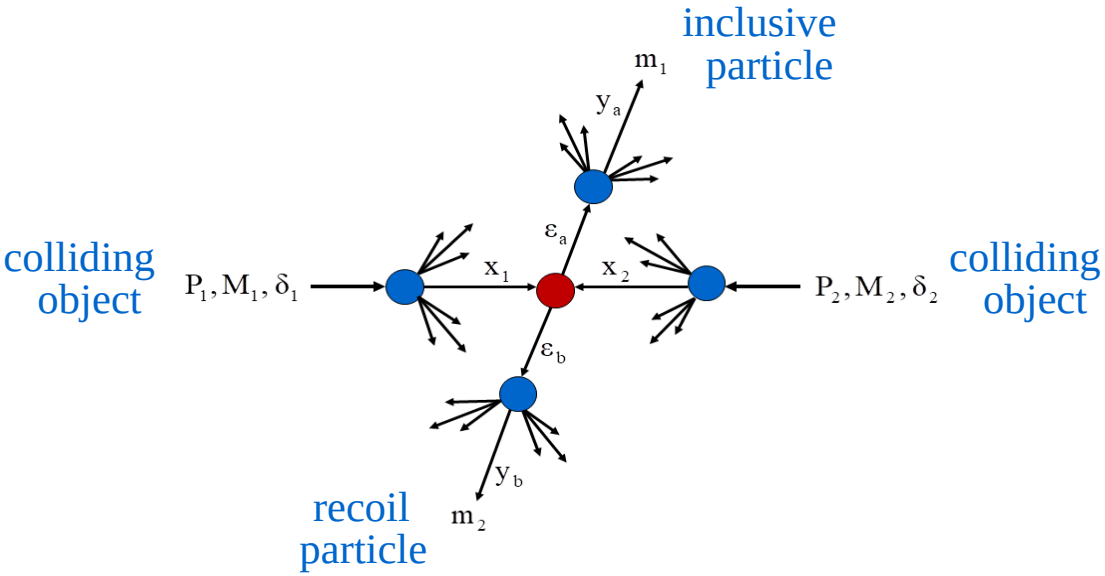
$Ed^3\sigma/dp^3$

Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable z.

$\Psi(z)$

Locality

Collisions of colliding objects
are expressed via interactions of their constituents



P_1, P_2, p – momenta of colliding and produced particles

M_1, M_2, m_1 – masses of colliding and produced particles

x_1, x_2 – momentum fractions of colliding particles carried by constituents

y_a, y_b – momentum fractions of scattered constituents carried by inclusive particle and its recoil

δ_1, δ_2 – fractal dimensions of colliding particles

ε_a, ε_b – fractal dimensions of scattered constituents (fragmentation dimensions)

m_2 – mass of recoil particle

Elementary sub-process:

$$(x_1M_1) + (x_2M_2) \rightarrow (m_1/y_a) + (x_1M_1 + x_2M_2 + m_2/y_b)$$

Momentum conservation law for sub-process

$$(x_1P_1 + x_2P_2 - p/y_a)^2 = M_X^2$$

Mass of recoil system

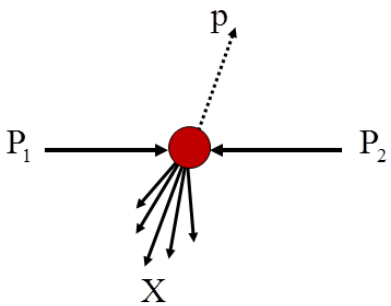
$$M_X = x_1M_1 + x_2M_2 + m_2/y_b$$

M.T., I.Zborovský
Yu.Panebratsev, G.Skoro
Phys.Rev.D54 5548 (1996)
Int.J.Mod.Phys.A16 1281 (2001)

Self-similarity

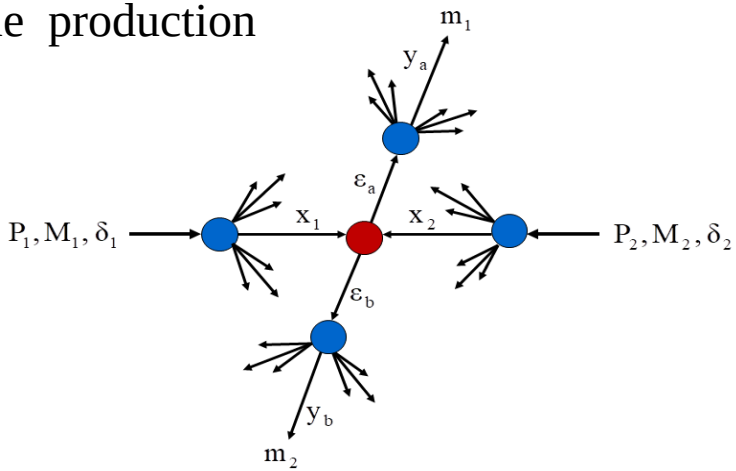
Interactions of constituents are mutually similar

The self-similarity parameter z is a dimensionless variable, expressed through the dimensional quantities P_1, P_2, p, M_1, M_2, m_1, m_2, characterizing the process of inclusive particle production



$$z = z_0 \cdot \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$



- Ω^{-1} is the minimal resolution at which a constituent sub-process can be singled out of the inclusive reaction
- $s_\perp^{1/2}$ is the transverse kinetic energy of the sub-process consumed on production of m_1 & m_2
- $dN_{ch}/d\eta|_0$ is the multiplicity density of charged particles at $\eta = 0$
- c is a parameter interpreted as a "specific heat" of created medium
- m_N is an arbitrary constant (fixed at the value of nucleon mass)

Fractality

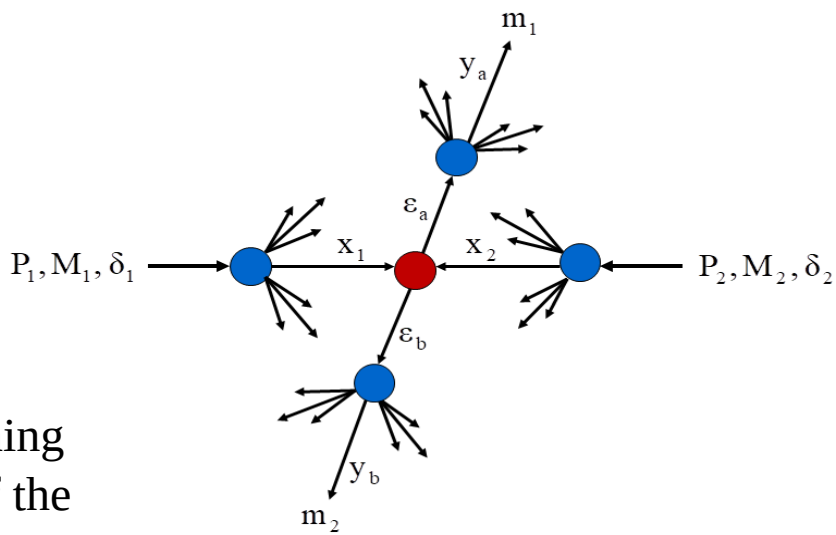
Self-similarity over a wide scale range

Fractal measure

$$z = z_0 \cdot \Omega^{-1}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$

$$0 < x_1, x_2 < 1 \qquad 0 < y_a, y_b < 1$$



Ω is relative number of configurations containing a sub-process with fractions x_1, x_2, y_a, y_b of the corresponding 4-momenta

δ_1, δ_2, ε_a, ε_b are parameters characterizing structure of the colliding objects and fragmentation process, respectively

Ω^{-1} (x_1, x_2, y_a, y_b) characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

The fractal measure z diverges as the resolution Ω^{-1} increases.

$$z(\Omega)\big|_{\Omega^{-1}\to\infty} \to \infty$$

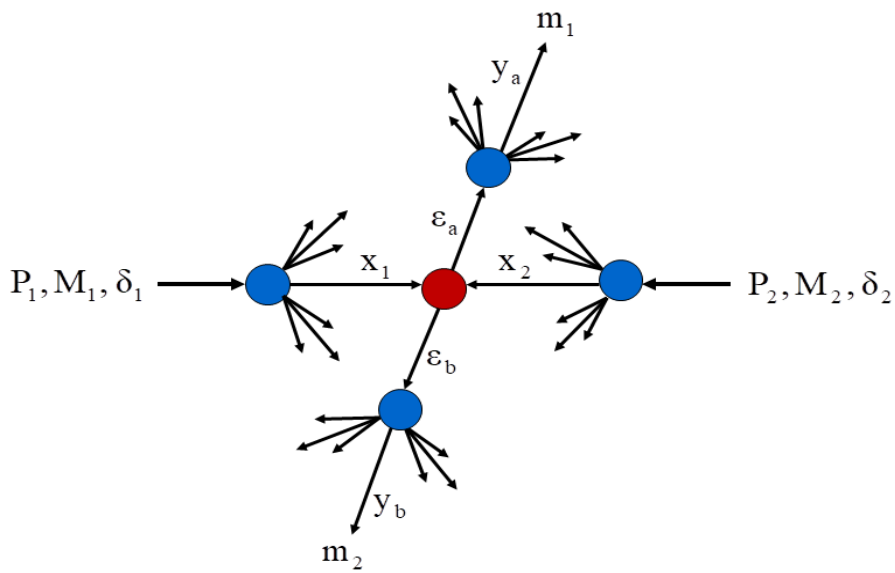
Momentum fractions: x_1, x_2, y_a, y_b

Principle of minimal resolution: The momentum fractions x_1, x_2 and y_a, y_b are determined in a way to minimize the resolution Ω^{-1} of the fractal measure z with respect to all constituent sub-processes taking into account 4-momentum conservation law.

Momentum conservation law

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

$$\begin{cases} \partial\Omega/\partial x_1 \,|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial x_2 \,|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial y_b \,|_{y_a=y_a(x_1,x_2,y_b)} = 0 \end{cases}$$



Resolution of sub-process

$$\Omega^{-1} = (1-x_1)^{-\delta_1}(1-x_2)^{-\delta_2}(1-y_a)^{-\varepsilon_a}(1-y_b)^{-\varepsilon_b}$$

Fractions x_1, x_2 ,y_a, y_b are expressed via Lorentz invariants – scalar products of 4-D momenta and particle masses.

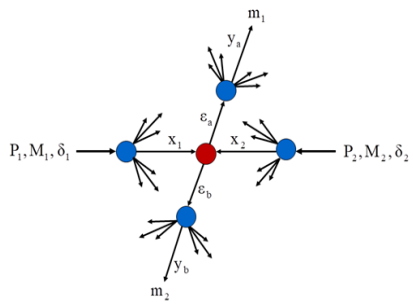
Mass of the recoil system

$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

Variable z & Fractal entropy $S_{\delta,\varepsilon}$

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$



$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$

Relative number of such constituent configurations which contain the configuration $\{x_1, x_2, y_a, y_b\}$

$$W = (dN_{ch}/d\eta|_0)^c \cdot \Omega$$

Statistical entropy

$$S = \ln W$$

Thermodynamical entropy for ideal gas

$$S = c_V \cdot \ln(T) + R \cdot \ln(V) + S_0$$

Fractal entropy

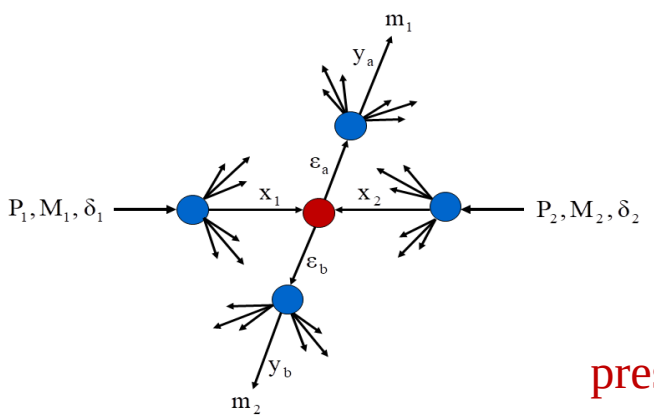
$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln(V_{\delta,\varepsilon}) + S_0$$

Entropy $S_{\delta,\varepsilon}$ for systems with structural constituents

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + S_0$$

- $dN_{ch}/d\eta|_0$ characterizes "temperature" of the colliding system.
- Provided local equilibrium, $dN_{ch}/d\eta|_0 \sim T^3$ for high temperatures and small μ.
- c has meaning of a "specific heat" of the produced medium.
- Fractional exponents $\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$ are fractal dimensions in the space of $\{x_1, x_2, y_a, y_b\}$.
- Entropy increases with $dN_{ch}/d\eta|_0$ and decreases with increasing resolution Ω^{-1}.

Scaling function $\Psi(z)$



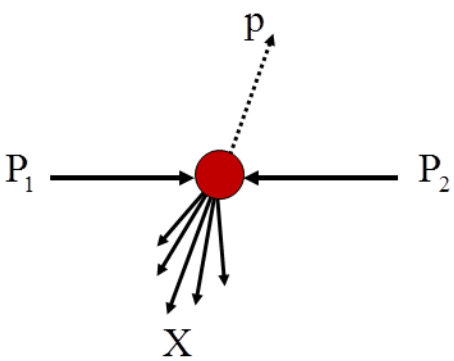
Normalization condition

$$\int_0^\infty \Psi(z)dz = 1$$

Scale transformation

$$z \to \alpha_F \cdot z \qquad \Psi \to \alpha_F^{-1} \cdot \Psi$$

preserves the normalization condition



$$\Psi(z) = \frac{\pi}{(dN/d\eta)\cdot\sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3} \quad \Longleftrightarrow \quad \int E\frac{d^3\sigma}{dp^3}dyd^2p_\perp = \sigma_{inel}\cdot < N >$$

- σ_{in} - the inelastic cross section
- $<N>$ - the average multiplicity
- $dN/d\eta$ - the multiplicity density
- $J(z,\eta;p_T^2,y)$ - the Jacobian
- $Ed^3\sigma/dp^3$ - the inclusive cross section

The scaling function $\Psi(z)$ is a probability density to produce the inclusive particle with the corresponding value of self-similarity variable z.

Data z-presentation
&
Self-similarity in inclusive pp and AA reactions.

Self-similarity & z-scaling

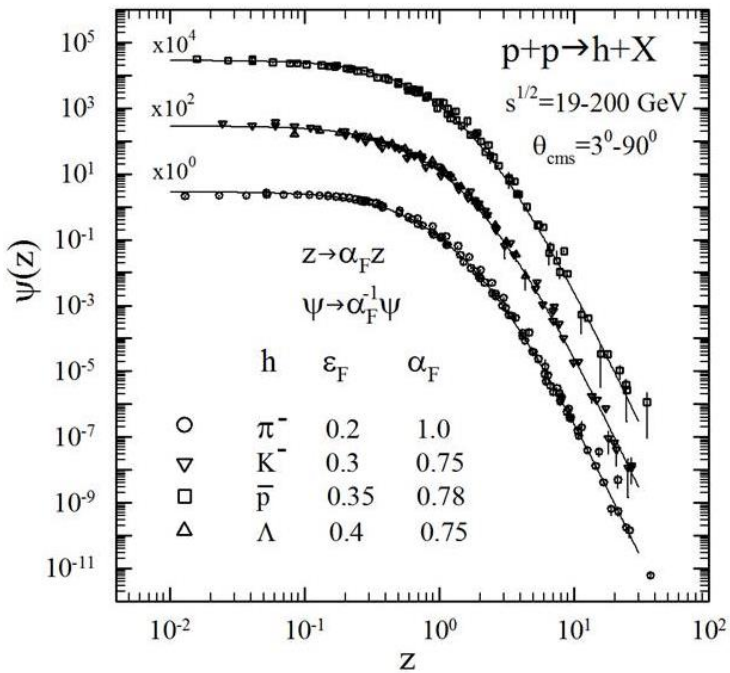
Inclusive cross sections of π^-, K^-, $\bar{p}$, Λ in pp collisions

FNAL:
PRD 75 (1979) 764

ISR:
NPB 100 (1975) 237
PLB 64 (1976) 111
NPB 116 (1976) 77
(low p_T)
NPB 56 (1973) 333
(small angles)

STAR:
PLB 616 (2005) 8
PLB 637 (2006) 161
PRC 75 (2007) 064901

Saturation at low z



Power law at high z

Energy scan of spectra
at U70, ISR, S$\bar{p}$pS, SPS, HERA, FNAL(fixed target), Tevatron, RHIC, LHC

MT & I.Zborovsky
T.Dedovich
Phys.Rev.D75,094008(2007)
Int.J.Mod.Phys.A24,1417(2009)
J. Phys.G: Nucl.Part.Phys. 37,085008(2010)
Int.J.Mod.Phys.A27,1250115(2012)
J.Mod.Phys.3,815(2012)
Int.J.Mod.Phys. A32,1750029(2017)
Nucl. Phys. A993 (2020) 121646

- Energy & angular independence
- Flavor independence (π, K, $\bar{p}$, Λ)
- Saturation for $z < 0.1$
- Power law $\Psi(z) \sim z^{-\beta}$ for high $z > 4$

Scaling – "collapse" of data points onto a single curve.
Universality classes – hadron species (ε_F, α_F).

Properties of $\Psi(z)$ in p+p collisions

- Energy independence of $\Psi(z)$ ($s^{1/2} > 20$ GeV)
- Angular independence of $\Psi(z)$ ($\theta_{cms}=3^0$-90^0)
- Multiplicity independence of $\Psi(z)$ ($dN_{ch}/d\eta=1.5$-26)
- Saturation of $\Psi(z)$ at low z ($z < 0.1$)
- Power law, $\Psi(z) \sim z^{-\beta}$, at high z ($z > 4$)
- Flavor independence of $\Psi(z)$ (π,K,φ,Λ,..,D,J/ψ,B,Υ,…, top)

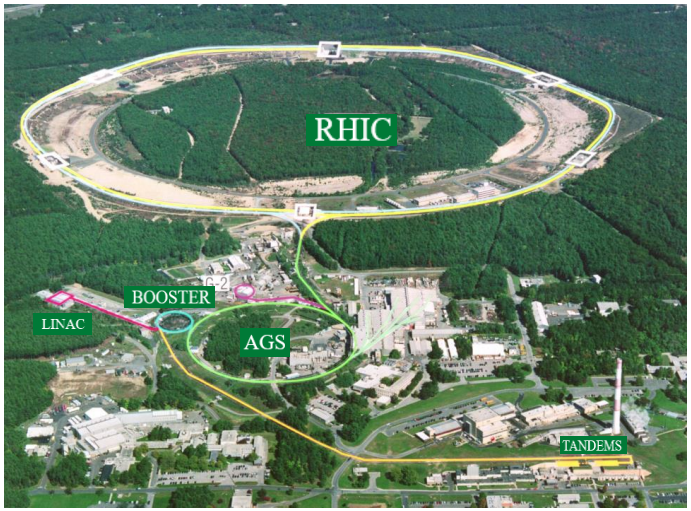
These properties reflect self-similarity, locality, and fractality of hadron interactions at a constituent level.

It concerns the structure of the colliding objects, constituent interactions and fragmentation process.

MT, I. Zborovský, A. O. Kechechyan, T. G. Dedovich,
Phys. Part. Nucl. 51, 141 (2020)

Strange particle production in p+p from RHIC

p+p @ RHIC



K^0_S, K^-, K^*, ϕ, Λ, Ξ, Ω, Σ^*, Λ^*

p+p is a benchmark for strangeness production in A+A collisions

M.T.& I.Zborovský
Int.J.Mod.Phys.
A32,1750029(2017)

Self-similarity of strangeness production in p+p

Universality: flavor independence of the scaling function

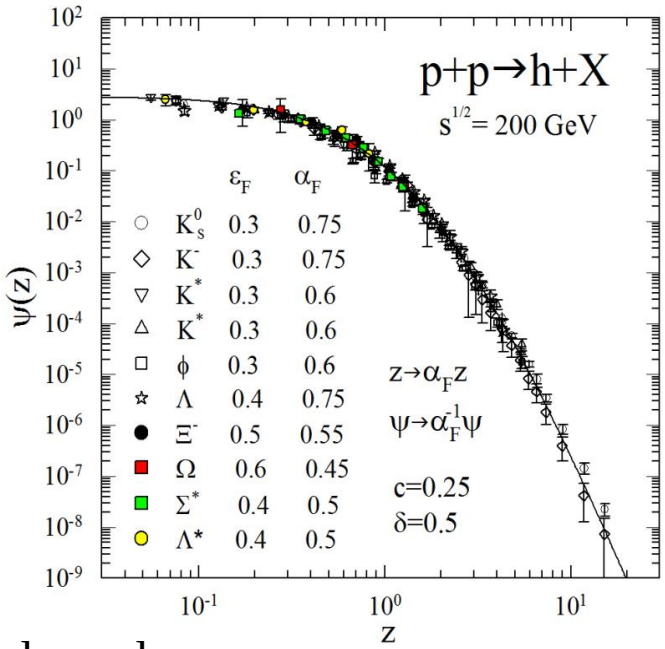
K^0_S, K^-, K^*, ϕ, Λ, Ξ, Ω, Σ^*, Λ^*

"Collapse" of data points onto a single curve

M.T.& I.Zborovský
Int.J.Mod.Phys.
A24,1417(2009)

Solid line for π^- meson is a reference frame

$\varepsilon_\pi = 0.2, \quad \alpha_\pi = 1$



STAR:
PRL 92 (2004) 092301
PRL 97 (2006) 132301
PLB 612 (2005) 181
PRC 71 (2005) 064902
PRC 75 (2007) 064901
PRL 108 (2012) 072302

PHENIX:
PRC 75 (2007) 051902
PRD 83 (2011) 052004
PRC 90 (2014) 054905

- Energy independence
- Angular independence
- Flavor independence
- Saturation for $z < 0.1$

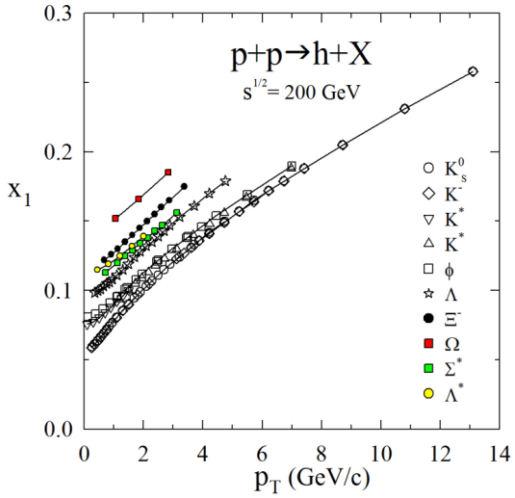
- Power law $\Psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$

Self-similarity of strangeness production in p+p

K^0_S, K^-, K^*, ϕ, Λ, Ξ, Ω, Σ^*, Λ^*

Constituent sub-process in terms of

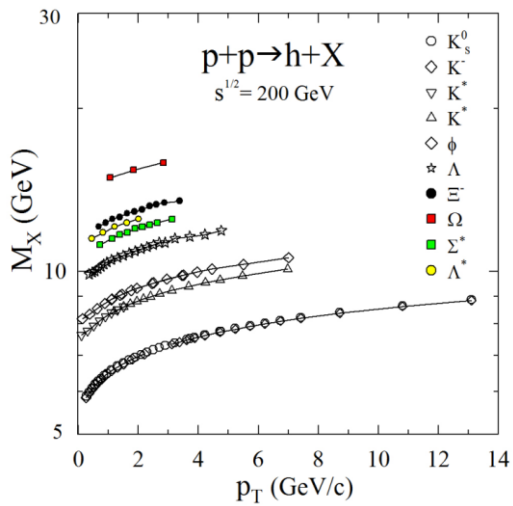
Momentum fraction



The more strangeness, the larger momentum fraction

$$x_1^{\Omega} > x_1^{\Xi} > x_1^{\Sigma} > x_1^{K}$$

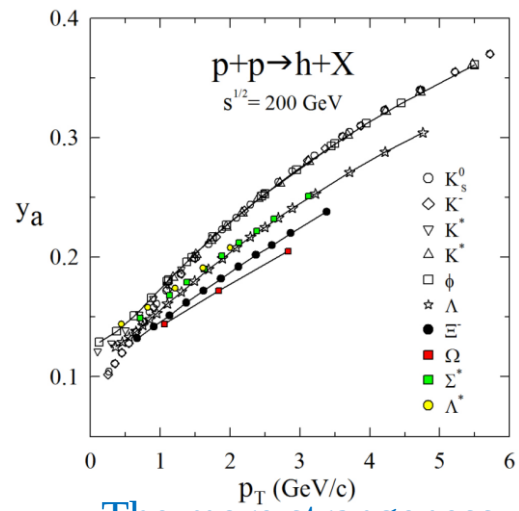
Recoil mass



The more strangeness, the larger recoil mass

$$M_X^{\Omega} > M_X^{\Xi} > M_X^{\Sigma} > M_X^{K}$$

Energy loss $\Delta E/E \sim (1-y_a)$



The more strangeness, the larger energy loss

$$\varepsilon_{\Omega} > \varepsilon_{\Xi} > \varepsilon_{\Sigma} > \varepsilon_{K}$$

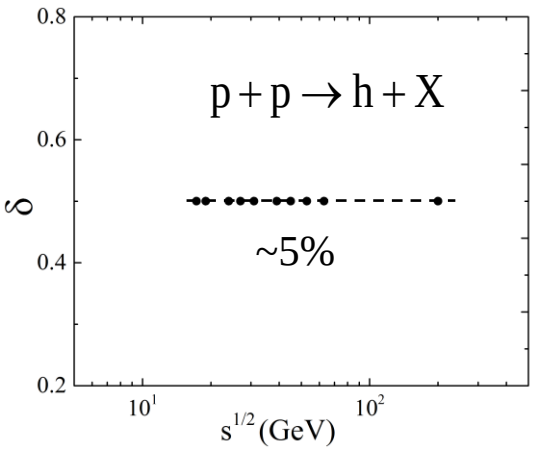
Smooth behavior of x_1, y_a, M_X vs. p_T.

Self-similarity dictates the properties of constituent sub-process.

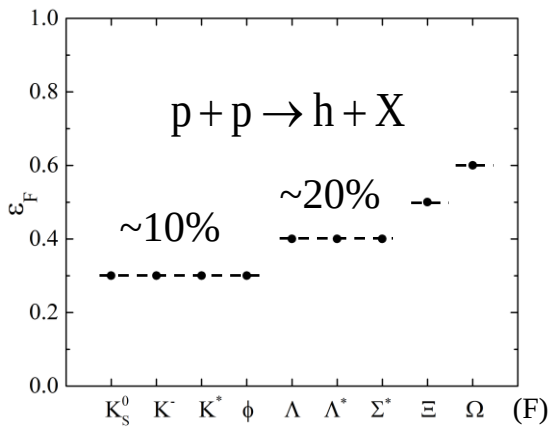
Model parameters: δ, ε_F , c for p+p

Parameters δ, ε_F , c are found from
the scaling behavior of Ψ as a function of self-similarity variable z

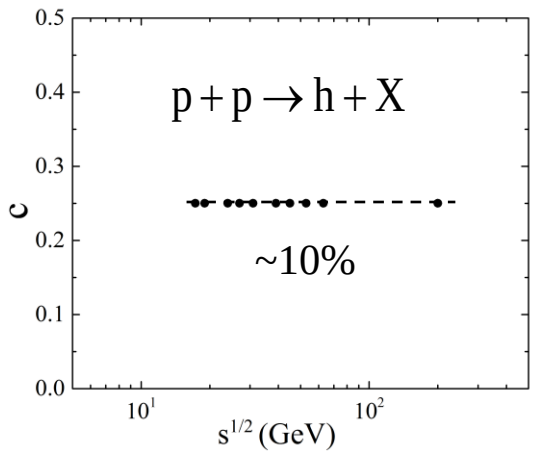
Proton fractal dimension δ



Fragmentation dimension ε_F



"Specific heat" c

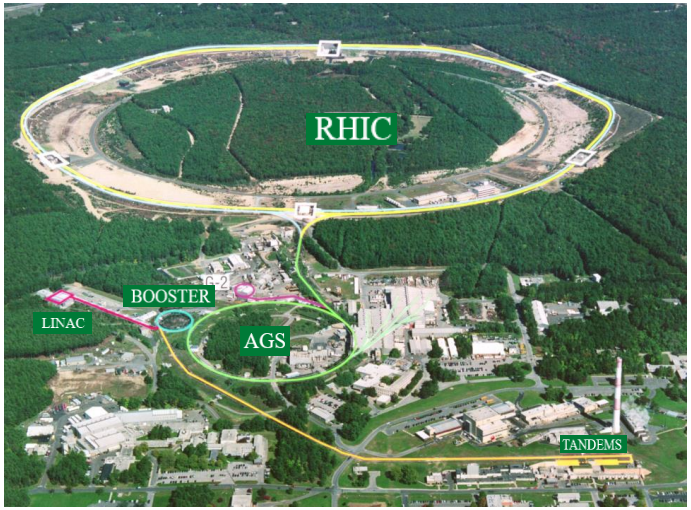


- δ, ε_F , c are independent of $\sqrt{s}$, p_T
- ε_F depends on flavor

- Self-similarity of proton sub-structure: δ = const
- Self-similarity of hadronization process: ε_F = const for F=const
- Constancy of "temperature" fluctuations: c = const

Strange particle production in Au+Au from RHIC

Au+Au @ BES-I



K^0_S, K^-, K^*, ϕ, Λ, Ξ, Ω, Σ^*, Λ^*

STAR Collaboration
J. Adam et al., Phys. Rev. C 102 (2020) 034909.
M. M. Aggarwal et al., Phys. Rev. C 83 (2011) 024901.
C. Adler et al., Phys. Lett. B 595 (2004) 143.
G. Agakishiev et al., Phys. Rev. Lett. 108 (2012) 072301.

MT & I. Zborovský
Nucl. Phys. A 1025 (2022) 122492

Self-similarity of K_S^0 production in Au+Au

Self-similarity parameter

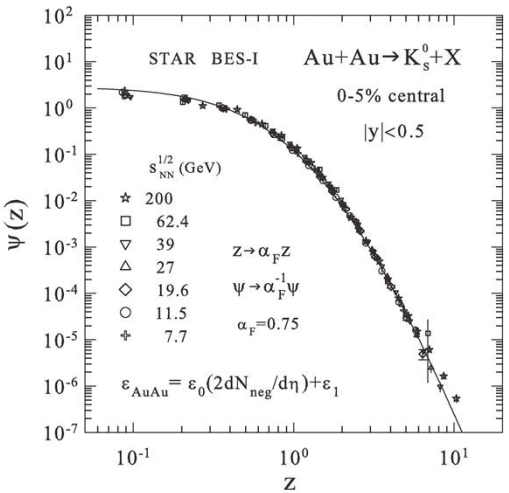
$$z = z_0 \Omega^{-1}$$

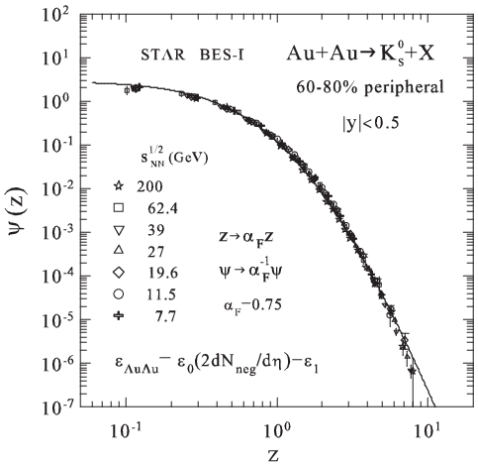
$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta\,|_0)^{c_{AA}} m_N}$$

$$\Omega = (1-x_1)^{\delta_A}(1-x_2)^{\delta_A}\,(1-y_a)^{\varepsilon_{AA}}(1-y_b)^{\varepsilon_{AA}}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c_{AA} - "specific heat" of bulk matter
- δ_A - nucleus fractal dimension
- ε_{AA} - fragmentation dimension

"Collapse" of data points onto a single curve





AA collisions:

$$\delta_A = A\delta$$

$$\varepsilon_{AA} = \varepsilon_0(2dN_{neg}^{AA}/d\eta) + \varepsilon_{pp}$$

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\,\sigma_{inel}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Dependence of ε_{AA} on multiplicity
- Power law at low- and high-z regions

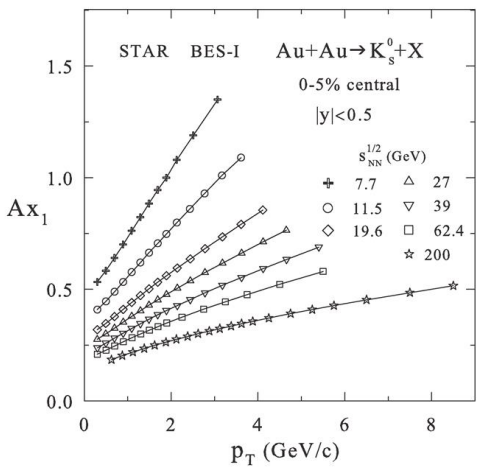
Indication of a decrease of δ for $\sqrt{s_{NN}} < 19.6$ GeV

M.T. & I.Zborovsky, Nucl. Phys. A993 (2020) 121646

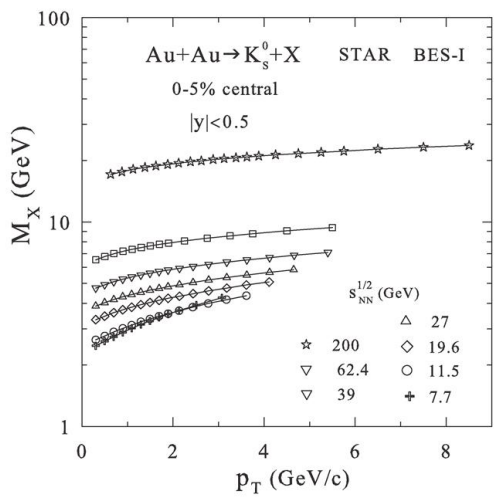
K_S^0 production in central Au+Au @ 7.7-200 GeV

Constituent sub-process in terms of

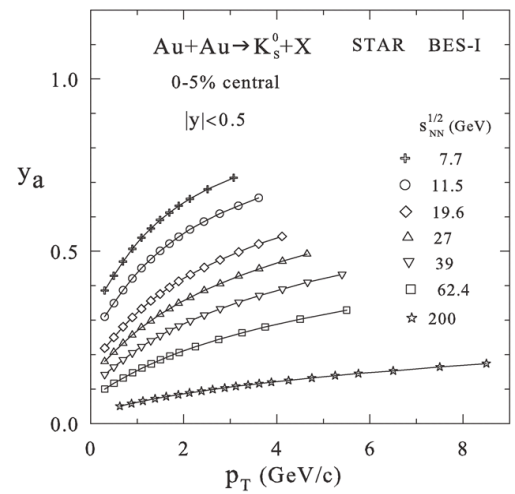
Momentum fraction Ax_1



Recoil mass M_X



Energy loss $\Delta E/E \sim (1-y_a)$



Momentum fraction
- increases with p_T
- decreases with $\sqrt{s_{NN}}$

Recoil mass
- increases with p_T
- increases with $\sqrt{s_{NN}}$

Energy loss
- decreases with p_T
- increases with $\sqrt{s_{NN}}$

Smooth behavior of x_1, y_a, M_X vs. p_T, centrality, collision energy

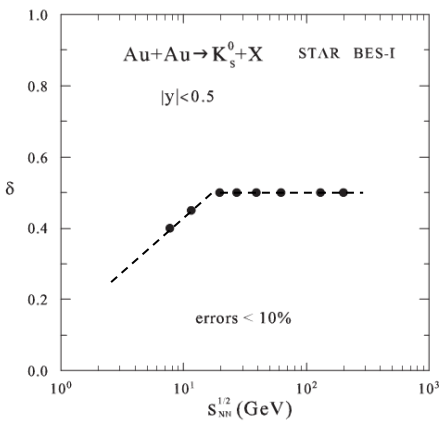
- High x_1 and p_T → compressed nuclear matter
- Large M_X → high density recoil system
- High y_a → small energy loss

Model parameters: δ_A, ε_{AA} , c_{AA} for Au+Au

Parameters δ_A, ε_{AA} , c_{AA} are found from
the scaling behavior of Ψ as a function of self-similarity variable z

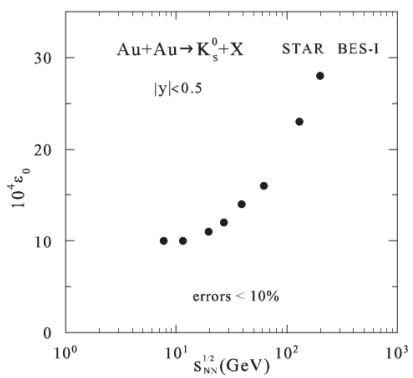
Nucleus fractal dimension

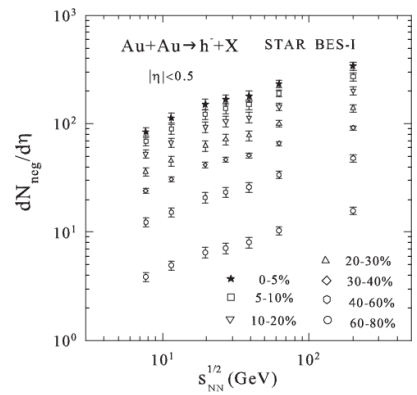
$$\delta_A = A \cdot \delta$$



Fragmentation dimension

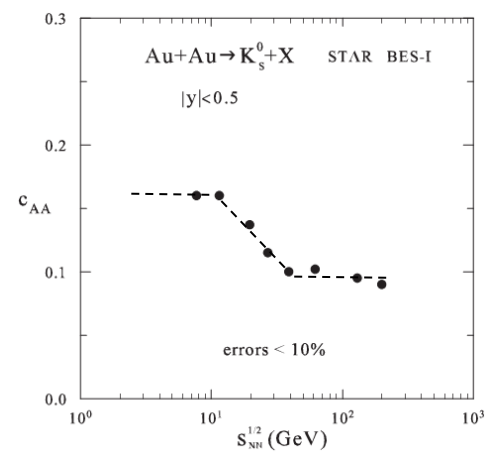
$$\varepsilon_{AA} = \varepsilon_0 (2dN_{neg}^{AA} / d\eta) + \varepsilon_{pp}$$





"Specific heat"

$$c_{AA}$$



- δ_A, ε_{AA} , c_{AA} depend on $\sqrt{s_{NN}}$
- ε_{AA} depends on flavor and multiplicity

Decrease of resolution with energy : $\delta = 0$ for point-like object.
Increase of energy loss vs. energy, multiplicity : $\varepsilon_{AA} = 0$ no energy loss.
Increase of temperature fluctuations with energy : decrease of specific heat c_{AA} .
Strange K_S^0 meson is a sensitive probe of the nuclear matter.

Signatures of Phase Transition and Critical Point

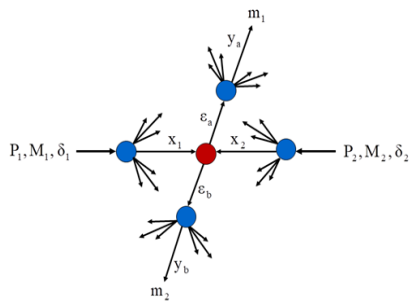
- Discontinuity or abrupt change of the model parameters: "specific heat"- c, fractal dimensions – δ, ε
- Enhancement of c-δ-ε correlations
- Energy loss is a contamination factor leading to smearing of the phase transition signatures

Fractal entropy $S_{\delta,\varepsilon}$ for systems with structural constituents

Variable z & Fractal entropy $S_{\delta,\varepsilon}$

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$



$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$

Relative number of such constituent configurations which contain the configuration $\{x_1, x_2, y_a, y_b\}$

$$W = (dN_{ch}/d\eta|_0)^c \cdot \Omega$$

Statistical entropy

$$S = \ln W$$

Thermodynamical entropy for ideal gas

$$S = c_V \cdot \ln(T) + R \cdot \ln(V) + S_0$$

Fractal entropy

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln(V_{\delta,\varepsilon}) + S_0$$

Entropy for systems with structural constituents

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + S_0$$

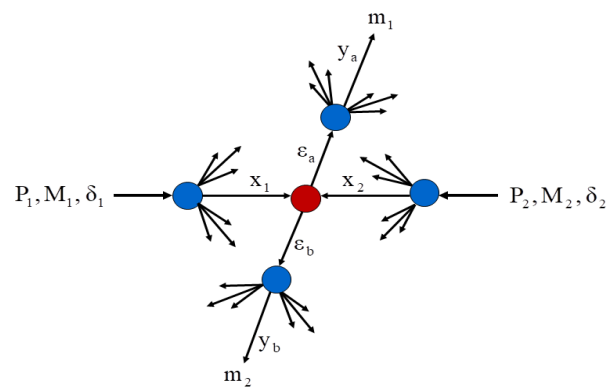
- $dN_{ch}/d\eta|_0$ characterizes "temperature" of the colliding system.
- Provided local equilibrium, $dN_{ch}/d\eta|_0 \sim T^3$ for high temperatures and small μ.
- c has meaning of a "specific heat" of the produced medium.
- Fractional exponents $\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$ are fractal dimensions in the space of $\{x_1, x_2, y_a, y_b\}$.
- Entropy increases with $dN_{ch}/d\eta|_0$ and decreases with increasing resolution Ω^{-1}.

Fractal entropy $S_{\delta,\varepsilon}$ and momentum fractions x_1, x_2, y_a, y_b

Principle of maximal entropy: The momentum fractions x_1, x_2 and y_a, y_b are determined in a way to maximize the entropy $S_{\delta,\varepsilon}$ with respect to all constituent sub-processes taking into account 4-momentum conservation law.

$$S_{\delta,\varepsilon} = c\cdot\ln\left(dN_{ch}/d\eta\big|_0\right) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + S_0$$

$$\begin{cases} \partial S_{\delta,\varepsilon}/\partial x_1\big|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial S_{\delta,\varepsilon}/\partial x_2\big|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial S_{\delta,\varepsilon}/\partial y_b\big|_{y_a=y_a(x_1,x_2,y_b)} = 0 \end{cases}$$



Momentum conservation law

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

Mass of the recoil system

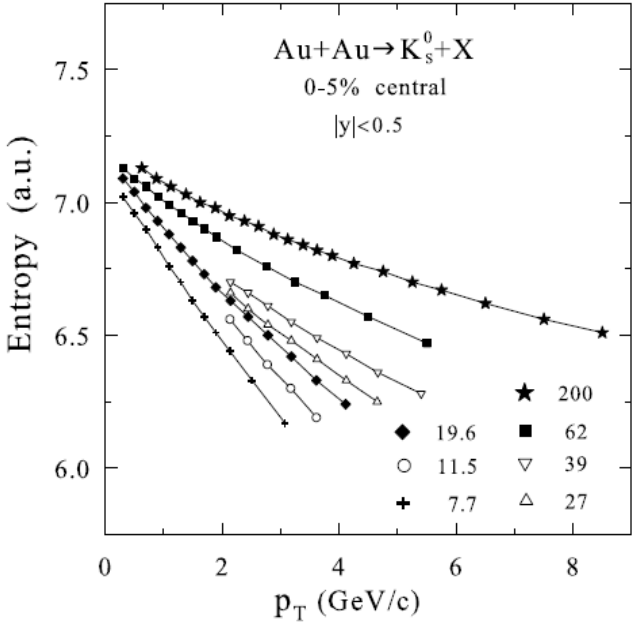
$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

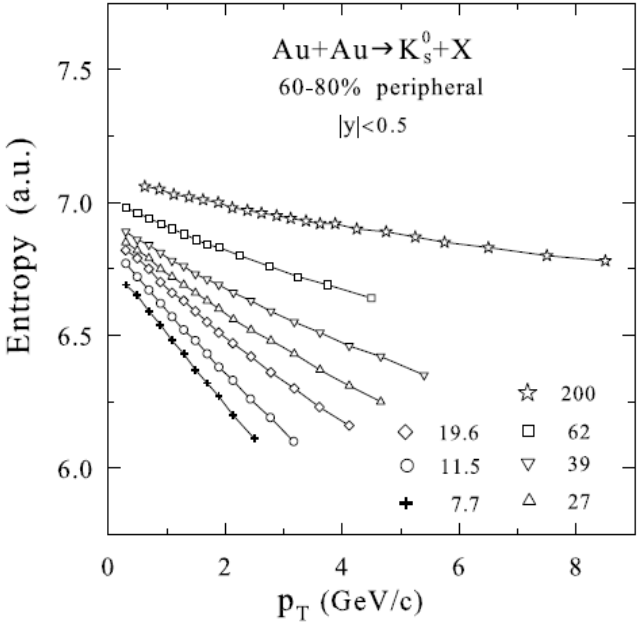
Fractions x_1, x_2, y_a, y_b are expressed via Lorentz invariants – scalar products of 4-D momenta and particle masses.

Maximal entropy $S_{\delta,\varepsilon}$ $\Leftrightarrow$ minimal resolution Ω^{-1} of the fractal measure z.

Fractal entropy $S_{\delta,\varepsilon}$ vs. $\sqrt{s_{NN}}$, centrality, p_T

$$S_{\delta,\varepsilon} = c \cdot \ln\left(dN_{ch}/d\eta\big|_0\right) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + S_0$$





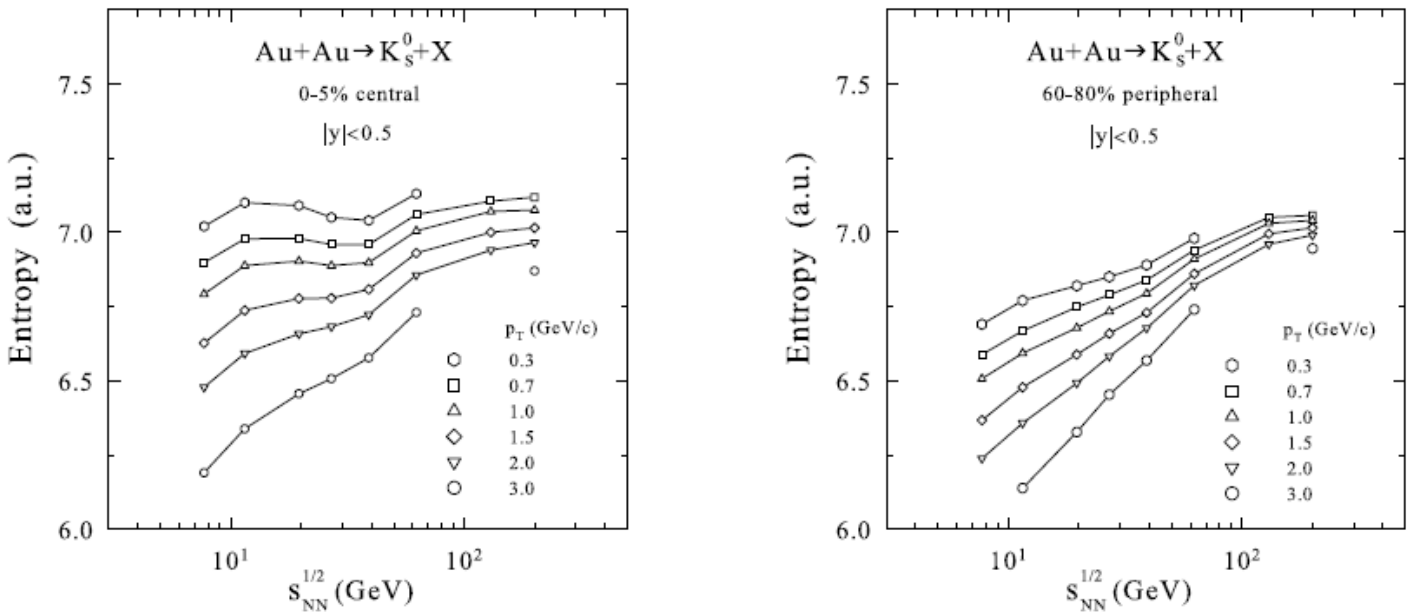
Fractal entropy $S_{\delta,\varepsilon}$

- is a smooth function of collision energy and p_T
- decreases as p_T increases
- increases with collision energy
- in central collisions is larger than in peripheral ones at low $p_T < 1$ GeV/c
- increases with multiplicity density $dN_{ch}/d\eta|_0$
- decreases with increasing resolution Ω^{-1}.

Fractal entropy $S_{\delta,\varepsilon}$ vs. $\sqrt{s_{NN}}$, centrality, p_T

Anomaly of $S_{\delta,\varepsilon}$ in the region $\sqrt{s_{NN}}$ = 11.5–39 GeV at low p_T

$$S_{\delta,\varepsilon} = c \cdot \ln\left(dN_{ch}/d\eta\big|_0\right) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + S_0$$



- The entropy reaches a local maximum at the energy $\sqrt{s_{NN}}$=11.5 -19.6 GeV and p_T=0.3 GeV/c.
- Decrease of $S_{\delta,\varepsilon}$ is seen at $\sqrt{s_{NN}}$=27 -39 GeV with a gradual increase at higher energies.
- Anomalous behavior of $S_{\delta,\varepsilon}$ is also visible at p_T=0.7 and 1.0 GeV/c in the same energy range.
- Monotonic growth of $S_{\delta,\varepsilon}$ is observed for all p_T in the peripheral collisions for all $\sqrt{s_{NN}}$.

Summary

- STAR BES-I data on transverse momentum spectra of K_S^0 mesons produced in Au+Au collisions at RHIC in mid-rapidity region were analyzed in the z-scaling approach.
- Self-similarity of strange K_S^0 - meson production in Au+Au collisions over a wide kinematic and centrality range was found.
- Constituent energy loss as a function of collision energy, centrality, and transverse momentum of K_S^0 meson was estimated.
- Model parameters - fractal dimensions and "specific heat", were found.
- Universality of Ψ vs. z and smooth behavior of x_1 , y_a , M_X vs. p_T , centrality, and collision energy were observed.
- The energy decrease of c_{AA} in the range $\sqrt{s_{NN}}$ = 11-39 GeV was found.
- Anomaly of fractal entropy $S_{\delta,\varepsilon}$ in the range $\sqrt{s_{NN}}$= 27 -39 GeV at low p_T as a possible signature of phase transition was considered.

Thank You for Your Attention !