Diffusion treatment

We will assume an isometric Maxwellian velocity distribution for the dark matter particles and a locally uniform dark matter density. Evolution of the stellar cluster of total energy \(E_{\text{star}}\) can be described following representation of collision integral in Fokker-Planck equation through diffusion coefficient as the following:

\[
D_{\text{coll}} \frac{d}{dt} \frac{\rho_{\text{DM}}}{M} - \frac{\sigma}{\rho_{\text{DM}}} v = \sum_{i} \left( \frac{\sigma}{\rho_{\text{DM}}} v - \sigma_{\text{DM}}(M_i)v \right). \tag{5}
\]

Here \(D_{\text{coll}}\) and \(D_{\text{DM}}\) are the diffusion coefficients of the squared stellar velocity and its radial component, \(\sigma_{\text{DM}}(M_i)\) is the formula for the mass respectively of star and MACHO (the objects on which the stars scatters as on wheels), \(\alpha\) is the density of MACHO, \(\sigma\) is their velocity dispersion, \(\beta\) is the Luminosity function. Function \(f(G)\) under assumed conditions is

\[
g_c(x) = \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{1-x^2}} \tag{6}
\]

with \(c\) being here star velocity, \(x = \frac{v}{\sqrt{\beta G M}}\), and \(\rho\) is the density of PBH clusters in the galaxy which is the total (averaged) PBH density in the galaxy. The potential energy per unit mass for Einsteins II's cluster is given by

\[
\rho_{\text{Einsteins II}} = \sqrt{\frac{\sigma}{\rho_{\text{DM}}}} v + \frac{\sigma_{\text{DM}}(M_i)}{\rho_{\text{DM}}} v, \tag{7}
\]

where \(\alpha = 0.36\) and \(\beta = 7.2\). Now replace \(\sigma_{\text{DM}}\) with \(\sigma_{\text{DM}}(M_i)\). Using usual theorem

\[
E = \frac{1}{\rho_{\text{DM}}} v. \tag{8}
\]

And equation (5) we obtain the equation for \(\rho_{\text{Einsteins II}}\). Now we differentiate equation (6) and divide both sides by \(\rho_{\text{Einsteins II}}\)

\[
E = \frac{\sigma}{\rho_{\text{DM}}} v + \frac{\sigma_{\text{DM}}(M_i)}{\rho_{\text{DM}}} v, \tag{9}
\]

\(E\) connected with diffusion coefficients in this way:

\[
E = \rho_{\text{Einsteins II}} \left( \frac{\sigma}{\rho_{\text{DM}}} v + \frac{\sigma_{\text{DM}}(M_i)}{\rho_{\text{DM}}} v \right). \tag{10}
\]

Let's look at cooling effect described by first-order diffusion coefficient:

\[
s_{\text{cool}} = \frac{4 \alpha v^2}{\rho_{\text{DM}}^2} \left( \frac{\sigma}{\rho_{\text{DM}}} v + \frac{\sigma_{\text{DM}}(M_i)}{\rho_{\text{DM}}} v \right), \tag{11}
\]

and

\[
s_{\text{cool}} = \frac{4 \alpha v^2}{\rho_{\text{DM}}^2} \left( \frac{\sigma}{\rho_{\text{DM}}} v + \frac{\sigma_{\text{DM}}(M_i)}{\rho_{\text{DM}}} v \right), \tag{12}
\]

where \(M_i\) is given by (13). It's obvious this is also negligible since \(M_{\text{cl}} < M_i\). Then replace the left side with (10). Since heating is much stronger than cooling we get

\[
\frac{\sigma}{\rho_{\text{DM}}} v + \frac{\sigma_{\text{DM}}(M_i)}{\rho_{\text{DM}}} v = 0. \tag{13}
\]

Conclusion

We have found that PBH cluster predominantly consisting of light PBHs (power law distribution is considered here) may allow circumventing constraints on PBH mass equal to the PBH cluster mass. We have achieved the weakening of constraints on MACHO dark matter, what is accumulated for (finite size) PBH cluster in this case. Dynamic friction of star on PBH plays significant role.

The next step is to find how long clusters of PBHs could live before complete disruption. In complete analysis of PBH and stellar clusters interaction, there should be the following effects to be taken into account:

1. Scattering of stars on PBH cluster as a whole (as on point-like one)
2. Time-dependent effects on PBH cluster density in the galaxy which is the total (averaged) PBH density in the galaxy.
3. Dynamic friction of stars inside PBH cluster.

Then third effect alse was taken into account and compared with first one [7]. That is we compared maximal and minimal limiting cases in sense of constraining PBH abundance. The work on complete analysis is in progress.

References