

$F(R)$ inflationary models with $R^{3/2}$ -term

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based on

V.R. Ivanov, S.V. Ketov, E.O. Pozdeeva, S.Yu. Vernov,
JCAP **2203** (2022) 058 [[arXiv:2111.09058](https://arxiv.org/abs/2111.09058)]
E.O. Pozdeeva, S.Yu. Vernov, [arXiv:2211.10988](https://arxiv.org/abs/2211.10988)

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$F(R)$ gravity models

The generic $F(R)$ gravity theories have the following action

$$S_F[g_{\mu\nu}] = \int d^4x \sqrt{-g} F(R), \quad (1)$$

with a differentiable function F .

The $F(R)$ gravity action can be rewritten as

$$S_J[g_{\mu\nu}, \sigma] = \int d^4x \sqrt{-g^J} [F_{,\sigma}(R - \sigma) + F], \quad (2)$$

where the new scalar field σ has been introduced, and $F_{,\sigma}(\sigma) = \frac{dF(\sigma)}{d\sigma}$. To avoid graviton as a ghost and scalaron (inflaton) as a tachyon one should put the following conditions:

$$F_{,\sigma} > 0 \quad \text{and} \quad F_{,\sigma\sigma} > 0, \quad (3)$$

that restrict possible values of parameters and R .

After the Weyl transformation of the metric $g_{\mu\nu}^E = \frac{2F_{,\sigma}(\sigma)}{M_{Pl}^2} g_{\mu\nu}^J$ one gets the following action in the Einstein frame:

$$S_E[g_{\mu\nu}, \sigma] = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R_E - \frac{h(\sigma)}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V \right], \quad (4)$$

where we have introduced the functions

$$h(\sigma) = \frac{3M_{Pl}^2}{2F_{,\sigma}^2} F_{,\sigma\sigma}^2 \quad \text{and} \quad V(\sigma) = M_{Pl}^4 \frac{F_{,\sigma}\sigma - F}{4F_{,\sigma}^2}. \quad (5)$$

Introducing the canonical scalar field ϕ instead of σ as

$$\phi = \sqrt{\frac{3}{2}} M_{Pl} \ln \left[\frac{2}{M_{Pl}^2} F_{,\sigma} \right] \quad (6)$$

allows one to rewrite the action S_E to the standard form:

$$S_E[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R_E - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \quad (7)$$

The Starobinsky R^2 inflationary model

Starobinsky model of inflation, whose action is given by

$$S_{\text{Star.}}[g_{\mu\nu}] = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g} \left(R + \frac{1}{6m^2} R^2 \right) , \quad (8)$$

includes the inflaton mass m .

A.A. Starobinsky, *Phys. Lett. B* **91** (1980) 99.

The action (8) is dual to the quintessence (or scalar-tensor gravity) action

$$S_{\text{quint.}}[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_{\text{Star.}}(\phi) \right] \quad (9)$$

in terms of the canonical scalar ϕ .

The induced scalar potential is given by

$$V_{\text{Star.}}(\phi) = \frac{3}{4} M_{Pl}^2 m^2 \left[1 - \exp \left(-\sqrt{\frac{2}{3}} \frac{\phi}{M_{Pl}} \right) \right]^2 . \quad (10)$$

The main cosmological parameters of inflation are given by the scalar tilt n_s and the tensor-to-scalar ratio r , whose values are constrained by the combined Planck, WMAP and BICEP/Keck observations of CMB as

$$n_s = 0.9649 \pm 0.0042 \quad (68\% \text{CL}) \quad \text{and} \quad r < 0.036 \quad (95\% \text{CL}).$$

BICEP, Keck Collaboration, P. A. R. Ade *et al.*, *Phys. Rev. Lett.* **127** (2021) 151301, arXiv:2110.00483 [astro-ph.CO]

The Starobinsky model has a good agreement to the observation data.

ϕ_i/M_{Pl}	5.2262	5.4971
n_s	0.961	0.969
r	0.0043	0.0027
N_e	49.258	62.335

The values of the inflationary parameters are sensitive to the duration of inflation and the initial value of the inflaton field, ϕ_i .

The only free parameter m , is fixed by the observable (CMB) value of the amplitude of scalar perturbations A_s :

$$m = 1.3 \left(\frac{55}{N_e} \right) 10^{-5} M_{Pl} = 3.2 \left(\frac{55}{N_e} \right) 10^{13} \text{GeV}, \quad (11)$$

where N is the number of e-foldings describing the duration of inflation.

In the spatially flat FLRW universe with the metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2),$$

the action (7) leads to the standard system of evolution equations:

$$6M_{Pl}^2 H^2 = \dot{\phi}^2 + 2V, \quad (12)$$

$$2M_{Pl}^2 \dot{H} = -\dot{\phi}^2, \quad (13)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \quad (14)$$

where $H = \dot{a}/a$ is the Hubble parameter, $a(t)$ is the scale factor, and the dots denote the derivatives with respect to the cosmic time t .

During inflation the e-foldings number $N_e = \ln\left(\frac{a_{\text{end}}}{a}\right)$, where a_{end} is the value of a at the end of inflation, is considered instead of the time variable.

Being motivated by the potential (10), we find useful to introduce the non-canonical dimensionless field

$$y \equiv \exp \left(-\sqrt{\frac{2}{3}} \frac{\phi}{M_{Pl}} \right) = \frac{M_{Pl}^2}{2F_{,\sigma}} > 0 \quad (15)$$

because it is (physically) *small* during slow-roll inflation.

In the Einstein frame, the slow-roll parameters are

$$\epsilon = \frac{M_{Pl}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 = \frac{y^2}{3} \left(\frac{V_{E,y}}{V_E} \right)^2 ,$$

$$\eta = M_{Pl}^2 \left(\frac{V_{,\phi\phi}}{V} \right) = \frac{2y}{3V_E} (V_{E,y} + yV_{E,yy}) .$$

The scalar spectral index n_s and the tensor-to-scalar ratio r are given by

$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon . \quad (16)$$

In the slow-roll approximation, the function $\phi(N_e)$ can be found as a solution of

$$\phi' \simeq \frac{M_{Pl}^2}{V} V_{,\phi} \quad \Leftrightarrow \quad y' = \frac{2y^2 V_{E,y}}{3V_E} . \quad (17)$$

when demanding that $\epsilon = 1$ corresponds to the end of inflation with $a = a_{\text{end}}$.

Main properties of the Starobinsky inflationary model

- ① The model includes only one parameter m , that is defined by A_s . Parameters n_s and r do not depend on m .
- ② The potential $V_{Star.}(\phi)$ is a monotonically increasing function at $0 < \phi < +\infty$.

$$V_{Star.}(\phi) = V_0(1 - y)^2 = V_0 \left[1 - \exp \left(-\sqrt{\frac{2}{3}} \frac{\phi}{M_{Pl}} \right) \right]^2,$$

where $V_0 = \frac{3}{4} m^2 M_{Pl}^2$.

- ③ Model is well-defined at $R > -3m^2$.

Generalizations of the Starobinsky model

There are 3 main ways of the Starobinsky model generalization:

- ① add a scalar field. R^2 -Higgs model

Y. Ema, Phys. Lett. B **770** (2017) 403

M. He, A.A. Starobinsky and J. Yokoyama, JCAP **1805** (2018) 064

F. Bezrukov, D. Gorbunov, C. Shepherd and A. Tokareva, Phys. Lett. B **795** (2019) 657

- ② add superstring-inspired corrections

S. V. Ketov and A. A. Starobinsky, Phys. Rev. D **83** (2011) 063512

A. S. Koshelev, L. Modesto, L. Rachwal and A. A. Starobinsky, JHEP **11** (2016) 067

S. V. Ketov, E. O. Pozdeeva and S. Yu. Vernov, arXiv:2211.01546

- ③ modify $F(R)$ function

J.D. Barrow and S. Cotsakis, Phys. Lett. B **214** (1988) 515

L. Sebastiani, G. Cognola, R. Myrzakulov, S. D. Odintsov and S. Zerbini, Phys. Rev. D **89** (2014) 023518

T. Miranda, J. C. Fabris and O. F. Piattella, JCAP **09** (2017) 041

The $(R + R^{3/2} + R^2)$ gravity model of inflation

Let

$$F(R) = \frac{M_{Pl}^2}{2} \left[R + \frac{1}{6m^2} R^2 + \frac{\delta}{m} R^{3/2} \right], \quad (18)$$

where we have introduced the dimensionless parameter δ . The condition $\delta > 0$ is necessary to get a stable $F(R)$ gravity model for all $R > 0$.

The $R^{3/2}$ term appears in the (chiral) modified supergravity

S.V. Ketov and A.A. Starobinsky, Phys. Rev. D 83 (2011) 063512
[1011.0240].

S.V. Ketov and S. Tsujikawa, Phys. Rev. D 86 (2012) 023529
[1205.2918].

The $R^{3/2}$ -term in $F(R)$ gravity arises in an approximate description of the Higgs field with a small cubic term in its scalar potential and a large non-minimal coupling to R

J.S. Martins, O.F. Piattella, I.L. Shapiro and A.A. Starobinsky,
2010.14639.

The corresponding scalar potential (5) is given by

$$V = \frac{4V_0\tilde{\sigma}(3\delta\sqrt{\tilde{\sigma}} + \tilde{\sigma})}{(6 + 9\delta\sqrt{\tilde{\sigma}} + 2\tilde{\sigma})^2} . \quad (19)$$

Equation (15) in this case is a quadratic equation on $\sqrt{\tilde{\sigma}}$, and its only real solution is

$$\tilde{\sigma} = \frac{3(1-y)}{y} + \frac{9\delta}{8y} \left[9\delta y - \sqrt{3y(27\delta^2y - 16y + 16)} \right] . \quad (20)$$

The potential can be rewritten as

$$V_E(y) = \frac{V_0}{2304y^2} (s + 3\delta y)(s - 9\delta y)^3 ,$$

where $s = \sqrt{3y(27\delta^2y - 16y + 16)}$.

When $\delta = 4\sqrt{3}/9$, the $F_{,\sigma}$ function is a perfect square, and the potential simplifies as

$$V_{\text{special}}(y) = \frac{V_0}{3} (3 + \sqrt{y})(1 - \sqrt{y})^3 , \quad (21)$$

or

$$V_{\text{special}}(\phi) = \frac{V_0}{3} \left(e^{\phi/(\sqrt{6}M_{Pl})} - 1 \right)^3 \left(1 + 3e^{\phi/(\sqrt{6}M_{Pl})} \right) e^{-2\sqrt{2/3}\phi/M_{Pl}} .$$

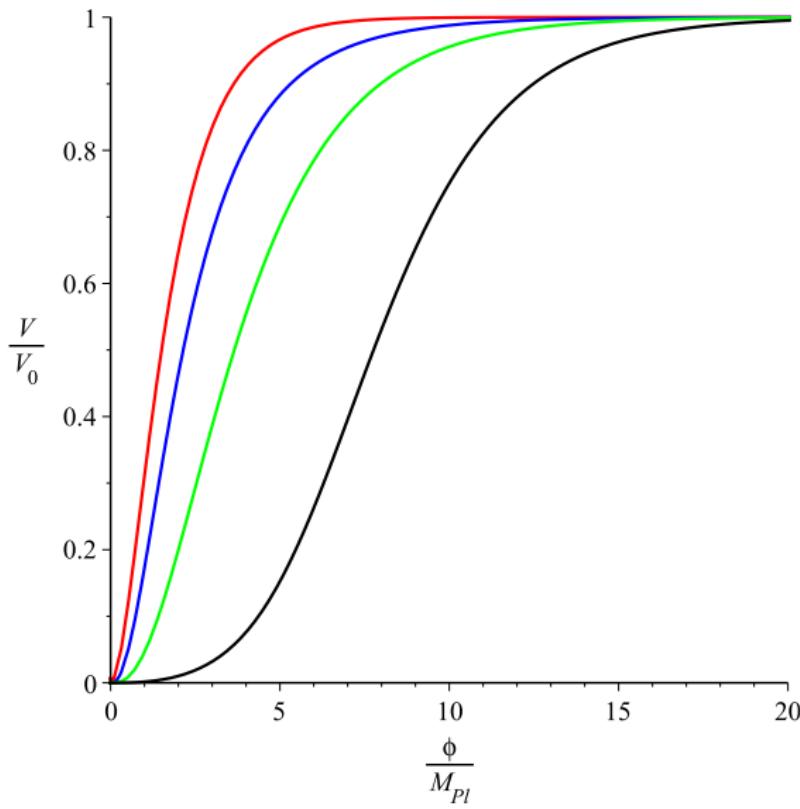


Figure: The potential $V(\phi)$ for $\delta = 0$ (red), $\delta = 1/5$ (blue), $\delta = 4\sqrt{3}/9$ (green), and $\delta = 5$ (black).

The inflationary parameters

The inflationary parameters are given by

$$n_s = 1 + \frac{8y (3s(3\delta(9\delta^2 - 16)s + 720\delta^2 - 256)y^2 - s^3\delta(39\delta y + s))}{(-9\delta y + s)^2 (3\delta y + s)^2 s} - \frac{8y [72\delta(4 - 9\delta^2)(27\delta^2 - 16)y^4 + [(768 - 1215\delta^4 - 432\delta^2)s - 144\delta(45\delta^2 - 16)]y^3]}{(-9\delta y + s)^2 (3\delta y + s)^2 s} \quad (22)$$

and

$$r = \frac{768 y^2 (-9\delta^2 y + s\delta + 8y)^2}{(-9\delta y + s)^2 (3\delta y + s)^2}. \quad (23)$$

The amplitude of scalar perturbations is given by

$$A_s = \frac{(-9\delta y + s)^5 (3\delta y + s)^3}{3538944y^4\pi^2 (-9\delta^2 y + s\delta + 8y)^2 m^2}. \quad (24)$$

The observed value of A_s determines the value of the parameter m .

The slow-roll evolution equation allows us to relate N_e with y at the end of inflation,

$$N_e = \left(\frac{9}{8} - \delta^{-2} \right) \ln [9\delta^3 (9\delta y - s) + 24(1 - 4y)\delta^2 + 8(\delta s + 4y)] \\ + \left(\delta^{-2} - \frac{3}{8} \right) \ln y + \frac{s}{4\delta y} - N_0 ,$$

where the integration constant N_0 is fixed by the condition $N_e(y_{end}) = 0$. The analytic formula for $N_0(\delta)$ is obtained by substituting $N_e = 0$ and $y = y_{end}$.

The condition $\epsilon = 1$ gives

$$y_{end} = \frac{3(4 - 3\delta^2 + \sqrt{3}\delta^2) - \sqrt{9(4 - 3\delta^2 + \sqrt{3}\delta^2)^2 - 72(2 - 3\delta^2)}}{2(2 - 3\delta^2)(3 + 2\sqrt{3})} .$$

It is worth noticing that this solution has no singularity at $\delta = \sqrt{2/3}$, while $y_{end}(\delta)$ is a smooth monotonically decreasing function.

Table: The values of y , N_e and r corresponding to $n_s = 0.961$ and $n_s = 0.969$, respectively, and the values of y_{end} for some values of the parameter δ .

δ	y_{end}	$y_{in, n_s=0.961}$	$y_{in, n_s=0.969}$	$N_{e, 0.961}$	$N_{e, 0.969}$	$r_{n_s=0.961}$	$r_{n_s=0.969}$
0	0.464	0.0140	0.0112	49.3	62.3	0.0043	0.0027
0.2	0.395	0.00682	0.00505	45.0	56.8	0.0096	0.0065
$\frac{4\sqrt{3}}{9}$	0.299	0.00146	0.000968	48.1	60.9	0.0152	0.0099
1	0.279	0.000939	0.000616	49.4	62.4	0.0157	0.0102
5	0.205	$4.32 \cdot 10^{-5}$	$2.75 \cdot 10^{-5}$	56.3	69.7	0.0168	0.0108
10	0.199	$1.08 \cdot 10^{-5}$	$6.91 \cdot 10^{-6}$	58.7	72.0	0.0168	0.0108
25	0.197	$1.74 \cdot 10^{-6}$	$1.11 \cdot 10^{-6}$	61.4	74.8	0.0169	0.0108
50	0.196	$4.34 \cdot 10^{-7}$	$2.77 \cdot 10^{-7}$	63.5	76.9	0.0169	0.0108
100	0.196	$1.09 \cdot 10^{-7}$	$6.92 \cdot 10^{-8}$	65.5	79.1	0.0169	0.0108

Inflationary models with $(R + R_0)^{3/2}$ term

Let us consider the following modification of the action (18):

$$F = \frac{M_{Pl}^2}{2} \left[\left(1 - \frac{3}{2}\beta\delta\right) R + \frac{R^2}{6m^2} + \frac{\delta}{m} \left(R + \beta^2 m^2\right)^{3/2} - m^2 \beta^3 \delta \right]. \quad (25)$$

This model includes two dimensionless parameters δ and β . To construct new one-parametric generalizations of the Starobinsky model, we plan to connect these parameters.

The first derivative

$$F^{(1)} = \frac{M_{Pl}^2}{4} (2 - 3\delta\beta) + \frac{M_{Pl}^2}{6m^2} R + \frac{3M_{Pl}^2\delta}{4m} \sqrt{\beta^2 m^2 + R} > \frac{M_{Pl}^2}{2} \quad (26)$$

for any $R > 0$ and $F^{(1)}(0) = M_{Pl}^2/2$ as in the Starobinsky model. The second derivative $F^{(2)}(R) > 0$ for any $R > -\beta^2 m^2$ and $\delta \geq 0$.

So, the model is well-defined for any values of parameters $\beta \neq 0$ and $\delta > 0$ at $R > R_0$, where $R_0 < 0$.

The function $F(R)$ has a correct GR limit at $R \ll m^2$:

$$F = \frac{M_{Pl}^2}{2} R \left[1 + \frac{4\beta + 9\delta}{24\beta} \tilde{\sigma} - \frac{\delta}{16\beta^3} \tilde{\sigma}^2 + \frac{3\delta}{128\beta^5} \tilde{\sigma}^3 + \mathcal{O}(\tilde{\sigma}^4) \right],$$

where $\tilde{\sigma} = R/m^2$.

To get inflationary parameters we construct the corresponding scalar potential (5):

$$V_E(\tilde{\sigma}) = \frac{4V_0 \left(6\beta^3\delta + 3\delta\sqrt{\beta^2 + \tilde{\sigma}} (\tilde{\sigma} - 2\beta^2) + \tilde{\sigma}^2 \right)}{\left(9\delta\beta - 9\delta\sqrt{\beta^2 + \tilde{\sigma}} - 2\tilde{\sigma} - 6 \right)^2}, \quad (27)$$

To obtain $V_E(y)$, we solve Eq. (15):

$$y + \frac{6}{9\delta\beta - 9\delta\sqrt{\beta^2 + \tilde{\sigma}} - 2\tilde{\sigma} - 6} = 0, \quad (28)$$

and get

$$\tilde{\sigma} = \frac{3(1-y)}{y} + \frac{9\delta}{8y} (4\beta y + 9\delta y \pm s), \quad (29)$$

where $s = \sqrt{(72\delta\beta y + 81\delta^2 y + 16\beta^2 y - 48y + 48)y}$. To get $\sigma = 0$ at $y = 1$ for all δ and β we choose solution (29) with "-".

A new one-parametric generalization

Let us consider the case of $\beta = \sqrt{3} - \frac{9}{4}\delta$.

$$\sigma(y) = -\frac{3}{2y} \left(3\delta\sqrt{3y} - 3\sqrt{3}\delta y + 2y - 2 \right). \quad (30)$$

The corresponding potential has a simple form:

$$V_E(y) = V_0(y-1)^2 - \sqrt{3}V_0\delta\sqrt{y}(\sqrt{y}+2)(\sqrt{y}-1)^2 \quad (31)$$

that is equivalent

$$V(\phi) = V_{\text{Star.}} + \sqrt{3}V_0\delta e^{-\phi/(\sqrt{6}M_{Pl})} \left(e^{-\phi/(\sqrt{6}M_{Pl})} + 2 \right) \left(e^{-\phi/(\sqrt{6}M_{Pl})} - 1 \right)^2.$$

The potential $V_E(y)$ has an extremum at

$$y_{\text{extr}} = \frac{\sqrt{3} - 6\delta + \sqrt{3 - 12\sqrt{3}\delta + 27\delta^2}}{2(\sqrt{3} - 3\delta)}. \quad (32)$$

$0 < y_{\text{extr}} < 1$ for $\delta > 4\sqrt{3}/9$ and $y_{\text{extr}} = 1$ at $\delta = 4\sqrt{3}/9$.

So, to get a positive potential without any extremum for $0 < y < 1$ one should put the condition $\delta \leq 4\sqrt{3}/9$ that is equivalent to $\beta \geq 0$.

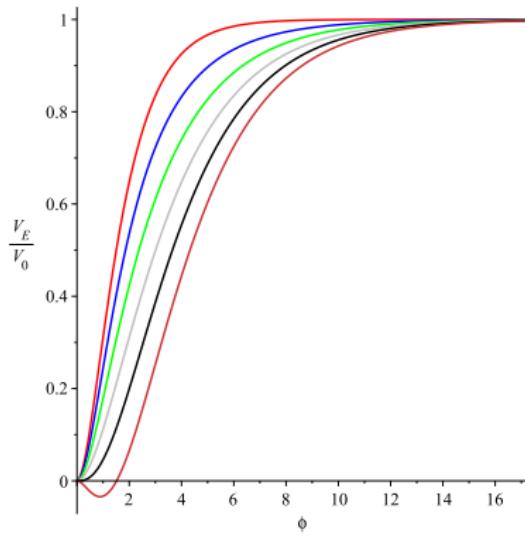
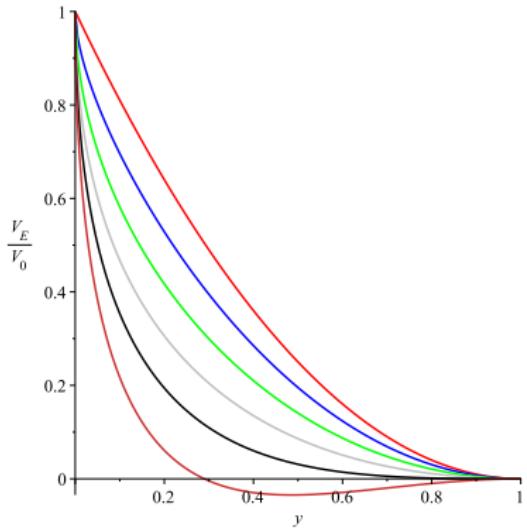


Figure: The potential $V_E(y)$ (left) and the corresponding $V(\phi)$ (right) in the case of in the case of $\beta = \sqrt{3} - \frac{9}{4}\delta$ for $\delta = 0$ (red), $\delta = \sqrt{3}/9$ (blue), $\delta = 2\sqrt{3}/9$ (green), $\delta = \sqrt{3}/3$ (grey), $\delta = 4\sqrt{3}/9$ (black), and $\delta = 1$ (orange).

The special case $\delta = 1/\sqrt{3}$

At $\delta = 1/\sqrt{3}$, that corresponds to $\beta = \sqrt{3}/4$, we obtain the potential

$$V_E(y) = V_0(1 - \sqrt{y})^2, \quad V(\phi) = V_0 \left(1 - e^{-\phi/(\sqrt{6}M_{Pl})}\right)^2 \quad (33)$$

and simple expressions for slow-roll and inflationary parameters:

$$\epsilon = \frac{y}{3(1 - \sqrt{y})^2}, \quad \eta = \frac{\sqrt{y}(2\sqrt{y} - 1)}{3(1 - \sqrt{y})^2} \quad (34)$$

$$n_s = 1 - \frac{2\sqrt{y}(\sqrt{y} + 1)}{3(\sqrt{y} - 1)^2}, \quad r = \frac{16y}{3(1 - \sqrt{y})^2}. \quad (35)$$

The conditions $\epsilon(y_{\text{end}}) = 1$ and $y_{\text{end}} < 1$ gives the following solution:

$$y_{\text{end}} = \frac{1}{4} \left(3 - \sqrt{3}\right)^2. \quad (36)$$

The slow-roll evolution equation (17),

$$y' = \frac{2y^{3/2}}{3(\sqrt{y} - 1)} \quad (37)$$

allows us to express N_e via y :

$$N_e = \frac{3}{\sqrt{y}} + \frac{3}{2} \ln(y) + N_0, \quad (38)$$

where the integration constant N_0 fixed by the condition $N_e(y_{\text{end}}) = 0$:

$$N_0 = -\frac{3}{\sqrt{y_{\text{end}}}} - \frac{3}{2} \ln(y_{\text{end}}). \quad (39)$$

Using Eq. (35) we calculate the value of y_{in} for suitable values of n_s , namely for $n_s = 0.961$, $n_s = 0.965$, and $n_s = 0.969$. It allows us to calculate the corresponding values of r and number of e-folding during inflation. The same procedure can be made for an arbitrary δ . The results of calculations for different values of δ are presented in Table 22. One can see that the tensor-to-scalar ratio $r(\delta)$ is an increasing function and $r(1/\sqrt{3})$ is almost in four times more than in the Starobinsky model.

Table: The values of y , N_e and r corresponding to $n_s = 0.961$, $n_s = 0.965$ and $n_s = 0.969$, respectively, and the values of y_{end} for some values of the parameter δ .

δ	0	0.1	$\frac{\sqrt{3}}{9}$	$\frac{2\sqrt{3}}{9}$	$\frac{1}{\sqrt{3}}$	$\frac{4\sqrt{3}}{9}$
y_{end}	0.464	0.460	0.455	0.439	0.402	0.299
$y_{in, n_s=0.961}$	0.0140	0.0114	0.00895	0.00479	0.00253	0.00146
$y_{in, n_s=0.965}$	0.0126	0.0101	0.00777	0.00402	0.00209	0.00120
$y_{in, n_s=0.969}$	0.0112	0.00878	0.00660	0.00329	0.00168	0.000968
$N_e, n_s=0.961$	49.3	45.2	44.0	45.3	47.4	48.1
$N_e, n_s=0.965$	55.0	50.4	49.1	50.7	53.0	53.8
$N_e, n_s=0.969$	62.3	57.0	55.6	57.6	60.1	60.9
$r_{n_s=0.961}$	0.0043	0.0074	0.010	0.014	0.015	0.015
$r_{n_s=0.965}$	0.0035	0.0061	0.0084	0.0114	0.012	0.012
$r_{n_s=0.969}$	0.0027	0.0049	0.0068	0.0091	0.0097	0.0098

CONCLUSIONS

We studied several extensions of the Starobinsky inflation model of the $(R + R^2)$ gravity in the context of $F(R)$ gravity.

- The modification of the Starobinsky model by the $R^{3/2}$ term has a significant impact on the value of the tensor-to-scalar ratio r .
- A new one-parameter generalization of the Starobinsky model has been constructed, describing by

$$F = \frac{M_{Pl}^2}{2} \left[\frac{1}{8} \left(8 - 3\delta(4\sqrt{3} - 9\delta) \right) R + \frac{R^2}{6m^2} + \frac{\delta}{m} \left[R + \left(\sqrt{3} - \frac{9}{4}\delta \right)^2 m^2 \right]^{3/2} - \Lambda \right],$$

where $\Lambda = m^2 \delta \left(\sqrt{3} - \frac{9}{4}\delta \right)^3$. For any $0 \leq \delta \leq 4\sqrt{3}/9$, this model is consistent with cosmological observations.

- At $\delta = 1/\sqrt{3}$, we obtain the most simple form of the potential

$$V(\phi) = V_0 \left(1 - e^{-\phi/(\sqrt{6}M_{Pl})} \right)^2 \quad \text{and}$$

$$F = \frac{M_{Pl}^2}{2} \left[\frac{5}{8} R + \frac{R^2}{6m^2} + \frac{\sqrt{3}}{3m} \left[R + \frac{3}{16} m^2 \right]^{3/2} - \frac{3}{64} m^2 \right]. \quad (40)$$

The tensor-to-scalar ratio r is almost in four times more than in the Starobinsky model.